THE ELEMENTS

OF

ALTERNATING CURRENTS

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BY

W. S. FRANKLIN AND R. B. WILLIAMSON

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PREFACE TO SECOND EDITION.

Two years' use of Franklin and Williamson's Elements of Alternating Currents, and many valuable suggestions from friends, make it possible to issue a greatly improved edition, which is now offered to the public. It is not expected that this text will be found easy reading, but it is certain that any one of moderate ability can gain by serious study of this text a clear understanding of the principles of alternating currents and of the theory of the various types of alternating current machinery.

Aside from the minor changes which occur on almost every page of the book, this new edition differs from the old in having a very complete series of practical problems with answers. The alterations in the chapters on the Alternator, on the Transformer, on the Synchronous Motor, on the Rotary Converter and on the Induction Motor, are especially noteworthy.

The last five chapters have been added to give the student a general description of some of the ordinary types of alternating current apparatus and the conditions under which it is operated. These chapters are intended to be read in connection with the corresponding chapters that treat on the theory of the apparatus under consideration.

The authors' thanks are due to Professor Morgan Brooks for many valuable suggestions, to Messrs. C. M. Crawford and H. W. Brown for assistance in preparing copy, in solving problems and in reading proof, and to the manufacturing companies who have kindly furnished a number of the cuts used in the chapters that have been added.

W. S. F.
R. B. W.

SOUTH BETHLEHEM,
July 30, 1901.

PREFACE TO FIRST EDITION.

THIS book represents the experience of seven years' teaching of alternating currents, and almost every chapter has been subjected repeatedly to the test of class-room use. The authors have endeavored to include in the text only those things which contribute to the fundamental understanding of the subject and those things which are of importance in the engineering practice of to-day.

It may be taken for granted that the authors are deeply indebted to Mr. C. P. Steinmetz, whose papers are unique in their close touch with engineering realities.

W. S. F.

SOUTH BETHLEHEM,
June, 1899.

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SYMBOLS.

i	instantaneous value of current.
$\boldsymbol{I}$	maximum value of an harmonic alternating current.
I	effective value of an alternating current.
e	instantaneous value of electromotive force.
$\boldsymbol{E}$	maximum value of an harmonic alternating electromotive force.
E	effective value of an alternating electromotive force.
r, R,	resistance (r sometimes used for radius).
L	inductance.
C	electrostatic capacity.
t	time.
N	turns of wire.
n	speed in revolutions per second.
f	frequency in cycles per second.
ω	frequency in radians per second.
μ	magnetic permeability.
l	length.
s	sectional area.
Φ	magnetic flux.
Φ_1	flux-turns.
H	magnetic field intensity.
j	square root of minus one, $\sqrt{-1}$.
x, X,	reactance.
z, Z,	impedance.
ϵ	base of Naperian logarithms.

THE

ELEMENTS OF ALTERNATING CURRENTS.

CHAPTER I.

INTRODUCTION.

INDUCTANCE AND CAPACITY.

1. **Magnetic flux.**—Let s be the area of a surface at right angles to the velocity of a moving fluid and let v be the velocity of the fluid. Then sv is the flux of fluid across the area in units volume per second. Similarly, the product of the intensity, H, of a magnetic field into an area, s, at right angles to H is called the *magnetic flux* across the area. That is

$$\Phi = Hs \tag{i}$$

in which Φ is the magnetic flux across an area s, which is at right angles to a magnetic field of intensity H.

The unit of magnetic flux is the flux across one square centimeter of area at right angles to a magnetic field of unit intensity. This unit flux is called a *line of force** or simply a *line*. For example, the intensity of the magnetic field in the air gap between

* A line of force is a line drawn in a magnetic field so as to be in the direction of the field at each point. The term *line of force* is used for the unit flux for the following reason: Consider a magnetic field. Imagine a surface drawn across this field. Suppose this surface to be divided into *parts* across each of which there is a unit flux. Imagine lines of force drawn in the magnetic field so that one line of force passes through each of the *parts* of our surface. Then the magnetic flux across any area anywhere in the field will be equal to the *number of these lines* which cross the area.

the pole face of a dynamo and the armature core is, say, 5,000 units, and this field is normal to the pole face of which the area

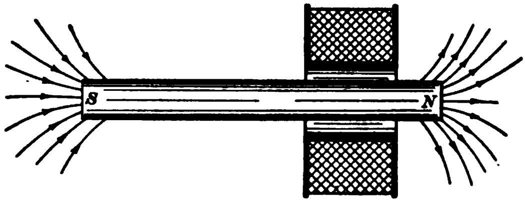


Fig. 1.

is 300 square centimeters, so that 1,500,000 lines of magnetic flux pass from the pole face into the armature core.

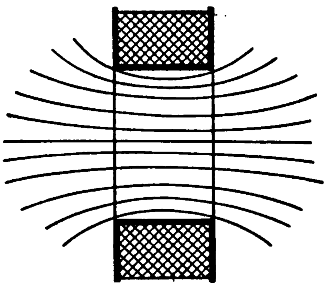


Fig. 2.

The trend of the lines of force near the poles of a magnet is shown in Fig. 1. In Fig. 2 is shown the trend of the lines of force through a coil of wire in which an electric current is flowing.

2. Induced electromotive force.—When a bundle of N wires connected in series moves across a magnetic field so as to cut the lines of force, in each wire an electromotive force is induced which is equal to the rate, $d\Phi/dt$, at which lines of force are cut, and the total electromotive force induced in the bundle of wires is

$$e = -N\frac{d\Phi}{dt} \tag{ii}$$

Similarly, when the magnetic flux through a coil changes, an electromotive force is induced in the coil, such that

$$e = -N\frac{d\Phi}{dt} \tag*{(ii) \textit{bis}}$$

in which N is the number of turns of wire in the coil, $d\Phi/dt$ is the rate of change of flux, and e is the induced electromotive force. The negative sign is chosen for the reason that an increasing

positive flux produces a left-handed electromotive force in the coil.*

Examples.—(*a*) A conductor on a dynamo armature cuts the 1,500,000 lines of force from one pole face in, say, $\frac{1}{50}$ second, that is, at the rate of 75,000,000 lines per second; and this is the electromotive force (in c.g.s. units) induced in the conductor.

(*b*) A coil, having N turns of wire surrounds a magnet NS, through which there are Φ lines of flux, as shown in Fig. 1. The coil is quickly removed from the magnet, reversed, and replaced, the whole operation being accomplished in t seconds. The flux Φ, being reversed with respect to the coil, is to be considered as changing from $+\Phi$ to $-\Phi$, the total change being therefore 2Φ. Dividing this total change of flux by the time t gives $2\Phi/t$, which is the average value of $d\Phi/dt$ during the time t, so that the average electromotive force induced in the coil is $N \cdot 2\Phi/t$. This electromotive force is expressed in c.g.s. units, and is to be divided by 10^8 to reduce it to volts.

3. The magnetic field as a seat of kinetic energy.—The magnetic field is a kind of obscure motion of an all-pervading medium, the ether; and this motion represents energy. The amount of energy in a given portion of a magnetic field is proportional to the square of the intensity of the field. This is analogous to the fact that the kinetic energy of a portion of a moving liquid is proportional to the square of the velocity of the liquid.

4. Kinetic energy of the electric current in a coil. *Definition of inductance.*—The kinetic energy of an electric current is the energy which resides in the magnetic field produced by the current. The kinetic energy is, at each point, proportional to the square of the field intensity, that is, to the square of the current. Therefore the total kinetic energy of the field is proportional to the square of the current. That is

$$W = \tfrac{1}{2}Li^2 \qquad (1)$$

* This, although an inadequate statement, must suffice; especially inasmuch as the sign in equation (ii) is of no practical importance.

in which W is the total energy of a current i in a given coil, and $(\frac{1}{2}L)$ is the proportionality factor. The quantity L is called the *inductance* of the given coil.

Units of inductance.—When in equation (1) W is expressed in joules and i in amperes, then L is expressed in terms of a unit called the *henry*. When W is expressed in ergs and i in c.g.s. units of current, then L is expressed in c.g.s. units of inductance. The c.g.s. unit of inductance is called the *centimeter*, for the reason that the square of a current must be multiplied by a length to give energy or work; that is, inductance is expressed as a length and the unit of inductance is, of course, the unit of length. The henry is equal to 10^9 centimeters of inductance.

Example.—A given coil with a current of 0.8 c.g.s. unit produces a magnetic field of which the total energy is 6,400,000 ergs, so that the value of L for this coil is 20,000,000 centimeters. If the current is expressed in amperes and energy in joules then the total energy corresponding to 8 amperes would be 0.64 joules and the value of L would be 0.02 henry.

Non-inductive circuits.—A circuit of which the inductance is negligibly small is called a *non-inductive circuit*. Since the inductance of a circuit depends upon the energy of the magnetic field, a non-inductive circuit is one which produces only a weak field, or a field which is confined to a very small region. Thus, the two wires, Fig. 3, constitute a non-inductive circuit, espe-

Fig. 3.

cially if they are near together; for, these two wires with opposite currents produce only a very feeble magnetic field in the surrounding region. The wires used in resistance boxes are usually arranged non-inductively. This may be done by doubling the wire back on itself and winding this double wire on a spool. In this case the electromotive force between adjacent wires may be great and they may have considerable electrostatic

capacity. In order to make a non-inductive resistance coil without this defect, the wire may be wound, in one layer, on a thin paper cylinder so as to bring the terminals as far apart as possible. This cylindrical coil is then flattened so as to reduce the region (inside) in which the magnetic field is intense. This gives a non-inductive coil of which the electrostatic capacity is inconsiderable.

5. Moment of inertia, analogue of inductance.—The kinetic energy of a rotating wheel resides in the various moving particles of the wheel. The velocity (linear) of each particle of the wheel is proportional to the speed (angular velocity) of the wheel, and the energy of each particle is proportional to the square of its velocity, that is, to the square of the speed. Therefore, the total kinetic energy of the wheel is proportional to the square of the speed. That is,

$$W = \tfrac{1}{2} K\omega^2 \tag{2}$$

in which W is the total energy of a wheel rotating at angular velocity ω and $(\tfrac{1}{2}K)$ is the proportionality factor. The quantity K is called the *moment of inertia* of the wheel.

6. Proposition.—*The inductance of a coil wound on a given spool is proportional to the square of the number of turns N of wire.* For example, a given spool wound with No. 16 wire has 500 turns and an inductance of, say, 0.025 henry; the same spool wound with No. 28 wire would have about ten times as many turns and its inductance would be about 100 times as great, or 2.5 henrys.

Proof.—To double the number of turns on a given spool would everywhere double the field intensity for the same current, and therefore the energy of the field would everywhere be quadrupled for a given current so that the inductance would be quadrupled according to equation (1).

7. Proposition.—*The inductance of a coil of given shape is proportional to its linear dimensions, the number of turns of wire being unchanged.* For example, a given coil has an inductance of 0.022 henry, and a coil three times as large in length, diameter, etc., has an inductance of 0.066 henry.

8. Electromotive force required to make the current in a coil change.—A current once established in a coil of zero resistance would continue to flow without the help of an electromotive force to maintain it just as a wheel when once started continues to turn, provided there is no resistance to the motion of the wheel. To increase the speed of the wheel a torque must act upon it in the direction of its rotation, and to increase the current in the coil an electromotive force must act on the coil in the direction of the current.

When an electromotive force e (over and above the electromotive force required to overcome the resistance of the coil) acts upon a coil the current is made to increase at a definite *rate*, $\frac{di}{dt}$, such that

$$e = L\frac{di}{dt} \tag{3a}$$

Proof of equation (3).—Multiplying both members of this equation by the current i we have $ei = Li\frac{di}{dt}$. Now ei is the rate, $\frac{dW}{dt}$, at which work is done on the coil, in addition to the work used to overcome resistance, and this must be equal to the rate at which the kinetic energy of the current of the coil increases. Differentiating equation (1) we have $\frac{dW}{dt} = Li\frac{di}{dt}$. Therefore, equation (3) is proven.

Torque required to make the speed of a wheel increase.—When a torque T (over and above the torque required to overcome the frictional resistance) acts upon a wheel, then the angular velocity ω of the wheel is made to increase at a definite *rate*, $\frac{d\omega}{dt}$, such that

$$T = K\frac{d\omega}{dt} \tag{4}$$

Proof of equation (4).—Multiplying both members of this equation by the angular velocity ω of the wheel we have $T\omega = K\omega\frac{d\omega}{dt}$. Now $T\omega$ is the rate, $\frac{dW}{dt}$, at which work is done on the wheel; and this must be equal to the rate at which the kinetic energy of the wheel increases. Differentiating equation (2) we have $\frac{dW}{dt} = K\omega\frac{d\omega}{dt}$. Therefore equation (4) is proven.

9. Magnetic flux and flux-turns.—In dealing with a coil of wire

it is frequently necessary to consider *the product of the magnetic flux through the coil multiplied by the number of turns of wire in the coil.* This product is called the *flux-turns*, and it is represented by the single symbol Φ_1. That is:

$$\Phi_1 = N\Phi \tag{5}$$

in which Φ is the flux through the coil (strictly the flux through a mean turn of the coil), N is the number of turns of wire in the coil, and Φ_1 is the flux-turns.

Proposition.—The flux-turns Φ_1 through a coil due to a current i in the coil is

$$\Phi_1 = Li \qquad (6)^*$$

in which L is the inductance of the coil. This proposition is proven in the next article.

10. Self-induced electromotive force. *Reaction of a changing current.*—When one pushes on a wheel, causing its speed to increase, the wheel reacts and pushes back against the hand. This reacting torque is equal and opposite to the acting torque, $K \cdot \frac{d\omega}{dt}$, [equation (4)], which is causing the increase of speed. Thus, when the speed of the wheel is increasing, the reacting torque is in a direction opposite to the speed, and, when the speed is decreasing, the reacting torque is in the same direction as the speed.

Similarly when an electromotive force acts upon a circuit,† causing the current to increase, the increasing current reacts. The reacting electromotive force is equal and opposite to the acting electromotive force $L\frac{di}{dt}$ [equation (3)], which is causing the current to increase. This reacting electromotive force is called a self-induced electromotive force. The self-induced electromotive force is therefore

$$e = -L\frac{di}{dt} \tag{3b}$$

* In this equation L and i must be expressed in c.g.s. units because the unit of flux corresponding to the ampere-henry *is not much used.*

† Supposed to have zero resistance for the sake of simplicity of statement.

When a current is increasing $\left(\frac{di}{dt}\text{ positive}\right)$ the self-induced electromotive force is opposed to the current, and when a current is decreasing $\left(\frac{di}{dt}\text{ negative}\right)$ the self-induced electromotive force is in the direction of the current, exactly as in the case of a rotating wheel.

Proof of equation (6).—If the current i in a coil is changing, then, from equation (6) we have $d\Phi_1/dt = L \cdot di/dt$, and from equation (5) we have $d\Phi_1/dt = N \cdot d\Phi/dt$. But $-N \cdot d\Phi/dt$ is the electromotive force, e, induced in the coil by the changing flux, or by the changing current. Therefore $e = -L \cdot di/dt$, which, being identical to equation (3*b*), shows that equation (6) is true.

11. Calculation of inductance in terms of flux-turns per unit current.—According to equation (6) the inductance of a coil is equal to the quotient Φ_1/i, where Φ_1 is the flux-turns through the coil,* due to the current i in the coil. There are important cases in which the flux through a coil, due to a given current, may be easily calculated and, therefore, the inductance of such a coil is easily determined.

Long solenoid.—Consider a long cylindrical coil of wire of radius r, of length l and having N turns of wire. The field intensity in the coil is $H = 4\pi Ni/l$ and the area of the opening of the coil is πr^2, so that the flux through the opening is $4\pi^2r^2Ni/l$ $(= \Phi)$. The coil has N turns, so that $\Phi_1 = N\Phi = 4\pi^2r^2N^2i/l$; dividing this by i we have, according to equation (6),

$$L = \frac{4\pi^2r^2N^2}{l} \tag{7}$$

in which L is inductance in centimeters, r is the radius of the coil in centimeters and l the length of the coil in centimeters. This equation is strictly true only for very long coils on which the wire is wound in a thin layer; the equation is, however, very useful in enabling one to calculate easily the approximate inductance of even short thick coils.

* That is, the flux through a mean turn multiplied by the number of turns of wire.

Coil wound on an iron core.—A coil of N turns of wire is wound on an iron ring l cm. in circumference (mean) and s cm.^{2} in sectional area, as shown in Fig. 4. The coil produces through the ring a magnetic flux $\Phi = m.m.f./m.r.$, where $m.m.f.$ ($= 4\pi Ni$) is the magnetomotive force due to the coil, and $m.r.$($= l/\mu s$) is the magnetic reluctance of the iron core, i being the current in the coil and μ the permeability of the iron. Therefore,

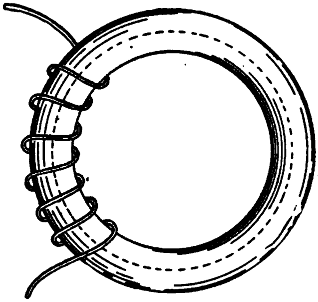


Fig. 4.

$$\Phi_1 = N\Phi = \frac{4\pi\mu s N^2 i}{l} = Li$$

or

$$L = \frac{4\pi\mu s N^2}{l} \tag{8}$$

Remark.—The permeability, μ, of iron decreases with increasing magnetizing force. Therefore, the inductance of a coil wound on an iron core is not a definite constant as in case of a coil without an iron core.

12. Growth and decay of current in an inductive circuit.—When a torque is applied to a wheel the wheel gains speed until the whole of the applied torque is used to overcome the resistance of the air, etc. While the speed is increasing part of the applied torque overcomes this resistance and the remainder causes the speed to increase.

When an electromotive force is applied to a circuit the current in the circuit increases until the whole of the applied electromotive force is used to overcome the resistance of the circuit. While the current is growing part of the applied electromotive force overcomes resistance and the remainder causes the current to increase. Therefore,

$$E = Ri + L\frac{di}{dt} \tag{9}$$

in which E is the applied electromotive force, i is the instantane-

ous value of the growing current, R is the resistance of the circuit and L its inductance. Ri is the part of E used to overcome resistance and $L\frac{di}{dt}$ is the part of E used to make the current increase.

If a circuit of inductance, L, and resistance, R, with a given current is left to itself without any electromotive force to maintain the current, the current dies away or decays, and the electromotive force Ri, which at each instant overcomes the resistance, is the self-induced electromotive force $-L\frac{di}{dt}$; so that at each instant $Ri = -L\frac{di}{dt}$ or

$$0 = Ri + L\frac{di}{dt} \qquad (10)$$

Examples.—An electromotive force of 110 volts acts on a coil, of which the inductance is 0.04 henry and the resistance is 3 ohms. At the instant that the electromotive force begins to act, the actual current i in the coil is zero, and the whole of the electromotive force acts to increase the current, so that 110 volts $= 0.04$ henry $\times\, di/dt$ or $di/dt = 2750$ amperes per second. When the growing current has reached a value of 30 amperes, Ri is equal to 90 volts, and the remainder of the 110 volts acts to cause the current to increase, that is, 20 volts $= 0.04$ henry $\times$ di/dt or $di/dt = 500$ amperes per second.

If a current is established in this coil and the coil left to itself, short circuited, without any electromotive force to maintain the current; then, as the decaying current reaches a value of, say, 30 amperes, the electromotive force Ri is 90 volts, and this electromotive force is equal to $-L\frac{di}{dt}$, so that $\frac{di}{dt}$ is -2250 amperes per second.

13. Problem I.—An inductive circuit with a current flowing in it is left to itself, short circuited. At a certain instant, from which time is to be reckoned ($t = 0$), the value of the current is I. It is required to find an expression for the decaying

current at each succeeding instant; the resistance R and the inductance L of the circuit being given.

Let i be the value of the current at the instant t. Then

$$i = I\varepsilon^{-\frac{R}{L}\cdot t} \tag{11}$$

Proof.—To establish the truth of equation (11) it is sufficient to show that $i = I$ when $t = 0$, and that equation (10) is satisfied. Substituting $t = 0$ in equation (11) we have $i = I$. Differentiating equation (11) we have

$$\frac{di}{dt} = -\frac{R}{L} I\varepsilon^{-\frac{R}{L}\cdot t}$$

or

$$\frac{di}{dt} = -\frac{R}{L}\cdot i$$

or

$$Ri + L\frac{di}{dt} = 0$$

which is equation (10).

The ordinates of the curve, Fig. 5, show a decaying current.

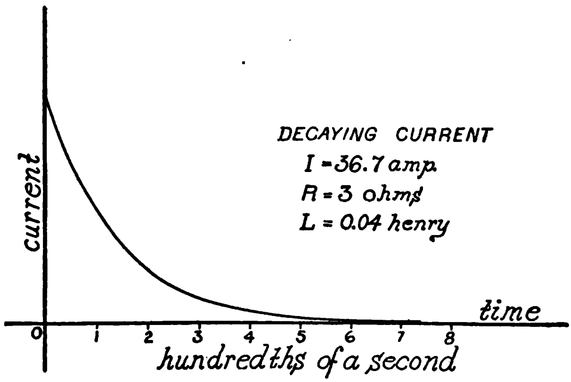


Fig. 5.

14. Problem II.—A constant electromotive force E is connected to a circuit of resistance R and inductance L. Required an expression for the growing current t seconds after the electromotive force is connected to the circuit.

The required expression is

$$i = \frac{E}{R} - \frac{E}{R}\cdot\varepsilon^{-\frac{R}{L}\cdot t} \tag{12}$$

Proof.—To establish the truth of equation (12) it is sufficient to show that $i = 0$ when $t = 0$ and that equation (9) is satisfied.

The ordinates of the curve, Fig. 6, show the values of a growing current.

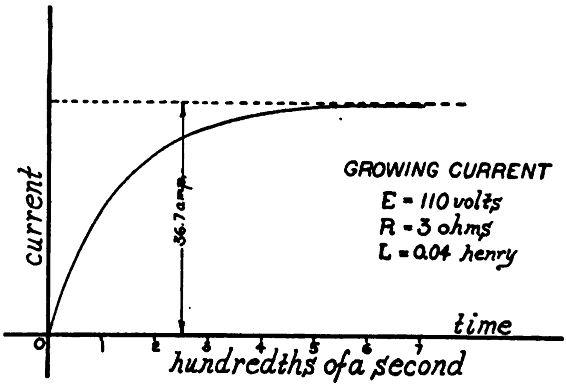


Fig. 6.

15. Electric charge.—The electric current in a wire is looked upon as a transfer of electric charge along the wire. The amount of electric charge q, which in t seconds passes a given point of wire conveying a current i is

$$q = it \tag{13}$$

or the rate dq/dt at which the charge passes a given point on a wire is

$$\frac{dq}{dt} = i \tag{14}$$

Units charge.—When i in equation (13) is expressed in amperes and t in seconds, q is expressed in terms of a unit called the *coulomb*. That is, the coulomb is the amount of electric charge which passes in one second along a wire carrying one ampere. When i is expressed in c.g.s. units and t in seconds, q is expressed terms of the c.g.s. unit charge.

Measurement of electric charge.—An electric charge may be determined by measuring the current i which it will maintain during an observed time, t. Then q may be calculated from equation (13). The charge capacity of storage batteries is determined in this way. A very small charge cannot be measured by measuring the current i and the time t, for such a charge cannot maintain a steady measurable current for a sufficient time. A

small electric charge is measured by allowing it to pass quickly through a galvanometer and observing the throw of the needle. The charge is sensibly proportional to the throw. A galvanometer used in this way is called a ballistic galvanometer.

16. Condensers. Electrostatic capacity.—When the terminals of a battery are connected to two metal plates, as shown in Fig. 7, a momentary current flows as indicated by the arrows and the electric charge which passes along the wire during this momentary current is stored upon the plates, for upon disconnecting the battery and connecting the plates with a wire a momentary reversed current may be observed. If a ballistic galvanometer be included in the circuit the amount of charge which passes into the plates may be measured. This amount of charge is proportional to the electromotive force e of the battery (other things being equal) that is

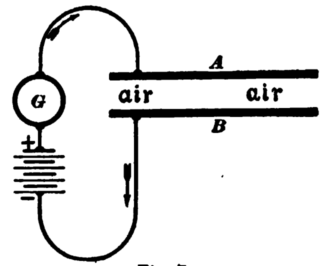


Fig. 7.

$$q = Ce \qquad (15)$$

in which q is the electric charge which flows along the wire into the plates, e is the electromotive force of the battery and C the proportionality factor. Two plates arranged in this way constitute what is called a *condenser* and factor C is called the *electrostatic capacity* of the condenser. If, in the equation (15), q is expressed in coulombs and e in volts, then C is expressed in terms of a unit called a *farad*. That is, a condenser has a capacity of one farad when one coulomb of electric charge is pushed into it by a battery of which the electromotive force is one volt. The unit of capacity which is commonly used to express the capacities of condensers, electric cables, etc., is the microfarad. The microfarad is one millionth of a farad. The microfarad is used because the farad is too large a unit to use conveniently.

Condensers to have a large capacity (as much as a microfarad)

are usually made up of alternate sheets of tinfoil and waxed paper or mica, as indicated in Fig. 8. Alternate metal sheets are connected together as shown, thus practically forming two plates of large area.

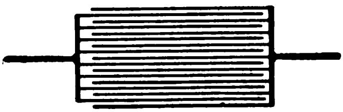


Fig. 8.

17. Hydrostatic analogue of the condenser.—Consider a chamber with water-tight compartments, A and B, Fig. 9, separated by an elastic diaphragm, DD, of rubber. If a pump P is connected to the compartments as shown, a definite quantity of water q will be forced through the pipe, out of one compartment into the other, and this quantity will be proportional to the difference of pressure e generated in A and B by the pump. That is

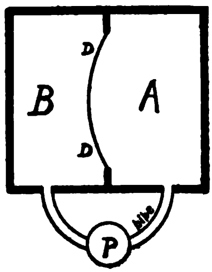


Fig. 9.

$$q = Ce$$

in which C is the proportionality factor. The diaphragm separating A and B is subject to mechanical stress very much as the insulator or dielectric between the plates of a condenser is subject to electrical stress.

18. Inductivity of dielectric.—The material between the plates of a condenser is called a *dielectric*. The capacity of a condenser of given dimensions depends upon the material which is used as the dielectric. The quotient, capacity of a condenser with a given dielectric divided by its capacity with air as the dielectric, is called the *inductivity* of the given dielectric.

TABLE OF INDUCTIVITIES.

Air equal to unity.

Glass	3.00 to 10.00	Mica	4.00 to 8.00
Vulcanite	2.50	Shellac	2.95 to 3.60
Paraffine	1.68 to 2.30	Turpentine	2.15 to 2.43
Beeswax	1.86	Petroleum	2.04 to 2.42

The capacity of a condenser is given by the equation:

$$C_{\text{farads}} = 885 \times 10^{-16} \times \frac{ka}{x} \qquad (16)$$

in which a is the combined area in square centimeters of all the leaves of dielectric between the condenser plates, x is the thickness in centimeters of the dielectric leaves, and k is the inductivity of the dielectric used.

19. Mechanical and electrical analogies.—The analogy between moment of inertia and inductance as pointed out in the discussion of inductance is but a small part of an extended analogy between pure mechanics and electricity. This extended analogy is here briefly outlined.

$x = vt$ (1) in which x is the distance traveled in t seconds by a body moving at velocity v.	$\phi = \omega t$ (2) in which Φ is the angle turned in t seconds by a body turning at angular velocity ω.	$q = it$ (3) in which q is the electric charge which in t seconds flows through a circuit carrying a current i.
$W = Fx$ (4) in which W is the work done by a force F in pulling a body through the distance x.	$W = T\phi$ (5) in which W is the work done by a torque T in turning a body through the angle ϕ.	$W = Eq$ (6) in which W is the work done by an electromotive force E in pushing a charge q through a circuit.
$P = Fv$ (7) in which P is the power developed by a force F acting upon a body moving at velocity v.	$P = T\omega$ (8) in which P is the power developed by a torque T acting on a body turning at angular velocity ω.	$P = Ei$ (9) in which P is the power developed by an electromotive force E in pushing a current i through a circuit.
$W = \frac{1}{2} mv^2$ (10) in which W is the kinetic energy of a mass m moving at velocity v.	$W = \frac{1}{2} K\omega^2$ (11) in which W is the kinetic energy of a wheel of moment of inertia K turning at angular velocity ω.	$W = \frac{1}{2} Li^2$ (12) in which W is the kinetic energy of a coil of inductance L carrying a current i.
$F = m\frac{dv}{dt}$ (13) in which F is the force required to cause the velocity of a body of mass m to increase at the rate $\frac{dv}{dt}$	$T = K\frac{d\omega}{dt}$ (14) in which T is the torque required to cause the angular velocity of a wheel of moment of inertia K to increase at the rate $\frac{d\omega}{dt}$	$E = L\frac{di}{dt}$ (15) in which E is the electromotive force required to cause a current in a coil of inductance L to increase at the rate $\frac{di}{dt}$
$x = aF$ (16)	$\phi = bT$ (17)	$q = CE$ (18)
$\frac{4\pi^2 m}{\tau^2} = \frac{1}{a}$ (19)	$\frac{4\pi^2 K}{\tau^2} = \frac{1}{b}$ (20)	$\frac{4\pi^2 L}{\tau^2} = \frac{1}{C}$ (21)

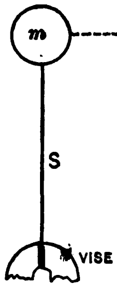


Fig. a.

A body of mass m is supported by a flat spring S, clamped in a vise as shown in Fig. a. A force F pushing sidewise on m moves it a distance x, which is proportional to F, according to equation (16). When started the body m will continue to vibrate back and forth and the period τ of its vibrations is determined by equation (19).

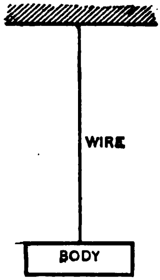


Fig. b.

A body of moment of inertia K is hung by a wire as shown in Fig. b. A torque T acting on the body will turn the body and twist the wire through an angle ϕ, which is proportional to T, according to equation (17). When started, the body will vibrate about the wire as an axis and the period τ of its vibrations is determined by equation (20).

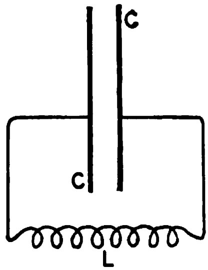


Fig. c.

A condenser C is connected to the terminals of a coil of inductance L as shown in Fig. c. An electromotive force E acting anywhere in the circuit pushes into the condenser a charge q, which is proportional to E, according to equation (18). When started the electric charge will surge back and forth through the coil, constituting what is called an oscillatory current and the period of one oscillation is determined by equation (21).

PROBLEMS.

1. The intensity of the magnetic field in the air gap between the pole face and the armature core of a dynamo is 5,000 c.g.s. units and the pole face is 10 cm. × 20 cm. Required the magnetic flux from pole face to armature core. Ans. 1,000,000 lines.

2. A coil of an alternator armature has 20 turns of wire and engages the whole 1,500,000 lines which flow from a pole of the field magnet. In $\frac{1}{250}$ second this coil moves from a north pole to an adjacent south pole of the field magnet, when the flux is

reversed. Calculate the average electromotive force in the coil during this interval. Ans. 150 volts.

3. The core of an induction coil when magnetized has a flux of 120,000 lines passing through it. When the primary circuit is broken the flux through the core drops to 15,000 lines in $\frac{1}{500}$ second. What is the average value during this interval of the electromotive force which is induced in the secondary coil which has 150,000 turns of wire? Ans. 78,750 volts.

4. Find the approximate inductance, in henrys, of a cylindrical coil 25 cm. long and 5 cm. mean diameter, wound with one layer of wire containing 150 turns. Ans. 0.00022 henry.

5. Calculate the kinetic energy of a current of 20 amperes in the above coil. Ans. 0.044 joule.

6. The above coil is connected to 110-volt mains. Find the rate, in amperes per second, at which the current begins to increase in the coil. Ans. 500,000 amperes per second.

7. Calculate the rate at which the current (problem 6) is increasing when it has reached the value of 10 amperes, the resistance of the coil being 2.5 ohm. Ans. 386,262.6 amperes per second.

8. A coil of which the resistance is 2.5 ohms and the inductance 0.04 henry has a current started in it. The coil is then short circuited and the current left to die away. Calculate the rate, in amperes per second, at which the current is decreasing as it passes the value of 10 amperes. Ans. -625 amperes per second.

9. A coil of wire has an inductance of 0.035 henry. Calculate the magnetic flux-turns through the coil due to a current of 5 c.g.s. units in the coil. The coil has 1,500 turns of wire; calculate the number of lines of flux through a mean turn. Ans. 175,000,000 line-turns, 116,666 lines.

10. A condenser has a capacity of 1.2 microfarads. Calculate the charge which is pushed into this condenser by an electromotive force of 1,000 volts, and calculate the time during which

this charge would maintain a current of one ampere. Ans. 0.0012 coulomb, 0.0012 second.

11. A condenser is built up of leaves of mica, each 0.1 millimeter thick, between sheets of tinfoil 7 x 10 centimeters. Find the number of mica leaves required to give a capacity of one microfarad. (The inductivity of mica may be taken as 6.) Ans. 269 leaves.

12. A condenser, consisting of two sheets of tinfoil 30 x 30 centimeters pasted on the two sides of a pane of glass 2 millimeters thick (inductivity of glass equals 6), is discharged through a coil consisting of 250 turns of wire wound on glass tube 3 centimeters in diameter and 30 centimeters long. Find the approximate periodic time of the electrical oscillations. (See Article 19.) Ans. 0.00000417 second.

13.* The field coils of a shunt dynamo have a resistance of 100 ohms and an inductance of 20 henrys. An electromotive force of 500 volts is applied. Calculate the time required for the current to reach a value of 4 amperes. Ans. 0.322 second.

14. A telegraph line has 3.6 henrys inductance and 2,500 ohms resistance. Calculate the time required for the current to reach ⅔ of its full value, E/R, after the circuit is closed. Ans. 0.00157 second.

* This problem applies to the case in which the field magnet is laminated. When the field magnet is made of solid iron the eddy currents produced in it during magnetization permit the current in the field coils to grow very quickly to nearly its full value; while, at the same time, the magnetization of the core grows more slowly than when the iron is laminated.

CHAPTER II.

THE SIMPLE ALTERNATOR.

20. The alternator. *Definition of alternating electromotive force and of alternating current.*—The *alternator* is an arrangement by means of which mechanical energy or work is used to cause the magnetic flux from a magnet to pass through the opening of a coil of wire first in one direction and then in the other direction. This varying magnetic flux induces an electromotive force in the coil first in one direction and then in the other direction. This electromotive force, called an *alternating electromotive force*, produces an *alternating current* in the coil and in the circuit which is connected to the terminals of the coil.

Examples.—In the common type of alternator the above-mentioned magnet and coil move relatively to each other. Fig. 10 shows the essential features of such an alternator. The poles of a multipolar magnet, called the *field magnet*, project radially inwards toward the passing teeth of a rotating mass of laminated iron A, and upon these teeth are wound the coils of wire in which the alternating electromotive force is induced. This rotating mass of iron with its windings of wire is called the *armature*. On the armature shaft, at the one end of the armature, are mounted two insulated metal rings called *collecting rings*. These metal rings are connected to the ends of the armature wire, and metal *brushes* rub on these rings, thus keeping the ends of the armature winding in continuous contact with the terminals of the external circuit to which the alternator supplies alternating current. The electromotive forces induced in adjacent armature coils are in opposite directions at each instant and the coils are so connected together that these electromotive forces do not op-

pose each other. This is done by reversing the connections of every alternate coil, as indicated by the dotted lines connecting the coils in Fig. 10. The electromagnetic action of this type of alternator depends only upon the relative motion of field magnet

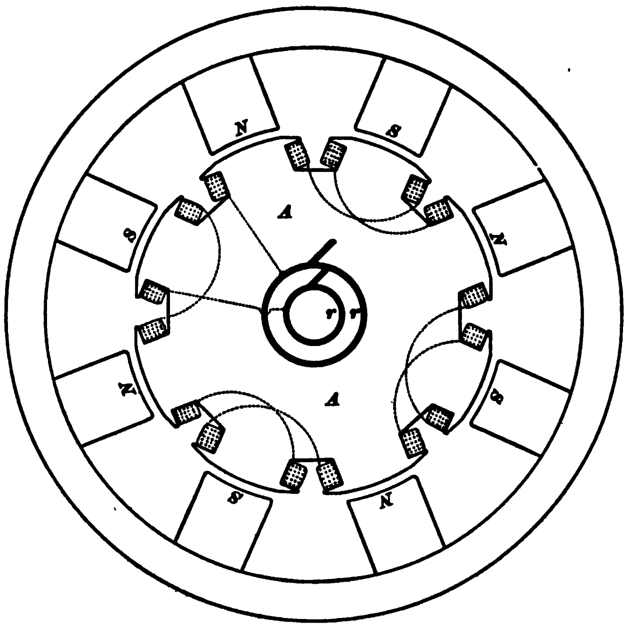


Fig. 10.

and armature, and large machines are often built with stationary armature and rotating field magnet.

In another type of alternator, called the *inductor alternator*, the magnet and the wire in which the alternating electromotive force is induced are both stationary, and a moving mass of laminated iron causes the magnetic flux from the magnet to pass through the stationary coils in the desired manner. Fig. 11 shows the essential features of the inductor alternator. *NSNS*, etc., are the field magnet poles. The armature wire is wound on the intermediate projections *AAA*, and the inductors *III* are supported by a spider keyed to a rotating shaft.

The exciter.—The field magnet of an alternator is usually an electromagnet which is excited by a continuous electric current supplied by an independent generator, generally by an auxiliary continuous current dynamo called the *exciter*. The exciting current flows through coils of wire wound on the projecting poles *NSNS* in Figs. 10 and 11.

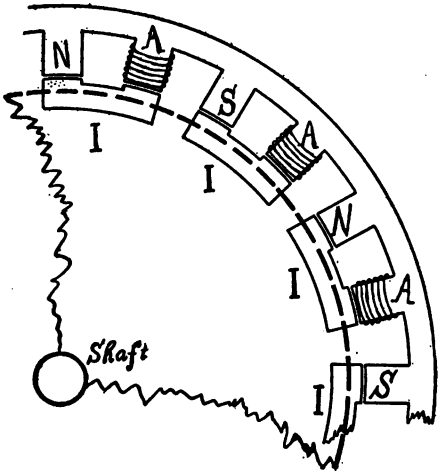


Fig. 11.

Armature cores and armature windings. — The type of armature core shown in Fig. 10 is called the *toothed* armature core, and the winding is said to be *concentrated*, that is, the armature conductors are grouped in a few heavy bunches. Armature cores are also made with many small slots, in which the armature conductors are grouped in small bunches. This type of core is called a *multi-slotted* core, and the winding is said to be *distributed.* The various types of armature windings are described in Chapter IX.

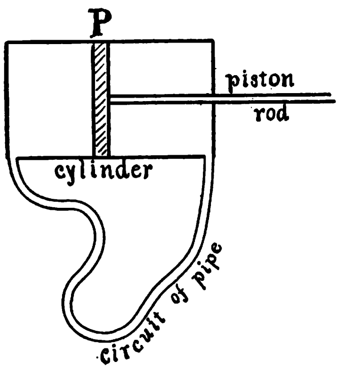


Fig. 12.

In some of the earlier types of alternators the armature core consisted of a smooth, cylindrical mass of laminated iron, upon the face of which the conductors were arranged in bands side by side, one layer or more in depth. This type of armature is called the *smooth core* armature.

The hydraulic analogue of the alternator.—Consider a valveless pump *P*, Fig. 12, of which the piston is pushed rapidly back and

forth. This to-and-fro motion of the piston produces an alternating hydrostatic pressure-difference between the outlet and inlet of the pump, and causes the water to surge back and forth through the circuit of pipe.

21. Advantages and disadvantages of alternating currents.— The electric transmission of a given amount of power may be accomplished by a large current at low electromotive force, or by a small current at high electromotive force. In the first case very large and expensive transmission wires must be used, or the loss of power in the transmission line will be excessive. In the second case comparatively small and inexpensive transmission wires may be used. Thus it is a practical necessity to employ high electromotive forces in long-distance transmission of power.

High electromotive forces are dangerous under the conditions which ordinarily obtain among users of electric light and power, and many types of apparatus, such as incandescent lamps, operate satisfactorily only with medium or low electromotive forces. Therefore, means must be provided at a receiving station for transforming the power which is delivered, from high electromotive force and small current, to low electromotive force and large current if long-distance transmission is to be successful. This is called *step-down transformation.* The advantage of alternating current over direct current lies almost wholly in the cheapness of construction, the cheapness of operation and the high efficiency of the alternating current as compared with the direct-current apparatus that is required for transformation.

In step-down transformation of direct current a motor takes a small current from the high electromotive force transmission mains, and drives a dynamo which delivers large current to service mains at low electromotive force. This apparatus, or its equivalent, the dynamotor, is expensive to construct, it requires attention in operation, and its efficiency is never, perhaps, above 90 per cent.

The step-down transformation of alternating currents is accomplished by means of the alternating current transformer,

which is described in Chapter X. The alternating current transformer is very much cheaper than a dynamo and motor of the same output, it requires no attention in operation, and its efficiency under full load is usually greater than 97 per cent.

Alternating current has some minor advantages over direct current, on account of the fact that alternating current machines are frequently simpler in construction than direct current machines. In particular, the commutator is not an essential part of an alternating current machine. In the case of the inductor alternator and the induction motor, the rotating part may not have any sliding electrical contacts whatever.

The simple alternating current is not suited to motor service, for the alternating current motor does not start satisfactorily. For uninterrupted service the synchronous motor is frequently used, the starting being effected by an auxiliary engine or other independent mover. The synchronous motor is described in Chapter XII.

The synchronous motor is not satisfactory when frequent starting is necessary. For such service the induction motor is used. The induction motor is described in Chapter XIV. The induction motor, to start satisfactorily, must be supplied with two or more distinct alternating currents, transmitted to the motor over separate lines. This is called the polyphase system of transmission. It is described in Chapter VIII.

For some purposes, especially for the electrolytic processes, which are used on a large scale in electro-chemical works, direct current only can be used. When power, transmitted by alternating current, is to be delivered in the form of direct current, the conversion is effected by means of the rotary converter, which is described in Chapter XIII.

The rectifier is sometimes used for converting alternating current into direct current. The rectifier is described in Article 28. The rectifier is used in American practice only in connection with the compounding of the field of the alternator as described in Article 94.

22. Characteristic features of alternating-current problems.—Alternating-current problems differ from direct-current problems chiefly for two reasons, as follows: (*a*) The rapid changing of the alternating current produces an electromotive force reaction in any circuit which has inductance, and a portion of the electromotive force which acts upon the circuit is necessarily used in overcoming this reaction. (*b*) The rapid changing of an alternating electromotive force causes the various parts of a circuit to become alternately charged and discharged with electricity, and a portion of the alternating current which is delivered to a circuit does not flow through the entire circuit, but charges and discharges the various parts of the circuit. In short, alternating-current phenomena differ from direct-current phenomena because of the effects of inductance and capacity.

In many of the practical problems in alternating currents the capacity is concentrated at one part of the circuit in the form of a condenser. These problems, which are comparatively simple, are treated in Chapters V., VI. and VII. In the most general case the capacity is distributed throughout the circuit, or in other words all portions of the circuit are charged and discharged perceptibly as the alternating electromotive force which acts upon the circuit pulsates. The phenomena exhibited by long transmission lines depend very materially upon the effects of distributed capacity. These phenomena are discussed in Chapter XV. A clear idea of the effects of distributed capacity may be obtained by considering Fig. 12. Imagine the circuit of pipe in this figure to be made of a distensible rubber tube. Then at a given instant the flow of water through the tube at one point will not be the same as the flow through the tube at another point; and the difference of flow at the two points will be accommodated by expansion and contraction of the intervening portion of the tube.

23. Speed and frequency.—The electromotive force of an alternator passes through a set of positive values, while a given coil of the armature is passing from a south to a north pole of the

field magnet, and through a *similar* set of negative values, while the coil is passing from a north pole to a south pole, or *vice versa*. The complete set of values, including positive and negative values, through which an alternating electromotive force (or alternating current) repeatedly passes, is called a *cycle*. The number of cycles per second is called the *frequency*, f.

Let p be the number of *pairs* of field magnet poles, n the revolutions per second of the armature, and f the frequency of the electromotive force of the alternator. Then

$$f = pn \tag{17}$$

This is evident when we consider that the electromotive force passes through a complete cycle of values while an armature tooth is passing from a north pole to the next north pole, so there are p cycles of electromotive force for each revolution of the armature.

24. Electromotive force and current curves.—The successive instantaneous values of the electromotive force of an alternator may be represented by the ordinates of points on a curve, the abscissas representing time elapsed from some chosen epoch; the resulting curve is called the *electromotive force curve* of the alternator. In a similar manner the successive instantaneous values of an alternating current may be represented by ordinates and the elapsed times by abscissas giving a *current curve*. These curves are determined with the help of the *contact-maker* as explained in Article 30.

Examples.—The full-line curve, Fig. 13, represents the electromotive force of an alternator with a distributed armature winding; and the dotted curve represents the current which this electromotive force produces in a non-inductive circuit. This current is at each instant equal to the electromotive force divided by the resistance of the circuit, so that the current is a maximum when the electromotive force is a maximum. The current is then said to be in phase with the electromotive force, as is explained in Chapter IV.

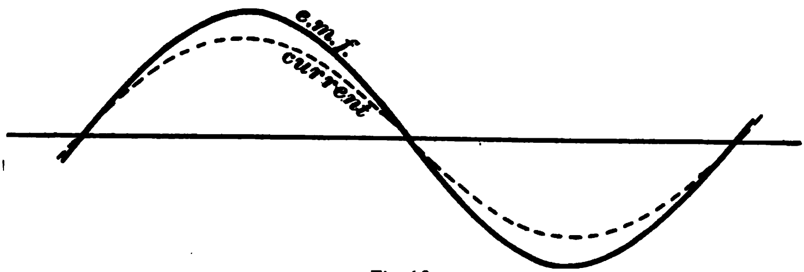


Fig. 13.

The full-line curve, Fig. 14, represents the electromotive force of an alternator with a distributed armature winding, and the

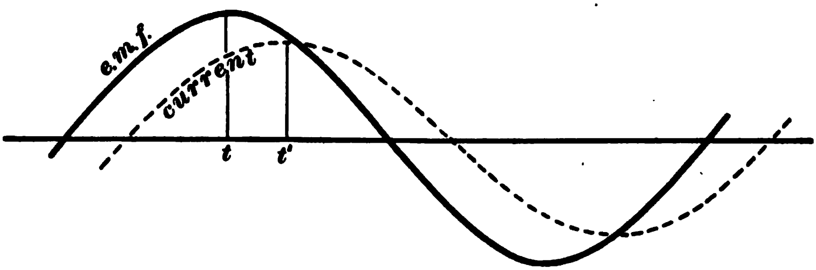


Fig. 14.

dotted curve represents the current which this electromotive force produces in an inductive circuit. In this case part of the electromotive force is, at each instant, used to cause the current

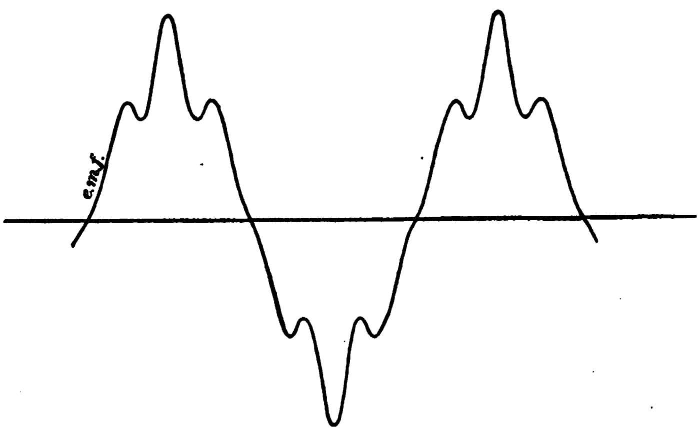


Fig. 15.

to increase or decrease. The part so used is $L\frac{di}{dt}$ according to equation (3), and the remainder, equal to Ri, is used to overcome the resistance of the circuit. When the current is zero then all the electromotive force is used to cause the current to change, since Ri is zero. When $\frac{di}{dt}$ is zero, the current is at its maximum or minimum value, and, at this instant, all the electromotive force is used to overcome the resistance of the circuit since $L \cdot di/dt$ is zero. The time t', Fig. 14, at which the current reaches its maximum value is later than the time, t, at which the electromotive force reaches its maximum value. In some cases, however, the current may reach its maximum value before the electromotive force.

The curve in Fig. 15 represents the electromotive force of an alternator with concentrated armature windings, the armature core being of the shape shown in Fig. 10.

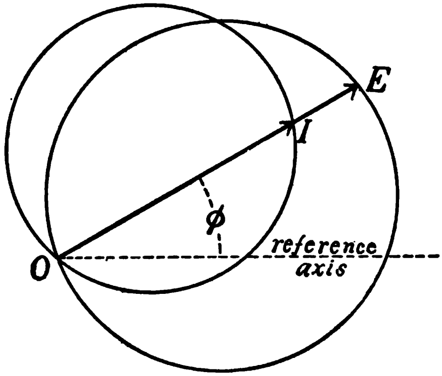


Fig. 16.

25. The representation of alternating electromotive force and current by polar coördinates.—The successive instantaneous values of an alternating electromotive force, or current, may be represented by the varying lengths of a rotating radius vector, elapsed time being represented by the angle between the moving radius vector and a fixed axis of reference. Thus, in Fig. 16, OE represents the value at a given instant of an alternating electromotive force and OI the value at the same instant of an alternating current, and the angle ϕ represents the time elapsed since a chosen instant. This Fig. 16 represents the same electromotive force and current as are represented by Fig. 14.

26. Instantaneous and average power delivered by an alternator.—Let e be the value at a given instant of the electromotive force of an alternator and i the value of the current at the same

instant. Then *ei* is the power in watts which is delivered by the alternator at the given instant, and the average value of *ei* is the average power delivered by the alternator.

Examples.—In Fig. 17 the full-line curve represents the electromotive force of an alternator and the dotted curve represents

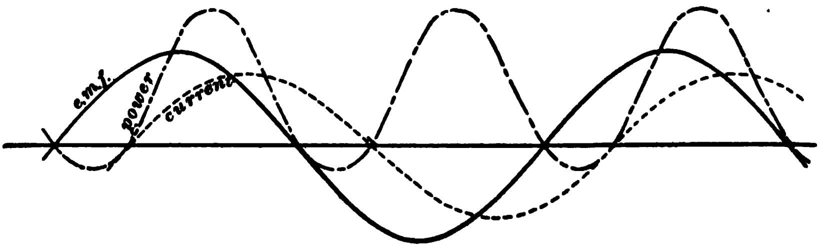


Fig. 17.

the current delivered by the alternator to a receiving circuit having inductance. The ordinates of the dot-dash curve represent the successive instantaneous values of the power, *ei*. As is shown in the figure, the power has both positive and negative values, the alternator does work on the circuit when *ei* is positive and the circuit returns power to the alternator when *ei* is negative, and, of course, while *ei* is negative, the dynamo is momentarily a motor, and may for the moment return power to the fly-wheel of the engine.

When the inductance of the receiving circuit is very large, the electromotive force and current curves are as shown in Fig. 18,

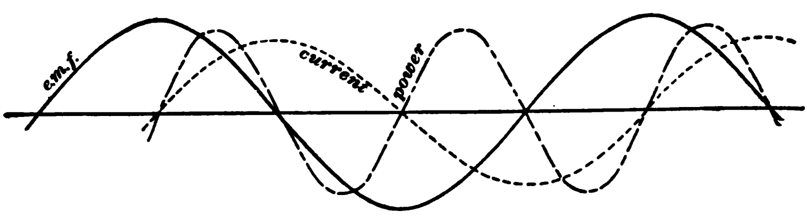


Fig. 18.

the instantaneous power *ei* passes through approximately similar sets of positive and negative values as shown by the dot-and-dash curve, and the average power is approximately zero.

27. Average values and effective values.—The average value of an alternating current or electromotive force is zero, inasmuch as similar sets of positive and negative values occur. The average value of an electromotive force or current *during the positive (or negative) part of a cycle* is usually spoken of briefly as the average of mean value, and is not zero.

Effective values.—Consider an alternating current, of which the instantaneous value is i. The rate at which heat is generated in a circuit through which the current flows is Ri^2, where R is the resistance of the circuit, and the *average* rate at which heat is generated in the circuit is R multiplied by the *average* value of i^2. A continuous current which would produce the same heating effect would be one of which the square is equal to the average value of i^2 or of which the actual value is equal to $\sqrt{\text{average } i^2}$. This square root of the average square of an alternating current is called the *effective* value of the alternating current. Similarly the square root of the average square of an alternating electromotive force is called the *effective value* of the alternating electromotive force.

Ammeters and voltmeters used for measuring alternating currents and alternating electromotive forces always give effective values, and in specifying an alternating electromotive force or current its effective value is always used.

Example.—Consider the successive instantaneous values (separated by equal time intervals) of an alternating current. The sum of these values divided by their number gives their average value. Square each instantaneous value. Add these squares, divide by their number and extract the square root, and the result is the square-root-of-average-square, or effective, value of the current.

Form factor of an alternating electromotive force.—The quotient, effective value of an alternating electromotive force, divided by the average value of the electromotive force during half a cycle, is called the *form factor* of the electromotive force, inasmuch as this ratio depends upon the shape of the electromotive force curve.

In the case of a sine-curve-electromotive-force the form factor is equal to 1.11, as is shown in Chapter IV. The form factor of the electromotive force represented by the curve in Fig. 15 is greater than 1.11.

28. The alternating current rectifier is an arrangement for reversing the connections of a receiving circuit with each reversal of the current from the alternator, so that the current may flow always in the same direction in the receiving circuit. The rectifier is frequently used on alternators for rectifying the main current in the series coils on the field for the purpose of providing increase of field excitation with increase of current output of the alternator. In this case the rectifier is a commutator mounted on the armature shaft. This commutator has as many bars as there are poles of the field magnet of the alternator. These bars are wide and separated by quite narrow spaces filled with mica. Let these bars be numbered in order around the commutator. The even-numbered bars are connected together and the odd-numbered bars are connected together. The connecting wire leading from one terminal of the alternator armature to one of the collector rings is cut and the two ends thus formed are connected, one to the even-numbered bars of the rectifying commutator and the other to the odd-numbered bars. The circuit which is to receive the rectified current is connected to two brushes which rub on the rectifying commutator, these brushes being so spaced that one touches an odd-numbered bar when the other touches an even-numbered bar. These brushes are carried in a rocker arm, which is moved forwards or backwards until the brushes are passing from one bar to the next at the instant that the alternating current from the alternator is passing through the value zero. The proper adjustment of the brushes is indicated by a minimum of sparking.

29. The fundamental equation of the alternator.—The equation which expresses the effective value of the electromotive force of an alternator in terms of the armature speed n, the number of

pairs of field magnet poles p, the flux Φ from one pole of the field magnet, and the total number of armature conductors N which cross the face of the armature is called the fundamental equation of the alternator. This equation is important in designing. It is derived as follows for the case in which the armature conductors are concentrated in $2p$ slots, one to each field pole as shown in Fig. 19.

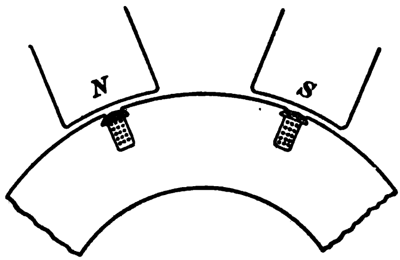


Fig. 19.

Let Φ be the lines of flux from one pole, then one armature conductor in one revolution cuts $2p\Phi$ lines, * and in one second it cuts $2p\Phi n$ lines, which is the *average* electromotive force (in c.g.s. units) induced in one armature conductor. We have, therefore,

Average † electromotive force of alternator in volts

$$= \frac{2p\Phi Nn}{10^8} \tag{18}$$

The ratio, effective electromotive force divided by average electromotive force, is, for commercial alternators, approximately equal to 1.11.‡ Therefore, the effective electromotive force of an alternator with concentrated armature winding is approximately

$$E = \frac{2.22 p\Phi Nn}{10^8} \tag{19}$$

or, since pn is the frequency according to equation (17) we have

$$E = \frac{2.22\Phi Nf}{10^8} \tag{20}$$

* Since we are concerned with the average value during half a cycle the change of sign during the two halves of a cycle is to be ignored and the flux from north and from south poles is to be treated without regard to sign.

† That is the average during half a cycle as explained in Article 27. The average during a whole cycle is zero.

‡ See Article 27.

in which Φ is the magnetic flux from one pole, and N the total number of conductors on the armature which are connected in series. Sometimes it is more convenient to have the equation given in terms of *armature turns* instead *of armature conductors.* The formula then becomes

$$E = \frac{4.44 \Phi T f}{10^8}. \tag{21}$$

T being the number of armature turns in series between the collector rings.

30. Experimental determination of electromotive force curves. *The contact-maker.*—A disc DD, Figs. 20 (a) and 20 (b), fixed

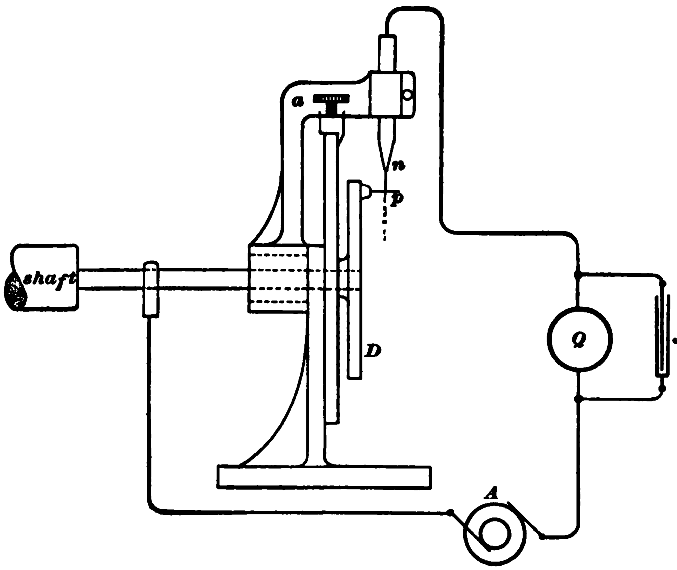


Fig. 20 (a).

to and rotating with the armature shaft, carries a pin p, which makes momentary electrical contact, once per revolution, with a jet of conducting liquid which issues from a nozzle n. This nozzle is carried on a pivoted arm a, and can be moved at will, its position being read off the divided circle cc. One terminal of an

electrostatic voltmeter Q is connected directly to one brush of the alternator, while the other terminal of the voltmeter is connected through the jet and pin to the other brush of the alternator as shown in Fig. 20 (a). The voltmeter then indicates the value of the electromotive force of the alternator at the instant of contact of jet and pin. By shifting the jet, step by step, around the

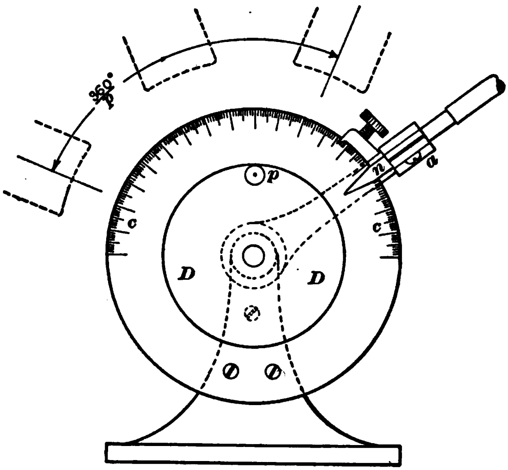


Fig. 20 (b).

circle successive instantaneous values of the electromotive force may be determined. The electromotive force passes through a complete cycle of values while the jet is shifted $\frac{1}{p}$ of a revolution, p being the number of pairs of poles of the alternator. In order that the electromotive force acting upon the electrostatic voltmeter may not fall off appreciably in the intervals between successive contacts of pin and jet, a condenser J is connected as shown in Fig. 20 (a). The indications of an electrostatic voltmeter are not accurate for small deflections and in using such an instrument for measuring a comparatively small electromotive force a battery of known electromotive force may be connected in the circuit so as to raise the electromotive force to an accurately measurable value.

In the determination of an alternating current curve, the current is sent through a non-inductive resistance R, Fig. 21, and the electromotive force between the terminals of this resistance is determined as before, the disc DD being fixed to the armature shaft of the alternator which is furnishing the current. The current at each instant is equal to the electromotive force divided by R.

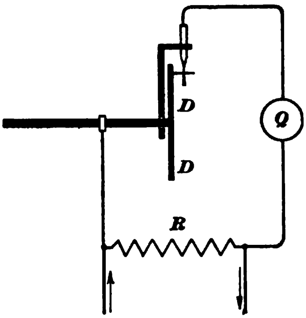


Fig. 21.

Remark.—A flat metal spring or brush is sometimes used instead of the liquid jet in Fig. 20. In this case the pin p is replaced by a strip of metal set in the edge of a circular disc of hard rubber and the spring rubs continuously upon the edge of this disc, touching the metal strip momentarily once per revolution of the disc.

PROBLEMS.

×15. An alternator has 16 poles and its speed is 900 revolutions per minute. What is the frequency of its electromotive force? What is the duration of one cycle? Ans. 120 cycles per second, $\frac{1}{120}$ second.

16. A four-pole alternator makes 1,800 revolutions per minute. Each pole face has 400 square centimeters of area and spans 54° of the circumference of the armature, the angle between the adjacent pole tips being 36°. The armature core is smooth, that is, not slotted, its length is 30 cm., and the field intensity in the gap space is 6,000 units. The armature is wound with four wires, one per field pole. These wires lie on the face of the armature parallel to the axis of the shaft, they are spaced 90° apart, and they are connected in series between the collecting rings. Calculate the electromotive force induced in the four wires when they are moving under the pole pieces. Plot the electro-

motive force curve of the alternator, and find its form factor. Ans. 9.6 volts, form factor 1.29.

17. The armature winding of the above alternator consists of 100 conductors arranged in four bands, each band 30° wide. These four bands replace the four single conductors described in problem 16, and all 100 conductors are in series between the collecting rings. Calculate the electromotive force of the alternator when the bands of conductors are wholly under the pole pieces; calculate the electromotive force when the bands are half under the pole pieces. Plot the electromotive force curve of the alternator and find its form factor. Speed and dimensions of alternator the same as in problem 16. Ans. 240 volts, 120 volts, form factor 1.165.

18. The field intensity in the gap space in problem 16 changes uniformly from 4,500 units under the leading pole tips to 7,500 units under the trailing pole tips. Plot the electromotive force curve of the four conductor winding and find its form factor. Speed and dimensions of alternator the same as in problem 16. Ans. form factor = 1.38.

19. The adjacent field pole tips of an alternator touch each other, the field intensity in the gap space is uniform, and the armature winding consists of one concentrated bundle of conductors for each field pole. What is the form factor? Ans. form factor = unity.

20. The electromotive force of an alternator passes through a complete cycle of values, while the magnetic flux, through an armature coil, passes through a cycle of values, starting out from any given value and coming back to that value again. Show that the electromotive force of an inductor alternator of the type shown in Fig. 11 passes through $2p$ cycles per revolution of the set of inductors, p being the number of pairs of field magnet poles, *NS*, *NS*, etc.

21. An alternator has 8 poles, and its speed is 900 revolutions per minute. The flux from one pole is 2,200,000 lines. The

armature winding is concentrated in 8 slots, and it consists of 1,000 conductors, all of which are connected in series. What is the effective electromotive force obtained between the collector rings? Assume the form factor to have the value 1.16. Ans. 3,060 volts.

22. An alternator has 10 poles, and runs at a speed of 1,500 revolutions per minute, generating 2,000 volts. The flux from one pole is 2,250,000 lines. How many turns must there be on the armature if they are all connected in series? Assume the form factor to be 1.16. Ans. 306.

23. The following are instantaneous values, in volts, of the electromotive force of an alternator, taken at equal intervals during an entire cycle: 0, 30, 60, 80, 90, 95, 90, 80, 60, 30, 0, – 30, – 60, – 80, – 90, – 95, – 90, – 80, – 60, – 30 and 0. The corresponding values, in amperes, of the current are: – 65, – 45, – 25, 0, 25, 45, 65, 75, 78, 75, 65, 45, 25, 0, – 25, – 45, – 65, – 75, – 78, – 75 and – 65. Find the instantaneous values of the power, plot the curves of electromotive force, of current and of power, and find the average power.

CHAPTER III.

ALTERNATING AMMETERS, VOLTMETERS AND WATTMETERS.

31. The hot-wire ammeter and voltmeter.*—In these instruments the current to be measured is sent through a stretched wire. The wire, heated by the current, lengthens and actuates a pointer which plays over a divided scale.

The hot-wire instrument, when calibrated by continuous currents, indicates effective values of alternating currents, and when calibrated by continuous electromotive forces it indicates effective values of alternating electromotive forces.

Proof.—Consider an alternating current and a continuous current C which give the same reading. *These currents generate heat in the wire at the same average rate.* This rate is RC^2 for the continuous current and $R \times$ average i^2 for the alternating current, i being the instantaneous value of the alternating current. Therefore $RC^2 = R \times$ average i^2 or $C^2 =$ average i^2 or $C = \sqrt{\text{average } i^2}$. Q. E. D.

The proof for electromotive forces is similar to this proof for currents.

Remark.—The readings of a hot-wire voltmeter cannot be reduced to current by dividing by a constant factor the resistance of the instrument, as can the readings of most other types of voltmeters, inasmuch as the resistance of the instrument varies greatly with the changing temperature of the wire.

32. The electro-dynamometer used as an ammeter.—The essential parts of the electro-dynamometer are shown in Figs. 22 (a) and 22 (b). These figures show the arrangement of the parts in

* All voltmeters, except the electrostatic voltmeter, are essentially ammeters. That is, the electromotive force to be measured produces a current which actuates the instrument. The scale over which the pointer plays may be arranged to indicate either the value of the current or the value of the electromotive force.

Siemens' type of instrument. The coil A is held stationary by the frame of the instrument, while the coil B is mounted at right angles to A and is hung from a suspension. This movable coil is provided with flexible or mercury-cup connections aa, and the current to be measured is sent through both coils in series. The force action between the coils is balanced by carefully twisting a helical spring b, one end of which is attached to the coil B and the other to the torsion head c. The observed angle of twist necessary to bring the swinging coil to its zero position is read off by means of the pointer d and the graduated scale c. The pointer f attached to the coil shows when it has been brought to its zero position. The observed angle of twist of the helical spring affords a measure of the force action between the coils and the current is proportional to the square root of this angle of twist. In other forms of electro-

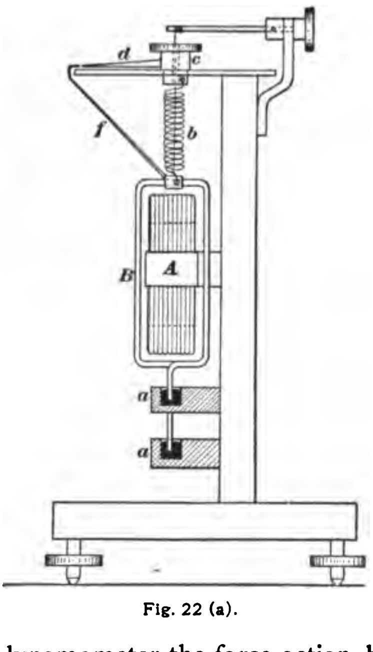


Fig. 22 (a).

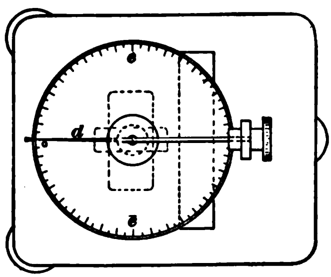


Fig. 22 (b).

dynamometer the force action between the coils moves the suspended coil and causes the attached pointer to play over a divided scale.

The electro-dynamometer when standardized by direct currents indicates effective values of alternating currents.

Proof.—A given deflection of the suspended coil depends upon a definite average or constant force action between the coils. The force action due to a constant cur-

rent c is kc^2 (proportional to c^2), and the average force action due to an alternating current is $k \times$ average i^2, so that if these currents gave the same deflection, we have $kc^2 = k \times$ average i^2, or $c^2 =$ average i^2, or $c = \sqrt{\text{average } i^2}$. Q. E. D.

Remark.—The electro-dynamometer is the standard instrument for measuring alternating currents, and it is always used in accurate measurements.

33. The electro-dynamometer used as a voltmeter.—When used as a voltmeter the coils of the electro-dynamometer are made of fine wire, and an auxiliary non-inductive resistance is usually connected in series with the coils.

When the inductance of the electro-dynamometer coils is small such an instrument, when calibrated by continuous electromotive forces, indicates effective values of alternating electromotive forces.

When it is certain that the inductance of an electro-dynamometer is negligibly small the instrument may be used in refined alternating electromotive force measurements.

Inductance error of the electro-dynamometer used as a voltmeter.—An electro-dynamometer which has been calibrated by continuous electromotive force indicates less than the effective value of an alternating electromotive force. The following discussion of this error for the case of harmonic electromotive force presupposes a knowledge of Chapters IV. and V. Let $\mathbf{E}$ be the reading of an electro-dynamometer voltmeter, when an alternating electromotive force (harmonic), of which the effective value is E, is connected to its terminals, that is, $\mathbf{E}$ is the continuous electromotive force, which gives the same deflection as E, and, since E gives the same deflection as $\mathbf{E}$, it follows that the effective current produced by E is equal to the continuous current produced by $\mathbf{E}$, that is,

$$\frac{\mathbf{E}}{R} = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \tag{22}$$

in which R is the total resistance of the instrument, L its inductance, and $\omega = 2\pi f$, where f is the frequency of the alternating electromotive force. Solving equation (22) for E we have

$$E = \frac{\sqrt{R^2 + \omega^2 L^2}}{R} \cdot \mathbf{E} \tag{23}$$

that is, the reading of the instrument must be multiplied by the factor

$$\frac{\sqrt{R^2 + \omega^2 L^2}}{R}$$

to give the true effective value of an harmonic alternating electromotive force.

Plunger type voltmeters have inductance errors also.

34. The electrostatic voltmeter.—Two insulated metal plates, which are connected to the terminals of a battery, or to any source of electromotive force, attract each other with a force which is strictly proportional to the square of the electromotive force. This principle is applied in the *electrostatic voltmeter*, which consists essentially of a fixed plate and a suspended plate to which a pointer is attached. The terminals of the electromotive force to be measured are connected to these plates.

Such an instrument, when calibrated by continuous electromotive force, indicates effective values of alternating electromotive force.

Proof.—A given deflection of the suspended plate depends upon a definite average or constant force action between the plates. The force action due to a constant electromotive force E is KE^2 (proportional to E^2), and the average force action due to an alternating electromotive force e is $K \times$ *average* e^2. If these electromotive forces give equal deflections the force KE^2 is equal to the average force $K \times$ *average* e^2 so that $E^2 =$ *average* e^2, or $E = \sqrt{\text{average } e^2}$. Q. E. D.

The electrostatic voltmeter is the standard instrument for measuring alternating electromotive forces, especially for the measurement of very high electromotive force. Further, with high electromotive forces the electrostatic attraction of parallel metal plates is great enough to be accurately measured by a balance and in this case the electromotive force between the plates (constant electromotive force, or effective value of an alternating electromotive force) may be calculated independently of calibrations of any kind. An instrument arranged for the absolute* measurement of electromotive force in this way is called an *absolute electrometer*.

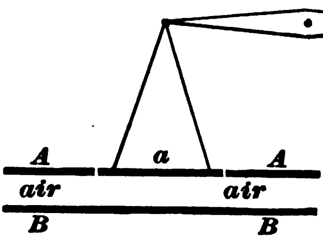


Fig. 23.

The absolute electrometer consists of two parallel metal plates, *AaA* and *BB*, Fig. 23. The central portion *a* of the upper

* That is, the measurement in terms of mechanical units of force, distance, etc.

plate, while remaining in electrical communication with AA, is detached and suspended from one arm of a balance beam as shown. The electromotive force E between AaA and BB, in volts, is given by the formula

$$E^2 = \frac{720{,}000\pi d^2 F}{a} \tag{24}$$

in which F is the observed downward pull on a in dynes, d is the distance apart of the plates in centimeters and a is the area in cm^2 of the detached portion a.

35. The spark gauge.—The high electromotive forces used in break-down tests are usually measured by means of the *spark gauge*. This consists of an adjustable air gap which is adjusted until the electromotive force to be measured is just able to strike across in the form of a spark. The electromotive force is then taken from empirical tables based upon previous measurements of the electromotive force required to strike across various widths of gap. In the spark gauge of the General Electric Co. the spark gap is between metal points, one of which is attached to a micrometer screw by means of which the gap space may be adjusted and measured. The striking distance in any spark gauge varies greatly with the condition of the points. It is, therefore, necessary to see that the points are well polished before taking measurements.

36. Plunger type ammeters and voltmeters.—In instruments of this type the current to be measured passes through a coil of wire which magnetizes and attracts a movable piece of soft iron to which the pointer is fixed.

A plunger meter (ammeter or voltmeter) should be calibrated under the conditions in which it is to be used. Thus, if a plunger instrument is to be used as an ammeter for alternating currents of a given frequency it should be calibrated by currents of this frequency, these currents being, for the purpose of the calibration, measured by a standard alternating current ammeter, such

as an electrodynamometer. The indications of a plunger instrument do not, however, vary greatly with frequency and such instruments are used for approximate measurements without regard to frequency.

The Thomson inclined coil meter of the General Electric Co. is of the plunger type. The essential parts of this instrument are shown in Fig. 24. A coil *A*, through which flows the current

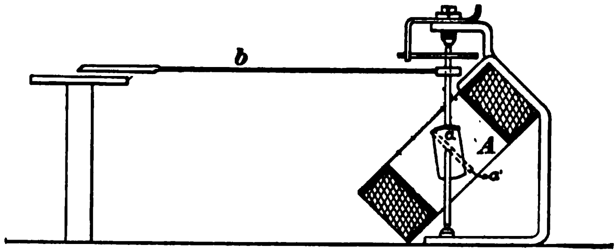


Fig. 24.

to be measured, is mounted with its axis inclined as shown. A vertical spindle mounted in jeweled bearings and controlled by a hair-spring passes through the coil, and to this spindle are fixed a pointer *b* and a vane of thin sheet-iron *a*. This vane of iron is mounted obliquely to the spindle. When the pointer is at the zero point of the scale the iron vane *a* lies nearly across the axis of the coil, and when a current passes through the coil the vane tends to turn until it is parallel to the axis of the coil, thus turning the spindle and moving the attached pointer over the calibrated scale.

37. The potential method for measuring alternating current.— The alternating current to be measured is passed through a known non-inductive resistance *R* and the electromotive force between the terminals of this resistance is measured by a voltmeter. The current (effective value) is then equal to the electromotive force (effective value) divided by the resistance.

The calorimetric method for measuring alternating current.— The current to be measured is passed through a known resist-

ance which is submerged in a calorimeter by means of which the heat H which is generated in the resistance in an observed interval of time t is determined. This heat being expressed in joules we have

$$H = I^2Rt \tag{25}$$

in which I is the effective value of the current.

Measurement of Power in Alternating Circuits.*

38. The three-voltmeter method.—A non-inductive resistance R, Fig. 25, is connected in series with the circuit bc in which the power P, to be determined, is expended. The electromotive forces E_1 between ab, E_2 between bc and E_3 between ac, are observed by means of a voltmeter as nearly simultaneously as possible. Then

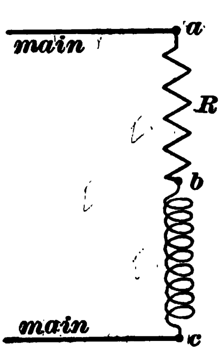


Fig. 25.

$$P = \frac{E_3^2 - E_1^2 - E_2^2}{2R} \tag{26}$$

Proof.—Let e_1, e_2 and e_3, be the instantaneous electromotive forces between ab, bc and ac, respectively, then

$$e_3 = e_1 + e_2 \tag{i}$$

or

$$e_3^2 = e_1^2 + 2e_1e_2 + e_2^2 \tag{ii}$$

or

$$\text{Average } e_3^2 = \text{average } e_1^2 + 2 \text{ average } e_1e_2 + \text{average } e_2^2 \dagger \tag{iii}$$

but $E_3^2 = \text{average } e_3^2$, $E_1^2 = \text{average } e_1^2$ and $E_2^2 = \text{average } e_2^2$. Further $\frac{e_1}{R}$ is the instantaneous current in abc, $\frac{e_1}{R} \cdot e_2$ is the instantaneous power expended in bc and average $\left(\frac{e_1}{R} \cdot e_2\right)$ or $\frac{1}{R} \times$ average (e_1e_2) is the average power P expended in bc so that average $(e_1e_2) = RP$. Therefore equation (iii) becomes

$$E_3^2 = E_1^2 + 2RP + E_2^2 \tag{iv}$$

or

$$P = \frac{E_3^2 - E_1^2 - E_2^2}{2R} \qquad \text{Q. E. D.}$$

* In alternating circuits power cannot be measured by means of an ammeter and a voltmeter as in the case of direct current for the reason that the power expended is in general less than the product of effective electromotive force into effective current on account of the difference in phase of the current and electromotive force.

† For proof of (iii) see proposition, Article 49.

39. The three-ammeter method for measuring power.—The circuit CC, Fig. 26, in which the power P to be measured is expended, is connected in parallel with a non-inductive resistance R, and three ammeters are placed as shown. Then

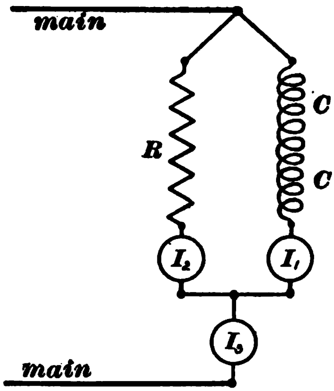


Fig. 26.

$$P = \frac{R}{2}(I_3^2 - I_1^2 - I_2^2) \qquad (27)$$

in which I_1, I_2 and I_3 are the currents indicated by the three ammeters.

Proof.—Let i_1, i_2 and i_3 be the instantaneous values of the currents I_1, I_2 and I_3. Then

$$i_3 = i_1 + i_2 \qquad \text{(i)}$$

or

$$i_3^2 = i_1^2 + 2i_1i_2 + i_2^2 \qquad \text{(ii)}$$

or

$$\text{average } i_3^2 = \text{average } i_1^2 + 2 \text{ average } (i_1i_2) + \text{average } i_2^2 \qquad \text{(iii)}$$

But

$$I_1^2 = \text{average } i_1^2, \quad I_2^2 = \text{average } i_2^2 \quad \text{and } I_3^2 = \text{average } i_3^2.$$

Further, the instantaneous electromotive force between the terminals of R or of CC is Ri_2, so that Ri_2i_1 is the instantaneous power expended in CC, and $R \times$ average (i_1i_2) is the average power P expended in CC. Therefore, average $(i_1i_2) = \frac{P}{R}$ and equation (iii) becomes

$$I_3^2 = I_1^2 + \frac{2P}{R} + I_2^2 \qquad \text{(iv)}$$

or

$$P = \frac{R}{2}(I_3^2 - I_1^2 - I_2^2) \qquad \text{Q. E. D.}$$

Combination method.—The three-ammeter method for measuring power may be modified by using the potential method for measuring I_2, Fig. 26. In this case the electromotive force between the terminals of R, Fig. 26, is measured by means of a voltmeter, so that $I_2 = \frac{E_2}{R}$ where E_2 is the voltmeter reading.

40. The wattmeter.—The wattmeter is an electrodynamometer, of which one coil a, Fig. 27, made of fine wire, is connected to the terminals of the circuit CC, in which the power to be

measured is expended. The other coil b, made of large wire, is connected in series with CC, as shown. The fine wire coil a is movable, and carries the pointer which indicates the watts expended in CC.

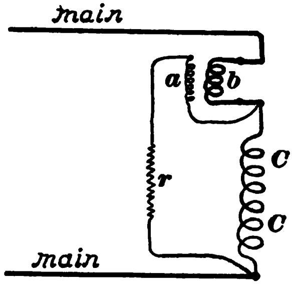


Fig. 27.

*Such an instrument when calibrated with continuous current and electromotive force indicates power accurately when used with alternating currents, provided the inductance of the circuit ar is small.**

Proof.—A given deflection of the movable coil a depends upon a certain average or constant force action between the coils. Consider a continuous electromotive force E which produces a current $\frac{E}{r}$ in a and a current C in CC and b. The force action between the coils is proportional to the product of the currents in a and b, that is, the force action is $k \cdot \frac{E}{r} \cdot C$, where k is a constant.

Consider an alternating electromotive force of which the instantaneous value is e: this produces a current $\frac{e}{r}$ through a (provided the inductance of a is zero) and a current i in CC and b. The instantaneous force action between thecoils is $k \cdot \frac{e}{r} \cdot i$ and the average force action is $\frac{k}{r} \cdot$ average (ei). If this alternating electromotive force gives the same deflection as the continuous electromotive force then

$$\frac{k}{r} \times \text{average } (ei) = EC\frac{k}{r}$$

or average $(ei) = EC$.

That is, the given deflection indicates the same power whether the currents are alternating or direct. Q. E. D.

Remark.—A good wattmeter is the standard instrument for measuring power in alternating-current circuits. The three-ammeter and the three-voltmeter methods are troublesome and slight errors of observation may in some cases lead to very great errors in the result.

The compensated wattmeter of the Weston Electrical Instrument Co.—In the above

* Small, that is, in comparison with $\frac{r}{2\pi f}$; where r is the total resistance of a and r, Fig. 27, and f is the frequency of the alternating current.

proof the current flowing through a and r, Fig. 27, is assumed to be negligible in comparison with the current in CC, so that the current in the coil b is sensibly the same as the current in CC. This is not, however, always the case. In fact the instrument indicates the total power expended in a, in r, and in CC. Let C be the current in CC and let a be the current in a and r. Then the current in b is $C + a$, and the force action upon the movable coil is proportional to the product $a(C + a)$, instead of being proportional to the product aC.

In the compensated wattmeter of the Weston Co. the wire leading over to the coil a, connected as shown, is laid alongside of each and every turn of wire in coil b. Then current $C + a$ flows down through b, current a flows back alongside of the wire of coil b, and the result is the same as if the current a were subtracted from the current $C + a$ so far as the magnetic action of the coil b is concerned.

41. The recording wattmeter is an instrument for summing up the total work or energy expended in a circuit.

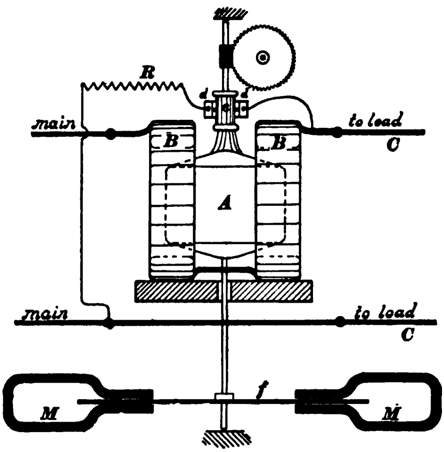


Fig. 28.

The Thomson recording wattmeter is a small electric motor without iron, the field and armature coils of which constitute an electrodynamometer. The field coils BB of this motor, Fig. 28, are connected in series with the circuit CC in which the work to be measured is expended.

The armature A, together with an auxiliary non-inductive resistance R, is connected between the terminals of the circuit CC, as shown. Current is led into the armature by means of the brushes dd pressing on a small silver commutator e.

Discussion of the Thomson recording wattmeter.—The driving torque, acting upon the armature, is proportional to the rate at which work is spent in the circuit CC (*i. e.*, to the power expended, as explained in Article 41). The instrument is so constructed that the speed of the armature is proportional to this driving torque or to the power spent in CC. That is, the rate of turning of the armature is proportional to the rate at which work is done in the circuit CC, so that the total number of revolutions turned by the armature is proportional to the total work expended in the circuit CC.

To make the armature speed proportional to the driving torque the armature is

mounted so as to be as nearly as possible free from ordinary friction and a copper disk *f*, Fig. 28, is mounted on the armature spindle so as to rotate between the poles of permanent steel magnets *MM*. To drive such a disk requires a driving torque proportional to its speed.

The starting coil.—In the above discussion it is assumed that the torque which opposes the motion of the armature *A*, Fig. 28, is proportional to the speed of the armature. In fact, however, this opposing torque may be considered in two parts: 1st, the torque to overcome friction, and 2d, the torque required to overcome the damping action of the magnets on the copper disk. The first part of the torque may be taken to be approximately constant, while the second part is accurately proportional to the speed. Therefore an arrangement for exerting on the armature a constant torque, sufficient to overcome friction, would largely eliminate errors due to friction. This is accomplished in the Thomson meter by supplementing the field coils *B*, Fig. 28, with an auxiliary field coil connected in the armature circuit. This auxiliary field coil is called a *starting coil*. So long as the electromotive force between the mains does not vary, the current in the starting coil is constant and it, therefore, exerts a constant torque upon the armature. If, however, the electromotive force between the mains varies, the torque due to the starting coil varies with the square of the electromotive force.

Remark.—The *induction wattmeter* is a sort of induction motor. It is described in Chapter XIV.

PROBLEMS.

24. The spring of a Siemens electrodynamometer is twisted through an angle of 220° to balance the force action of a current of 18.8 amperes. What current will require a twist of 165°? Ans. 9.35 amperes.

25. The angle of twist of a Siemens electrodynamometer can be read to ¼ of a degree. What are the relative errors in current due to an error of ¼ of a degree when the total twist is 10°

and when the total twist is 100°? Ans. The error, in amperes, is 3.16 times as great in case of the smaller deflection, and the percentage error is ten times as great.

26. The electromotive force e which produces a deflection d scale divisions of an electrostatic voltmeter is approximately: $e = k\sqrt{d}$. What are the relative errors in electromotive force due to an error of $\frac{1}{10}$ of a division in the reading when the total deflection is 10 divisions, and when the total deflection is 100 divisions? Ans. 10 : 1.

27. The fine wire coil of a wattmeter has 500 ohms resistance. The wattmeter indicates 62 watts when used to measure the power delivered to a 110-volt lamp, the fine wire coil being connected to the terminals of the lamp. What is the true power delivered to the lamp? Ans. 38 watts.

28. When a hot-wire voltmeter indicates 50 volts a current of 0.05 ampere flows through the instrument. When the same voltmeter indicates 100 volts, 0.07 ampere flows through the instrument. What is the resistance of the instrument in each case? Ans. 1,000 ohms, 1,428.6 ohms.

29. A Thomson wattmeter without a starting coil starts at 75 watts load. The wattmeter is adjusted to give true record when run at a 500-watt load. What will the instrument indicate when run on a constant load at 200 watts for 4 hours, running friction being assumed to be equal to half the starting friction? Ans. 702.6 watthours.

30. The above wattmeter is provided with a starting coil so as to start with a 40-watt load on 110-volt mains. At what load will the instrument start on 55-volt mains? Ans. 66.25 watts.

31. The above wattmeter with its starting coil is adjusted to read correctly at a load of 500 watts on 110-volt mains. At what load will it read correctly on 55-volt mains? Ans. 5,750 watts.

Suggestion.—Let x be required watts. The total driving torque in first case minus running friction is to be to total driving torque in second case minus running friction as 500 is to x.

CHAPTER IV.

HARMONIC ELECTROMOTIVE FORCE AND CURRENT.

42. Definition of harmonic electromotive force and current.—A line OP, Fig. 29, rotates at a uniform rate, f revolutions per second, about a point O, in the direction of the arrow gh. Consider the projection Ob of this rotating line upon the fixed line AB, this projection being considered positive when above O and negative when below O. *An harmonic electromotive force (or current) is an electromotive force (or current) which is at each instant proportional to the line* Ob, Fig. 29. The line Ob represents at each instant the actual value e of the harmonic electromotive force to a definite scale, and the length of the line OP (which is the maximum length of Ob) represents the maximum value $\boldsymbol{E}$ of the harmonic electromotive force to the same scale. The line Ob passes through a complete cycle of values during one revolution of OP, and so also does the harmonic electromotive force e. Therefore the revolutions per second f of the line OP is the frequency of the harmonic electromotive force e.

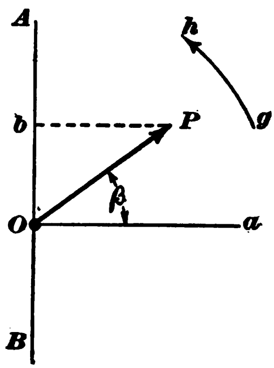


Fig. 29.

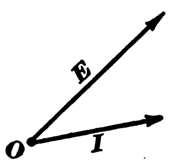


Fig. 30.

The rotating lines $\boldsymbol{E}$ and $\boldsymbol{I}$, Fig. 30, of which the projections on a fixed line (not shown in the figure) represent the actual instantaneous values e and i of an harmonic electromotive force and an harmonic current are said to *represent* the harmonic electromotive force and

current respectively. Of course, the rotation of the lines $\boldsymbol{E}$ and $\boldsymbol{I}$ is a thing merely to be imagined.

43. Algebraic expression of harmonic electromotive force and current.—The line OP, Fig. 29, makes f revolutions per second, and, therefore, it turns through $2\pi f$ radians per second, since there are 2π radians in a revolution, that is

$$\omega = 2\pi f \tag{28}$$

in which ω is the angular velocity of the line OP in radians per second. Let time be reckoned from the instant that OP coincides with Oa, then after t seconds OP will have turned through the angle $\beta = \omega t$, and from Fig. 29 we have

$$Ob = OP \sin \beta = OP \sin \omega t$$

But Ob represents the actual value e of the harmonic electromotive force at the time t and OP represents its maximum value $\boldsymbol{E}$, therefore

$$e = \boldsymbol{E} \sin \omega t \tag{29}$$

is an algebraic expression for the actual value e of an harmonic electromotive force at time t, $\boldsymbol{E}$ being the maximum value of e, and $\frac{\omega}{2\pi}$ being the frequency according to equation (28).

Similarly

$$i = \boldsymbol{I} \sin \omega t \tag{30}$$

is an algebraic expression for the actual value i of an harmonic current at time t, $\boldsymbol{I}$ being the maximum value of i.

Remark 1.—If time is reckoned from the instant that OP, Fig. 29, coïncides with the line Ob, then equations (29) and (30) become

$$e = \boldsymbol{E} \cos \omega t$$

$$i = \boldsymbol{I} \cos \omega t$$

Remark 2.—The curve which represents an harmonic electromotive force or an harmonic current (see Article 24) is a curve of sines.

Remark 3.—A great many alternators, especially those with distributed armature windings, generate electromotive forces which are very nearly harmonic. Calculations in connection with the design of alternating current apparatus are simple enough to be practicable only when the electromotive forces and currents are assumed to be harmonic. Hereafter, then, when speaking of alternating electromotive forces and currents, it will be understood that they are harmonic, unless the contrary is expressly stated.

44. Definitions.* *Cycle.*—A cycle is one complete set of values (positive and negative) through which an electromotive force or current repeatedly passes. The *frequency* is the number of cycles passed through per second. The *period* is the duration of one cycle. For example, an alternator generates electromotive force at a frequency of 60 cycles per second; the period is $\frac{1}{60}$ of a second and the angular velocity of the line OP, Fig. 29, is 60 revolutions per second or 377 radians per second ($= \omega$).

Synchronism.—Two alternating electromotive forces or currents are said to be in synchronism when they have the same frequency. Two alternators are said to run in synchronism when their electromotive forces are in synchronism.

45. Phase difference.—Consider two synchronous harmonic electromotive forces e_1 and e_2. Suppose that e_1 passes through its maximum value before e_2; then e_1 and e_2 are said to *differ in*

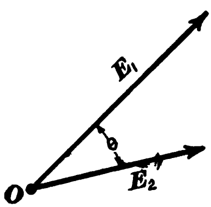


Fig. 31.

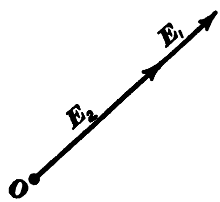


Fig. 32.

* The definitions of cycle and frequency given in Article 23 are here repeated for the sake of clearness. All definitions given in this article apply to alternating currents and electromotive forces of any character as well as to harmonic electromotive forces and currents.

phase. The line E_1, Fig. 31, which represents e_1 must be ahead of the line E_2 which represents e_2 as shown in the figure. The angle θ is called the *phase difference* of e_1 and e_2. The lines E_1 and E_2 are supposed to be rotating about O in a counter-clockwise direction as explained in Article 42.

When the angle θ, Fig. 31, is zero, as shown in Fig. 32, the electromotive forces e_1 and e_2 are said to be *in phase.* In this case the electromotive forces increase together and decrease together; that is, when e_1 is zero e_2 is also zero, when e_1 is at its maximum value so also is e_2, etc.

When $\theta = 90°$, as shown in Fig. 33, the two electromotive forces are said to be *in quadrature.* In this case one electromotive force is zero when the other is a maximum, etc.

When $\theta = 180°$, as shown in Fig. 34, the two electromotive forces are said to be *in opposition.* In this case the two electromotive forces are at each instant opposite in sign and when one is at its positive maximum the other is at its negative maximum, etc.

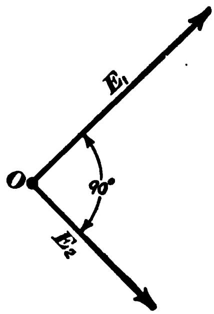


Fig. 33.

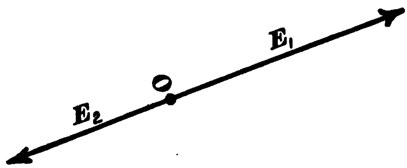


Fig. 34.

46. Composition and resolution of harmonic electromotive forces and currents. (*a*) *Composition.*—Consider two synchronous harmonic electromotive forces e_1 and e_2 represented by the lines E_1 and E_2, Fig. 35. The sum $e_1 + e_2$ is an harmonic electromotive force of the same frequency and it is represented by the line E. This is evident when we consider

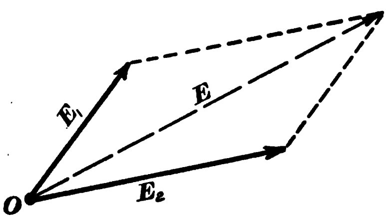


Fig. 35.

that the projection, on any line, of the diagonal of a parallelogram is equal to the sum of the projections of the sides of the parallelogram.

Corollary.—The sum of any number of synchronous electromotive forces (or currents) is another electromotive force (or current) of the same frequency which is represented in phase and magnitude by the line which is the vector sum of the lines which represent the given electromotive forces (or currents). Thus the lines E_1, E_2 and E_3, Fig. 36, represent three given synchronous harmonic electromotive forces and the line E (the vector sum of E_1, E_2 and E_3) represents an harmonic electromotive force which is the sum of the given electromotive forces.

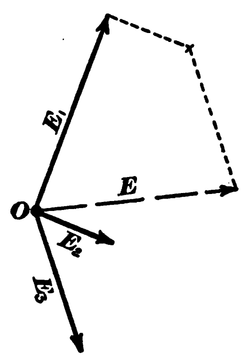


Fig. 36.

(*b*) *Resolution.*—A given harmonic electromotive force (or current) may be broken up into a number of harmonic parts of the same frequency by reversing the process of composition. For example, the line E, Fig. 36, represents a given harmonic electromotive force which may be split up into the three electromotive forces represented by the lines E_1, E_2 and E_3.

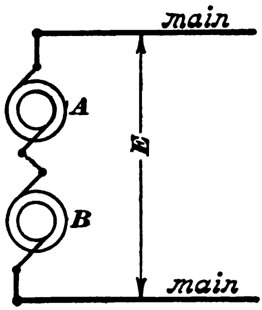


Fig. 37.

47. Examples of composition and resolution.

(*a*) Two alternators A and B running

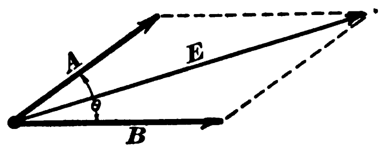


Fig. 38.

in synchronism are connected in series between the mains as shown in Fig. 37. If the electromotive forces of A and B are in phase the electromotive force between the mains will be simply the numerical sum of the electromotive forces of A and B. If,

on the other hand, the electromotive forces of A and B differ in phase the state of affairs will be as represented in Fig. 38; in which the lines A and B represent the electromotive forces of the alternators A and B respectively, θ is the phase difference of A and B, and the line $\boldsymbol{E}$ represents the electromotive force between the mains.

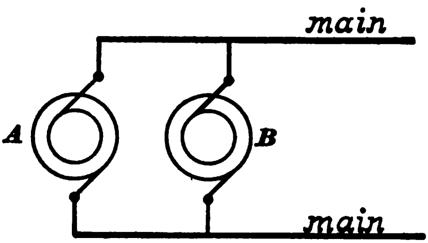


Fig. 39.

(*b*) Two alternators A and B running in synchronism are connected in parallel between the mains as shown in Fig. 39. Let the lines A and B, Fig. 40, represent the currents given by A and B respectively, the phase difference being θ; then the current in the main line is represented by $\boldsymbol{I}$.

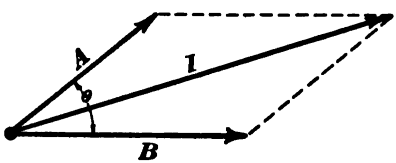


Fig. 40.

(*c*) Two circuits A and B are connected in series between the mains of an alternator as shown in Fig. 41. The line $\boldsymbol{E}$, Fig. 42, represents the electromotive force between the mains, the line A represents the electromotive force between the terminals of the circuit A, and the line B represents the electromotive force between the terminals of the circuit B. The circuits A and B are supposed to have inductance. If one of the circuits A or B contains a condenser, then the electromotive forces A and B, Fig. 42, may be nearly opposite to each other in phase,

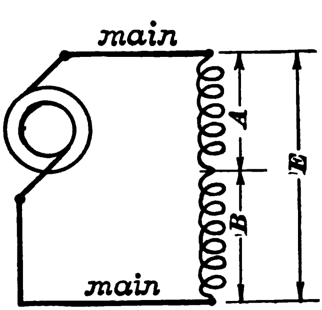


Fig. 41.

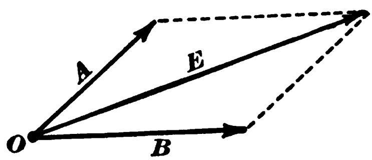


Fig. 42.

and A and B may each be indefinitely greater than the electromotive force E between the mains.

(d) Two circuits A and B, Fig. 43, are connected in parallel across the terminals of an alternator as shown. The current I from the alternator is related to the currents A and B as shown in Fig. 44. If one of the circuits A or B contains

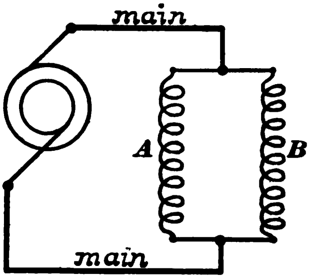


Fig. 43.

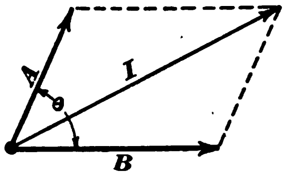


Fig. 44.

a condenser, then the currents A and B may be nearly opposite to each other in phase and the currents A and B may each be indefinitely greater than the current I from the alternator.

48. Rate of change of harmonic electromotive forces and currents.—Consider the harmonic current [see equation (30)]:

$$i = \boldsymbol{I} \sin \omega t \tag{a}$$

When this current is sent through an inductive circuit an electromotive force $L\frac{di}{dt}$ is at each instant required to make the current increase or decrease. In the study of alternating currents in inductive circuits it is, therefore, necessary to consider the rate of change $\frac{di}{dt}$ of the current.

Differentiating the above expression for i with respect to time we have

$$\frac{di}{dt} = \omega \boldsymbol{I} \cos \omega t \tag{b}$$

or

$$\frac{di}{dt} = \omega \boldsymbol{I} \sin (\omega t + 90) \tag{31}$$

This equation shows that the *rate of change* $\frac{di}{dt}$ of an harmonic current may be represented by the projection* of the line ωI, Fig. 45, which is 90° ahead of the line I which represents the current.

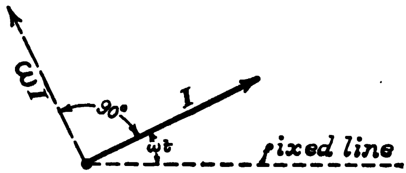


Fig. 45.

The relation of i and $\frac{di}{dt}$ is most clearly shown by the sine curve diagram. Thus the full-line curve, Fig. 46, represents the harmonic current i. The steepness of this curve at each point represents the value of $\frac{di}{dt}$. The steepness of this curve is greatest at the

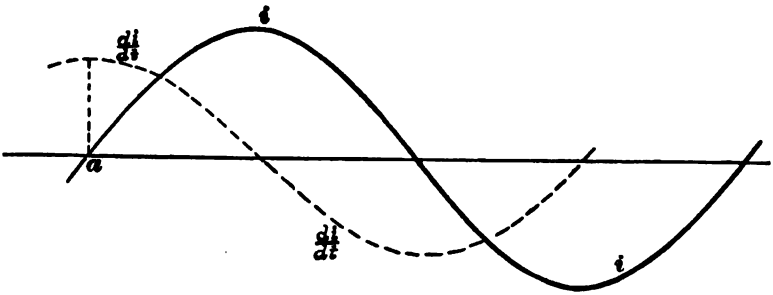


Fig. 46.

point a where the curve crosses the axis, hence the value of $\frac{di}{dt}$ is a maximum 90° before i reaches its maximum. The ordinates of the dotted curve represent the values of $\frac{di}{dt}$.

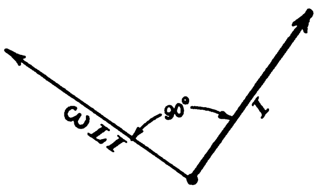


Fig. 47.

Remark.—It is to be noted that the portion $L\frac{di}{dt}$ of the total electromotive force acting on the circuit, which is used to cause the current to increase and decrease, is represented by the line ωLI, Fig. 47; the line I represents the current in the circuit.

* On a vertical fixed line not shown in the figure.

49. Average or mean value of an harmonic electromotive force or current.—Consider any varying quantity y. Its average value during an interval of time from t' to t'' is $\frac{\Sigma y \Delta t}{t'' - t'}$ the summation being extended throughout the interval. That is,

$$Av. y = \frac{1}{t'' - t'} \int_{t'}^{t''} y dt \qquad (32)$$

If the successive values of y be represented by the ordinates of the curve, Fig. 48, and the corresponding values of the time t be represented by the abscissas, then $\int_{t'}^{t''} y dt$ is the area of the shaded portion and $\frac{1}{t'' - t'} \int_{t'}^{t''} y dt$ is the height of a rectangle $t't''dc$ of the same area as the curve and having the same base. The average ordinate of such a curve as Fig. 48 may be obtained quite closely by measuring the lengths of a number of equidistant ordinates. The sum of these ordinates divided by the number of ordinates gives the average ordinate of the curve.

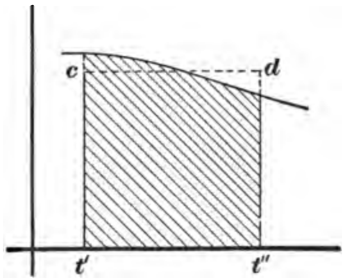


Fig. 48.

Proposition.—The average value of the sum of a number of quantities is equal to the sum of the average values of each.

Proof.—Let $x, y, z, \ldots$ be the quantities. Then, by definition we have

$$Av. (x + y + z + \ldots) = \frac{1}{t'' - t'} \int_{t'}^{t''} (x + y + z \ldots) dt \qquad \text{(i)}$$

but

$$\frac{1}{t'' - t'} \int_{t'}^{t''} (x + y + z \ldots) \, dt = \frac{1}{t'' - t'} \int_{t'}^{t''} x dt + \frac{1}{t'' - t'} \int_{t'}^{t''} y dt + \ldots$$
$$= Av. \, x + Av. \, y + \ldots \qquad \text{(ii)}$$

Therefore

$$Av. (x + y + z \ldots) = Av. \, x + Av. \, y + Av. \, z + \ldots \qquad (33)$$

Q. E. D.

50. Proposition.—The average value of an harmonic electromotive force or current during half a cycle

$$\left(\omega t = 0 \text{ to } \omega t = \pi, \text{ or } t = 0 \text{ to } t = \frac{\pi}{\omega}\right)$$

is

$$\frac{2 \; maximum \; value}{\pi}$$

Proof.—Let $e = E \sin \omega t$ be the harmonic electromotive force. Substitute $E \sin \omega t$ for y in equation (32) and we have

$$\text{Av. } e = \frac{E}{t'' - t'} \int_{t'}^{t''} \sin \omega t \, dt$$

Substituting x for ωt, and remembering that the limits are from $\omega t = 0$ to $\omega t = \pi$, we have

$$\text{Av. } e = \frac{E}{\pi} \int_0^{\pi} \sin x \, dx = \frac{E}{\pi} \Big[_0^{\pi} \cos x \cdot = \frac{2E}{\pi} \qquad (34)$$

or the average value of the harmonic electromotive force is twice the maximum value divided by π. Since $\frac{2}{\pi} = .636$, it may also be stated that the average value of an harmonic electromotive force, or current, is 0.636 times the maximum value.

Remark.—The average value of an harmonic electromotive force or current during one or more whole cycles is zero.

51. Proposition.—The square root of mean square, or effective value, of an alternating electromotive force or current during one or more whole cycles is equal to $\frac{\text{maximum value}}{\sqrt{2}}$ or $0.707 \times$ maximum.

Proof.—Let $e = E \sin \omega t$ be a harmonic electromotive force. To find the average value of $e^2 = E^2 \sin^2 \omega t$ it is necessary to find the average value of the square of the sine of the uniformly variable angle ωt. We have the general relation

$$\sin^2 \omega t + \cos^2 \omega t = 1 \qquad (a)$$

so that by equation (33)

$$\text{Av. } \sin^2 \omega t + \text{Av. } \cos^2 \omega t = 1 \qquad (b)$$

Now, the cosine of a uniformly variable angle passes similarly through the same set of values during a cycle as the sine, hence Av. $\sin^2 \omega t$ and Av. $\cos^2 \omega t$ are equal, so that from (b) we have:

$$2 \text{ Av. } \sin^2 \omega t = 1$$

or

$$\text{Av. } \sin^2 \omega t = \tfrac{1}{2}$$

The average value of e^2 is

$$\text{Av. } e^2 = E^2 \text{ Av. } \sin^2 \omega t$$

or

$$\text{Av. } e^2 = \frac{E^2}{2}$$

and

$$\sqrt{\text{Av. } e^2} = \frac{E}{\sqrt{2}} \qquad (35)$$

Q. E. D.

Note.—The square root of mean square value of an harmonic electromotive force or current is often spoken of as the *effective*

value of the electromotive force or current. When it is stated that an alternating current is so many amperes, the effective or square root of mean square value is always meant. The same is also true with regard to alternating electromotive forces. Hereafter the symbol E will be used to designate the effective value of an electromotive force and I to designate the effective value of an alternating current. *In case the currents and electromotive forces are harmonic* we have the relations

$$E = \frac{\mathbf{E}}{\sqrt{2}} \tag{36}$$

$$I = \frac{\mathbf{I}}{\sqrt{2}} \tag{37}$$

in which $\mathbf{E}$ and $\mathbf{I}$ are the maximum values of the electromotive force and current respectively.

Note.—The form factor of an harmonic electromotive force (see Article 27) is equal to $\frac{707}{636}$ or to 1.11.

52. Power.—As pointed out in Article 26, Chapter II., the power developed by an alternating electromotive force pulsates and in most practical problems it is the average power developed which is the important consideration. Let $e = \mathbf{E} \sin \omega t$ be an harmonic electromotive force acting on a circuit and $i = \mathbf{I} \sin (\omega t - \theta)$ the current produced in the circuit; θ being the difference in phase of the electromotive force and current as shown in Fig. 49. The power developed at a given instant is ei and in order to estimate the average power developed we must find an expression for the average value of ei. We have

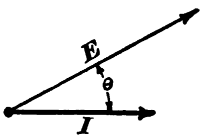


Fig. 49.

$$ei = \mathbf{EI} \sin \omega t \sin (\omega t - \theta)$$

or since $\sin (\omega t - \theta) = \sin \omega t \cdot \cos \theta - \cos \omega t \cdot \sin \theta$, we have

$$ei = \mathbf{EI} \cos \theta \cdot \sin^2 \omega t - \mathbf{EI} \sin \theta \cdot \sin \omega t \cdot \cos \omega t$$

Hence by equation (33)

Average $ei = \mathbf{EI} \cos \theta$ Av. $\sin^2 \omega t - \mathbf{EI} \sin \theta$ Av. $\sin \omega t \cos \omega t$.

The average value of sin ωt cos ωt is zero since it passes through positive and negative values alike. The average value of $\sin^2 \omega t$ is ½. Therefore,

$$\text{Average } ei = \text{Power} = \frac{EI}{2} \cos \theta \qquad (38)$$

It is more convenient to have this product expressed in terms of the effective values of the current and electromotive force. Hence substitute for E and I their values given by equations (36) and (37) and we have

$$\text{Power} = EI \cos \theta \qquad (39)$$

Power factor.—The factor cos θ which depends upon the inductance and resistance of the circuit which is receiving the power (see Article 54) is called the *power factor* of the circuit.

Remark.—The expression for the instantaneous power, namely ei, may be reduced to the form

$$p = P - A \cos (2\omega t - \theta)$$

in which p is the instantaneous power ei, P is the average power, and A is equal to $P \div \cos \theta$. Therefore p consists of a constant part and a part which alternates at a frequency twice as great as the frequency of e and i.

Steinmetz method of representing an harmonic alternating electromotive force or current. In Fig. 16 the successive instantaneous values of electromotive force (and of current) are represented by the varying length of a radius vector as explained in Article 25. The curve traced by the end of the radius vector is a circle when the electromotive force which is represented is harmonic. *Steinmetz uses the diameter of this circle to represent the alternating electromotive force; and when, in Steinmetz's notation, a current is behind an electromotive force in phase the line which represents the current is reached by a counter-clockwise rotation of the line which represents the electromotive force.*

The proof of the propositions concerning the composition and resolution of harmonic electromotive forces and harmonic currents (see Article 46) are somewhat more obscure in Steinmetz's

method of representation than they are in the method of representation used in this text.

PROBLEMS.

32. An harmonic alternating current of 65 amperes, frequency 133 cycles per second, flows through a circuit. What is the maximum rate of change of the current, and what is the value of the current at the instant of maximum rate of change? The circuit contains an inductance of 0.08 henry. What is the maximum value of the electromotive force which is required to make the current change? Ans. 76,800 amperes per second, 0 amperes, 6,144 volts.

33. What is the average value during half a cycle of an alternating current of maximum value 10 amperes? This alternating current, having a frequency of 60 cycles per second, flows into and out of a condenser. When the current passes through the value zero the charge on the condenser is $+q$, and when the current next reaches zero the charge on the condenser is $-q$. What is the value of q in coulombs? What would be the value of q if the frequency were twice as great? Ans. 6.37 amperes, 0.0264 coulomb, 0.0132 coulomb.

34. Two alternators A and B are connected in series. The electromotive force of A is 1,100 volts, and the electromotive force of B is 1,200 volts. The electromotive force of A is 90° ahead of the electromotive force of B in phase. What is their combined electromotive force? The two alternators give a current of 125 amperes which lags 30° behind their resultant electromotive force in phase. What is the power output of each alternator? Ans. 1,628 volts, 29,750 watts, 146,470 watts.

35. The electromotive force of alternator A, problem 34, is 135° ahead of the electromotive force of alternator B in phase. A current of 120 amperes flows through both alternators, lagging 30° behind their resultant electromotive forces. What is the out-

put of each alternator? Ans. 28,750 watts negative, 121,500 watts positive.

36. A harmonic electromotive force $e = E \sin \omega t$ produces in a circuit a current $i = I \sin (\omega t - \theta)$; that is, this current is $\theta°$ behind the electromotive force in phase. Show that the instantaneous power p is equal to

$$P - \frac{P}{\cos \theta} \cdot \cos (2\omega t - \theta)$$

where P is the average power.

37. An alternator supplies 20 amperes of current at 1,100 volts to a receiving circuit of which the power factor is 0.85 at a given frequency. Find the values of a, b, and θ in the expression: *instantaneous power* $= a - b \cos (2\omega t - \theta)$. Ans. $18{,}700 = a$, $22{,}000 = b$, $31.8° = \theta$.

38. An alternator having a harmonic electromotive force of 140 volts delivers 200 amperes to a circuit of which the power factor is 0.70. Find the maximum positive and maximum negative values of ei. Ans. 47,600 watts, − 8,400 watts.

39. An alternator delivers 200 amperes of current to glow lamps, and 75 amperes to start an induction motor. The power factor of the motor while starting is 0.3. Find the total current. Ans. 233.7 amperes.

40. Two alternators each give 120 volts effective electromotive force 90° apart in phase. The two machines are connected in series and they deliver 200 amperes of current to a receiving circuit. This current lags in phase 15° behind the resultant electromotive force of the two machines. Find the total power delivered by the two machines, and find the power delivered by each. Ans. 32,775, 12,000, 20,775 watts.

CHAPTER V.

FUNDAMENTAL PROBLEMS IN ALTERNATING CURRENTS.

53. Problem III.*—To determine the electromotive force required to maintain a harmonic alternating current in a non-inductive circuit. Let

$$i = I \sin \omega t \qquad (a)$$

be the given harmonic current. The required electromotive force e is used wholly to overcome the resistance R of the circuit and it is, therefore, equal to Ri so that

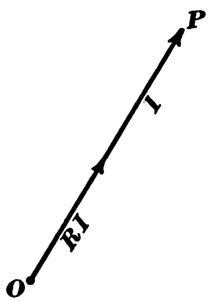


Fig. 50.

$$e = RI \sin \omega t \qquad (b)$$

This electromotive force is harmonic, its maximum value is RI and it is in phase with the current i. Thus the line I, Fig. 50, represents the given harmonic alternating current and the line RI represents the electromotive force required to maintain the given current in a non-inductive circuit.

54. Problem IV.—To determine the electromotive force required to maintain a harmonic alternating current in a circuit of resistance R and inductance L. Let

$$i = I \sin \omega t$$

be the given harmonic current. The required electromotive force consists of two parts, namely:

1. The part used to overcome the resistance of the circuit.

This part is at each instant equal to Ri; it is in phase with i and its maximum value is RI.

* Problems I. and II. are given in Chapter I.

2. The part used to make the current increase and decrease, or briefly to overcome the inductance. This part is at each instant equal to $L\frac{di}{dt}$ according to equation (3); it is 90° ahead of i in phase (see Article 48) and its maximum value is $\omega \boldsymbol{LI}$. Let the given harmonic alternating current i be represented by the line $\boldsymbol{I}$, Fig. 51. Then $R\boldsymbol{I}$ is the line which represents Ri; $\omega L\boldsymbol{I}$ is the line which represents $L\frac{di}{dt}$, and the line $\boldsymbol{E}$ represents the total electromotive force required to maintain the given current. From the diagram we have

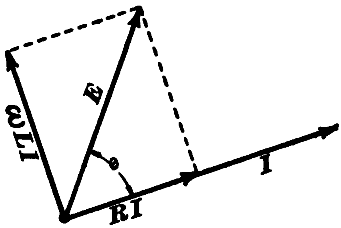


Fig. 51.

$$\boldsymbol{E} = \boldsymbol{I}\sqrt{R^2 + \omega^2 L^2} \tag{40}$$

in which $\boldsymbol{E}$ is the maximum value of the required electromotive force, and further

$$\tan\theta = \frac{\omega L}{R} \tag{41}$$

in which θ is the phase difference between the electromotive force and current.

The effective value of the electromotive force is $E = \frac{\boldsymbol{E}}{\sqrt{2}}$, and the effective value of the current is $I = \frac{\boldsymbol{I}}{\sqrt{2}}$ [by equations (36) and (37)], therefore substituting $\sqrt{2}E$ for $\boldsymbol{E}$ and $\sqrt{2}I$ for $\boldsymbol{I}$ in equation (40), we have

$$E = I\sqrt{R^2 + \omega^2 L^2} \tag{42}$$

When ωL is very small compared with R, the effect of inductance is negligible, and this problem reduces to problem III. When ωL is very large compared with R, the angle θ approaches 90° and the power $EI\cos\theta$ becomes very small, although E and I may both be considerable. In this case the current, lagging as it does 90° behind the electromotive force, is

called a *wattless current*. Thus the alternating current in a coil of wire wound on a laminated iron core is approximately wattless.

Corollary.—The current which is maintained in an inductive circuit by a *given* harmonic alternating electromotive force is a current of which the effective value is $\dfrac{E}{\sqrt{R^2+\omega^2L^2}}$ by equation (42), and which lags behind the electromotive force, by the angle of which the tangent is $\dfrac{\omega L}{R}$ by equation (41).

Remark.—The relation between maximum values of electromotive force and current (harmonic) is in every case the same as the relation between effective values, and henceforth effective values will, as a rule, be used in equations and diagrams. Maximum values will be indicated in the text by bold-faced letters, $\boldsymbol{E}$, $\boldsymbol{I}$, $\boldsymbol{Q}$, etc.; effective values by the letters E, I, etc., and instantaneous values by e, i, q, etc.

55. Problem V.—To determine the current in an inductive circuit immediately after an harmonic electromotive force, $\boldsymbol{E}\sin\omega t$, is connected to the circuit.

The current which can be maintained by the given electromotive force is

$$i' = \frac{\boldsymbol{E}}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\theta) \tag{a}$$

according to problem IV.; and the decaying current

$$i'' = C\epsilon^{-\frac{R}{L}t} \qquad (11)\ bis$$

can exist in the circuit independently of all outside electromotive force, C being a constant, as shown by Problem I., Chapter I. Therefore the current which can exist in an inductive circuit upon which an harmonic electromotive force acts $i = i' + i''$ or

$$i = \frac{\boldsymbol{E}}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t-\theta) + C\epsilon^{-\frac{R}{L}t} \tag{43}$$

in which ϵ is the Napierian base, θ is the angle defined by equation (41) and C is a constant. This constant C is determined by the condition that i is equal to zero at the instant when the electromotive force is connected to the circuit. Let t' be the given instant at which the harmonic electromotive force begins to act upon the circuit. Substitute the pair of values $\left\{\begin{matrix} t=t' \\ i=0 \end{matrix}\right\}$ in equation (43) and solve for C, the only unknown quantity; then substituting this value of C in equation (43) we have the expression for the actual current which flows in the circuit during the time that the maintained current is being established. In a very short time after the electromotive force is connected to the circuit the second term of equation (43) disappears and the

value of the current at each instant is given by the first term, which expresses the current which the given electromotive force can maintain.

56. Problem VI.—A coil of resistance R and inductance L, and a condenser of capacity C are connected in series across alternating current mains as shown in Fig. 52. An alternating current flows back and forth through the coil and charges the condenser in one direction and the other

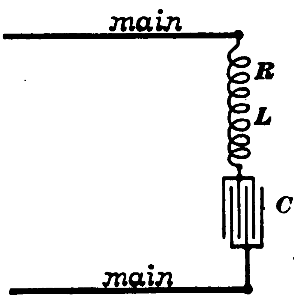


Fig. 52.

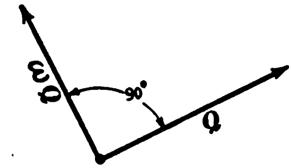


Fig. 53.

alternately. The problem of finding the relation between the current in the coil and the electromotive force between the mains is reduced to its simplest form as follows:

To determine the electromotive force necessary to make the charge q on the condenser vary so that

$$q = Q \sin \omega t \tag{a}$$

in which t is elapsed time, ωt is an angle increasing at a constant rate, and Q is the maximum value of the charge in the condenser. This varying charge may be represented by the projection of the rotating line Q, Fig. 53. The current in the circuit is the rate, $\frac{dq}{dt}$, at which the charge on the condenser changes. That is

$$i = \frac{dq}{dt} \tag{b}$$

or from equation (a) we have

$$i = \omega Q \cos \omega t \tag{c}$$

That is, the current is 90° ahead of q in phase, its maximum value is ωQ and it is represented by the line ωQ, Fig. 53. Using the symbol I for the maximum value of the current we have

$$I = \omega Q \tag{d}$$

The required electromotive force is at each instant used in part to overcome the resistance R of the coil, in part to cause the current to increase and decrease in the coil, and in part to hold the charge on the condenser.

1. The first part is equal to Ri at each instant. It is in phase with i, and its maximum value is $\boldsymbol{RI}$.

2. The second part is equal to $L\frac{di}{dt}$ at each instant. It is 90° ahead of the current and its maximum value is $\omega L\boldsymbol{I}$.

3. The third part is equal to $\frac{q}{C}$ * at each instant. It is in phase with q or 90° behind the current, and its maximum value is $\frac{\boldsymbol{Q}}{C}$ or $\frac{\boldsymbol{I}}{\omega C}$, from equation ($d$).

Let the line $\boldsymbol{I}$, Fig. 54, represent the current. Then $\boldsymbol{RI}$ represents the portion of the electromotive force used to overcome resistance, the line $\omega L\boldsymbol{I}$ 90° ahead of the current represents the electromotive force required to overcome inductance and the line $\frac{\boldsymbol{I}}{\omega C}$ 90° behind the current represents the electromotive force required to hold the charge on the condenser. The line $\boldsymbol{E}$ which is the vector sum of $\boldsymbol{RI}$, $\omega L\boldsymbol{I}$ and $\frac{\boldsymbol{I}}{\omega C}$ represents the total required electromotive force. From the right triangle of which the sides are $\boldsymbol{RI}$, $\omega L\boldsymbol{I} - \frac{\boldsymbol{I}}{\omega C}$, and $\boldsymbol{E}$, we have

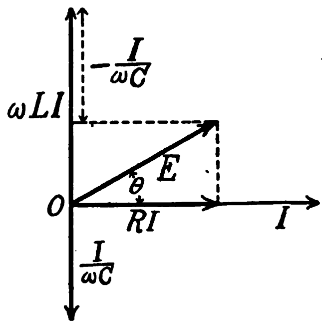


Fig. 54.

$$\boldsymbol{E} = \boldsymbol{I}\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

or

$$E = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{44}$$

* Since $q = Ce$ according to equation (15), Chapter I.

and

$$\tan\theta = \frac{\omega L - \dfrac{1}{\omega C}}{R} \qquad (45)$$

Corollary.—A given harmonic electromotive force acting upon a circuit containing a condenser of capacity C, inductance L and resistance R maintains a current of which the effective value is

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}} \qquad (46)$$

and which lags behind the electromotive force by the angle θ defined by equation (45). The quantity $\omega L - \dfrac{1}{\omega C}$ may be either positive or negative according as ωL or $\dfrac{1}{\omega C}$ is the greater so that the current may be either behind or ahead of the electromotive force in phase. In fact the limiting values of θ are $\pm\ 90°$.

Connection of condensers in series.—When a current I flows through two condensers in series the electromotive forces $I/\omega C$ and $I/\omega C'$ are in phase with each other, so that the total electromotive force required to overcome the reaction of the two condensers is

$$E = \frac{I}{\omega C} + \frac{I}{\omega C'}$$

The oscillatory current.—If $\omega L - \dfrac{1}{\omega C} = 0$ then the impressed electromotive force has only to overcome the resistance of the circuit, and problem VI. reduces in form to problem III. If the resistance of the circuit in this case were negligibly small then no electromotive force at all would be required to maintain the given harmonic current. Such a self-sustained harmonic current is called an *oscillatory current.* In this case, from $\omega L - \dfrac{1}{\omega C} = 0$, we have

$$\omega = \sqrt{\frac{1}{LC}} \qquad (47)$$

or since $\omega = 2\pi f$ [equation (28)] we have

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{48}$$

This equation expresses what is called the *proper frequency* of oscillation of the inductive circuit of a condenser. In case the resistance of the circuit is not zero, which is of course the only real case, then the only current which can exist in the circuit independently of any impressed electromotive force is a *decaying oscillatory current* the discussion of which is beyond the scope of this text. The character of this decaying oscillatory current is shown by the curve, Fig. 55. The ordinates of this curve represent the successive values of the current produced when a charged condenser is discharged through an inductive circuit.

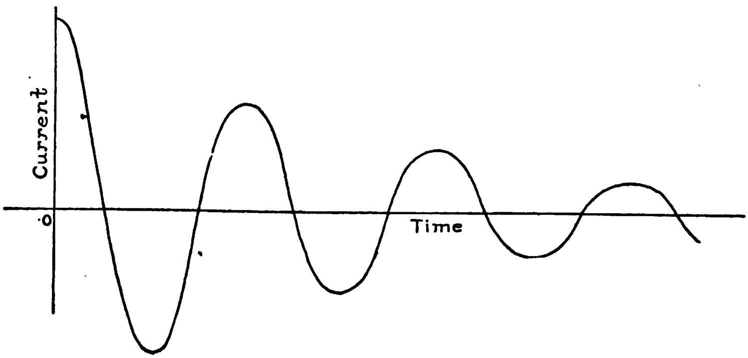


Fig. 55.

57. Electric resonance.—By inspecting equation (46) we see that an electromotive force of given effective value E produces the greatest current in the circuit, Fig. 52 (R, L and C given), when the frequency is such that $\omega L - \frac{1}{\omega C}$ is zero. *This production of a greatest current* by a given electromotive force at a *critical frequency* is called *electric resonance.* Thus the ordinates of the curve, Fig. 56, represent the values of effective current at various frequencies (abscissas). The curve is based on the values $E = 200$ volts, $R = 2$ ohms, $L = .352$ henry and $C = 20$

microfarads. The maximum point of the curve is not a cusp, as would appear from the figure, but the maximum is so sharply defined that it cannot be properly represented in so small a figure. When the frequency of the electromotive force is zero,

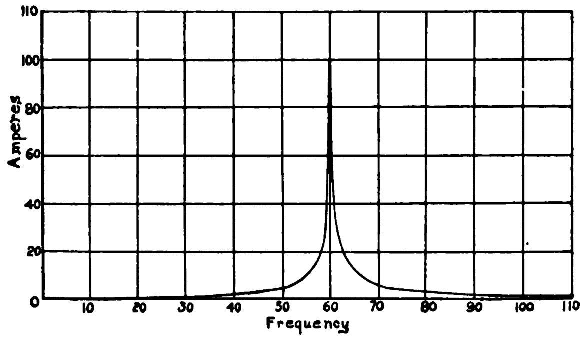


Fig. 56.

which is the case when a continuous electromotive force acts on the circuit, the current is zero except for the very slight current, which is conducted through the dielectric between the condenser plates. When the frequency of the electromotive force is very great the current approaches zero, inasmuch as a very small current of high frequency must increase and decrease at a very rapid rate, and to produce this rapid increase and decrease a very great electromotive force is required. At low frequency the current is kept down in value by the condenser, and at high frequency the current is kept down in value by the inductance.

Remark 1.—At critical frequency, $\omega L - \frac{1}{\omega C} = 0$ and equation (46) becomes simply $I = \frac{E}{R}$.

Remark 2.—While an electromotive force is being established between the plates of a condenser the dielectric is subjected to an increasing electrical stress and this increasing electrical stress is exactly equivalent, in its magnetic action, to an electric current flowing through the dielectric from plate to plate. Magnetically, therefore, a circuit containing a condenser is a complete circuit.

Increasing (or decreasing) electrical stress is called *displacement current.*

Multiplication of electromotive force by resonance.—When resonance exists in a circuit containing an inductance and a condenser in series, the alternating electromotive force ωLI between the terminals of the inductance, and the alternating electromotive force $I/\omega C$ between the terminals of the condenser, may each be much greater than the alternating electromotive force which acts upon the circuit. This fact is easily understood by means of the mechanical analogue. If even a very weak periodic force act upon a weight which is suspended by a spiral spring, the weight will be set into violent vibration, provided the frequency of the force is the same as the proper frequency of oscillation of the body. The *forces acting on the spring* may reach enormously greater values than the periodic force which maintains the motion of the system. Also the *forces which act upon the weight* to produce its up and down acceleration may greatly exceed in value the periodic force which maintains the motion.

Example.—A coil of 0.352 henry inductance and 2 ohms resistance and a condenser of 20 microfarads capacity, are connected in series between alternating current mains. The critical frequency of this circuit is 60 cycles per second, according to equation (48). The electromotive force between the mains is 200 volts and its frequency is 60 cycles per second. The current in the circuit is 100 amperes, according to equation (46), and the effective electromotive force between the condenser terminals is 13,270 volts ($= I/\omega C$).

Multiplication of current by resonance.—When resonance exists in a circuit containing an inductance and a condenser in parallel, as shown in Fig. 57, the alternating current in each branch may greatly exceed in value the alternating current I, which is delivered to the system. Let OE, Fig. 58, represent the electromotive force between the branch points a and b, Fig. 57; that is, the electromotive force of the alternator. The current I_2 in the condenser is nearly 90° ahead of E in phase, and the current I_1,

in the inductance, is nearly 90° behind E in phase, so that the resultant of I_1 and I_2 is small, as shown in Fig. 58.

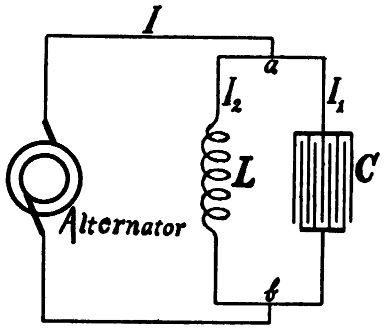


Fig. 57.

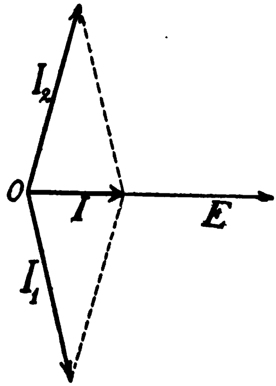


Fig. 58.

This multiplication of current by resonance is easily understood by means of the mechanical analogue as follows: A lever suspended at the center carries a weight L on one end, while the

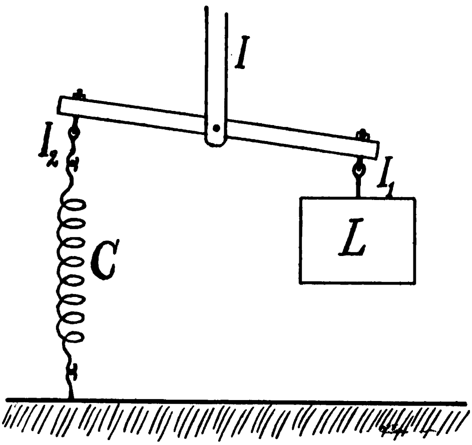


Fig. 59.

other end is held down by a spiral spring C. If the center of the lever is moved up and down to a very slight extent at the proper frequency, the system will be set into violent vibration

and the velocities I_1 and I_2 of the ends of the lever will greatly exceed in value the velocity I of the center of the lever, although half the sum of the velocities of the ends of the lever is at each instant equal to the velocity of the center of the lever, just as the sum of the currents I_1 and I_2, in Fig. 58, is at each instant equal to the current I.

PROBLEMS.

41. A circuit has inductance $L = 0.2$ henry and a resistance $R = 6$ ohms. Calculate the current produced by 100 volts, the frequency being 60 cycles per second. Calculate the phase difference between the electromotive force and current. Calculate power developed. Ans. 1.325 amperes, 85° 27′, 10.84 watts.

42. A circuit has 160 ohms resistance and 0.2 henry inductance. Calculate the power factor of the circuit for a frequency of 60 cycles per second. Ans. 0.9046.

43. An electromotive force of 20,000 volts acts on a receiving circuit of which the power factor is 0.85. Find the component of electromotive force parallel to the current and the component of electromotive force perpendicular to the current. Ans. 17,000 volts, 10,536 volts.

44. Show that $EI \cos \theta = RI^2$ in a circuit of resistance R, of inductance L, and containing a condenser of capacity C.

Suggestion.—Substitute in $EI \cos \theta$ the value of E in terms of I, R, L, C, and ω, and the value of $\cos \theta$ in terms of R, L, C, and ω.

45. A non-inductive resistance of 20 ohms, a resistanceless inductance of 0.06 henry, and a condenser of 105 microfarads capacity are connected in series to 110-volt 60-cycle mains. Find the electromotive force between the terminals of the resistance, between the terminals of the inductance, and between the terminals of the condenser. Find the same electromotive forces when the circuit is connected to 110-volt 120-cycle mains. Ans.

(*a*) 109.06 volts, 123.3 volts, 138 volts; (*b*) 57.5 volts, 130.1 volts, 126.3 volts.

46. An inductance of 0.1 henry and a resistance R are connected in series to 110-volt 60-cycle mains. Calculate the current values when R equals zero, when R equals one ohm, when R equals 5 ohms, when R equals 10 ohms, when R equals 20 ohms, when R equals 50 ohms, and when R equals 100 ohms, and plot a curve of which the abscissas represent R and the ordinates represent corresponding values of the current. Ans. 2.919 amperes, 2.917 amperes, 2.893 amperes, 2.821 amperes, 2.578 amperes, 1.757 amperes, 1.029 amperes.

47. A circuit contains a constant inductance and a variable resistance. Show in general that the current produced in the circuit by a given electromotive force is nearly independent of the resistance so long as the resistance is small compared with ωL.

48. A circuit contains a constant resistance and a variable inductance. Show in general that the current produced in the circuit by a given electromotive force is nearly independent of the inductance so long as ωL is small compared with R.

49. Alternating current mains deliver 100 amperes of current to a non-inductive circuit, to glow lamps for example. An inductive circuit of negligible resistance is then connected to the mains and it takes 10 amperes. What is the total current delivered by the mains? Ans. 100.5 amperes.

50. Two condensers, each of negligible resistance, have capacities of 0.5 and 0.05 microfarad respectively. The two condensers are connected in series to 1,100-volt alternating current mains. What is the electromotive force between the terminals of each condenser? Ans. 100 volts, 1,000 volts.

51. An electrostatic voltmeter has a capacity of 0.0006 microfarad when its deflection is a and 0.0008 microfarad when its deflection is b. The electromotive force between the terminals of the instrument is 75 volts when its deflection is a, and 125 volts when its deflection is b. An auxiliary condenser of 0.007

microfarad capacity is connected in series with the instrument. Find electromotive forces necessary to produce deflections *a* and *b*. Ans. 81.4 volts, 139.3 volts.

52. A direct-reading electrostatic voltmeter having 0.0006 microfarad capacity is connected through 50,000 ohms non-inductive resistance to 60 cycle mains. Find the percentage error* of the readings of the instrument and state whether it indicates too high or too low. Ans. 0.0064 per cent. too low.

53. The above electrostatic voltmeter is connected in series with an inductance of 2 henrys, resistance negligible. Find the percentage error of the voltmeter readings when a frequency of 60 cycles is used, and state whether it indicates too high or too low. Ans. 0.0017 per cent. too high.

54. An electrometer having an inductance of 0.02 henry and a resistance of 1,500 ohms gives the same deflection for a certain 60-cycle electromotive force as it does for 127.5 volts direct electromotive force. What is the effective value of the 60-cycle electromotive force? Ans. 127.5017 volts.

55. An electrodynamometer has a resistance of 500 ohms. When used as an alternating voltmeter at a frequency of 60 cycles per second its percentage error is $\frac{1}{10}$ per cent. What is its inductance? Ans. 0.0594 henry.

* In this and subsequent problems the percentage error of an instrument is understood to mean the difference between the instrument reading and the true value of the quantity measured, divided by the true value of the quantity.

CHAPTER VI.

THE USE OF COMPLEX QUANTITY. STEINMETZ'S METHOD.

58. Methods in alternating currents. *The graphical method and the trigonometrical method.*—All fundamental problems * in alternating currents may be solved by the graphical method, in which the various electromotive forces, currents, etc., are represented by lines in a diagram and the required results are measured off as in graphical statics. In practical problems, however, the different quantities under consideration differ so greatly in magnitude that it is difficult to scale off results with any degree of accuracy. The graphical method is, however, particularly useful for giving clear representations, and trigonometric formulas may be used in connection with graphical diagrams in every case.

The trigonometric formulas in the more complicated problems, however, become very unwieldy and are not suitable for easily obtaining numerical results.

Steinmetz's method.—Numerical results in alternating current calculations are most easily obtained by means of a method developed mainly by Steinmetz, in which *complex quantity* is used. This method is purely algebraic and is called by Steinmetz the *symbolic method.*

59. Simple quantity. Complex quantity.—A simple quantity is a quantity which depends upon a single numerical specification.

* The fundamental problems are those which treat of harmonic electromotive force and harmonic current. It is a mistake to suppose that differential equations furnish a method for treating alternating currents distinct from the three methods mentioned above. In the application of differential equations the first step is to derive one or more harmonic expressions for electromotive force and current and the subsequent development is precisely the one or the other of the above-mentioned methods.

Simple quantities are often called scalar quantities. A complex quantity requires two or more independent numerical specifications to entirely fix its value. For example, if wealth depends upon the possession of horses (h) and cattle (c), then, if no agreement exists as to the relative value of horses and cattle (in fact any such agreement is essentially arbitrary), the specification of the wealth of an individual would require the specification of both horses and cattle. Thus the wealth of an individual might be $5h + 100c$. The two or more numbers which go to make up a complex quantity are called the *elements* of the quantity.

Addition and subtraction.—Two complex quantities are added or subtracted by adding or subtracting the similar numerical elements of the quantities. Thus $5h + 100c$ added to $6h + 15c$ gives $11h + 115c$, and $2h + 25c$ subtracted from $5h + 100c$ gives $3h + 75c$.

Multiplication and division.—Consider two complex quantities $5h + 2c$ and $3h + 4c$ in which h and c are independent incommensurate units, say horses and cattle. Multiplying the first of these expressions by the latter, *using the ordinary rules of algebra*, we have:

$$(5h + 2c)(3h + 4c) = 15h^2 + 6hc + 20ch + 8c^2$$

Now in general the squares and products of units c^2, h^2, ch and hc have no meaning; so that the significance of the product of two complex quantities depends upon *arbitrarily chosen meanings for these squares and products of units.*

60. Vectors.—A vector is a quantity which has both magnitude and direction. A vector may be specified by giving its components in the direction of arbitrarily chosen axes of reference. In specifying a vector by its components, it is necessary to have it distinctly stated which is its x component and which is its y component.* This may be done either by verbal statement or by marking one of the components by a distinguishing index. Further it is allowable to connect the two components with the

* We are at present concerned only with vectors in one plane.

sign of addition. Thus $a + jb$ is a specification of the vector of which the x component is a and the y component is b; the index letter j being used to mark the y component. This expression of a vector is a complex quantity, the independent units of which are vertical and horizontal distances or components. For example, the vector OP, Fig. 60, is specified by the horizontal component a and the vertical component b. The vector may therefore be specified by the expression $a + jb$ the index j being used to show that b is the vertical component.

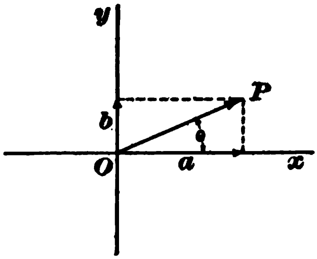


Fig. 60.

Numerical value and direction of a vector.—The numerical value of a vector is the square root of the sum of the squares of its components. Thus the numerical value of the vector $a + jb$ is $\sqrt{a^2 + b^2}$.

The angle between the vector and the x axis is the angle θ, of which the tangent is $\frac{b}{a}$, that is $\tan\theta = \frac{b}{a}$. This matter of value and direction of a vector is an important consideration in alternating current problems.

Addition and subtraction of vectors.—The sum of a number of vectors is a vector of which the x component is the sum of the x components of the several vectors, and of which the y component is the sum of the y components of the several vectors. Thus the sum of the vectors $a + jb$, $a' + jb'$, $a'' + jb''$ is

$$(a + a' + a'') + j(b + b' + b'')$$

The difference of the two vectors $a + jb$, $a' + jb'$ is

$$(a - a') + j(b - b')$$

Multiplication of vectors.—Consider the two vectors $a + jb$ and $a' + jb'$. Multiply these two expressions, using the formal rules of algebra, and we have for the product

$$aa' + jab' + ja'b + j^2bb'$$

Each term in this product must be interpreted arbitrarily or according to convention in order that this product may have a definite meaning.

In the first place, we may take aa', which is not affected by the index j, to be a horizontal quantity, and we may take ab' and $a'b$ to be vertical quantities or vertical components of a vector. As to the term j^2bb' we may note that the index letter j, used once, indicates that a quantity is vertical, while without the index j the quantity would be understood to be horizontal and to the right (see Fig. 60); that is, the letter j may be thought of as turning a quantity through 90° in the positive direction, that is, counter-clockwise. It is, therefore, convenient to think of the letter j when used twice ($jjbb'$ or j^2bb') as turning the quantity upon which it operates through 180°, or as reversing its direction, and, therefore, its algebraic sign. That is, j^2bb' is to be taken as equal to $-bb'$ or

$$j^2 = -1$$

so that the product of the two vectors $a + ja'$ and $b + jb'$ is to be interpreted as the vector

$$(ab - a'b') + j(ab' + a'b)$$

Quotient of two vectors.—Consider the quotient

$$\frac{a + jb}{a' + jb'}$$

multiply both numerator and denominator by $a' - jb'$, remembering that $j^2 = -1$, and we have

$$\frac{a + jb}{a' + jb'} = \frac{aa' + bb'}{a'^2 + b'^2} + j\left(\frac{a'b - ab'}{a'^2 + b'^2}\right)$$

which leads to the conception of the quotient of the two vectors as a vector of which the x component is

$$\frac{aa' + bb'}{a'^2 + b'^2}$$

and the y component is

$$\frac{a'b - ab'}{a'^2 + b'^2}$$

61. The fundamental equations in the application of complex quantity to alternating current problems.—The problem which forms the basis of nearly every discussion of alternating current phenomena is problem VI., which is discussed in Article 56. This problem deals with the relation between electromotive force and current in a circuit of resistance r, of inductance L and containing a condenser of capacity C. The electromotive force E has a component, rI, which is in phase with the current I, and a component $\left(\omega L - \frac{1}{\omega C}\right) I$, which is 90° ahead of the current in phase. Therefore, using the current vector as the reference axis, we have the following complex expression for the electromotive force:

$$E = rI + j\left(\omega L - \frac{1}{\omega C}\right) I \tag{49}$$

This equation is so often used that it is convenient to represent the factor $\left\{\omega L - \frac{1}{\omega C}\right\}$ by the single symbol x. That is

$$x = \omega L - \frac{1}{\omega C} \tag{50}$$

Using this abbreviation, equation (49) becomes

$$E = rI + jxI$$

or

$$E = (r + jx)\, I \tag{51}$$

The complex quantity $\{r + jx\}$ is called the *impedance* of the circuit. The factor x is called the *reactance* of the circuit.

Resistance and reactance are both expressed in ohms. The numerical value of impedance, namely, $\sqrt{r^2 + x^2}$, is expressed in ohms.

When ωL is greater than $1/\omega C$ the reactance x is positive. When $1/\omega C$ is greater than ωL the reactance is negative. When reactance is positive the component of E which is at right angles to I is 90° ahead of I in phase. When reactance is negative the component of E which is at right angles to I is 90° behind I in phase.

In some problems it is convenient to take the electromotive force vector as the reference axis, and to express current in terms of its components parallel to and perpendicular to E respectively. Thus solving equation (51) for I we have

$$I = \frac{E}{r + jx}$$

or

$$I = \frac{r - jx}{r^2 + x^2} \cdot E$$

or

$$I = \left(\frac{r}{r^2 + x^2} - j \cdot \frac{x}{r^2 + x^2} \right) E \qquad (52)$$

The complex quantity $\left\{ \frac{r}{r^2 + x^2} - j \cdot \frac{x}{r^2 + x^2} \right\}$ is called the *admittance* of the circuit.

The quantity $\left\{ \frac{r}{r^2 + x^2} \right\}$, by which E is multiplied to give the component of I parallel to E, is called the *conductance* of the circuit.

The quantity $\left\{ \frac{x}{r^2 + x^2} \right\}$, by which E is multiplied to give the component of I perpendicular to E is called the *susceptance* of the circuit.

Equation (52) is sometimes written:

$$I = (g - jb)E \qquad (53)$$

in which g is the conductance of the circuit and b is its susceptance.

Remark.—Algebraic developments based upon equations (51) and (53) *always give electromotive forces and currents in terms of their rectangular components.*

When a numerical value is required the square root of the sum of the squares of the components must be taken.

When the phase angle between a calculated electromotive force or current and the reference axis is desired it is found as the angle whose tangent is equal to the ratio of the components.

62. Expression for power.—Consider an electromotive force

$$E = a + ja' \qquad \text{(i)}$$

which maintains a current

$$I = b + jb' \qquad \text{(ii)}$$

in a circuit. The power developed by the electromotive force is

equal to the product of the numerical values of E and I into the cosine of the angular phase difference between E and I as explained in Article 52. When E and I are given in terms of their rectangular components, as in equations (i) and (ii) above, it is more convenient to calculate power by means of the formula

$$P = ab + a'b' \tag{54}$$

that is, the power is equal to the product of x-components of electromotive force and current plus the product of y-components of electromotive force and current. When a and b are opposite in direction their product is negative, and when a' and b' are opposite in direction their product is negative.

PROBLEMS.

56. Separate the components of the complex expression

$$\left(\frac{a + jb}{c + jd}\right)\left(e + jf\right)$$

find the numerical value of the expression, and find the tangent of the angle between it and the axis of reference, each in terms of a, b, c, d, e and f.

57. A current of which the components, referred to an arbitrary axis of reference, are 25 amperes and -10 amperes respectively, or of which the complex expression is $25 - 10j$, flows through a circuit of which the resistance is 8 ohms and the reactance is $+12$ ohms. Find the components of the electromotive force which maintains the current and find the power developed. Calculate the power by the use of equation (54) and also by multiplying the resistance of the circuit by the square of the numerical value of the current. Ans. x component $= 320$ volts, y component $= 220$ volts, 5,800 watts.

58. A circuit has 5 ohms resistance and $+3$ ohms reactance. What is the numerical value of the impedance in ohms? What is the inductance of the circuit in henrys, the frequency being 133

cycles per second, no condenser being connected in the circuit? Ans. 6 ohms, 0.00359 henry.

59. An electromotive force of which the components are 50 volts and 75 volts respectively, that is, the electromotive force is expressed by $50 + 75j$, produces a current through a circuit of which the resistance is 10 ohms and the reactance is -6 ohms. Find the components of the current, and find the power developed. Calculate the power by the use of equation (54) and also by multiplying the resistance of the circuit by the square of the numerical value of the current. Ans. x-component of current $= 0.368$ amperes, y-component of current $= 7.72$ amperes, power $= 597.4$ watts.

60. Separate the components of the complex expression

$$E = r_1 I_1 + j\omega L_1 I_1 + \frac{\omega^2 M^2 I_1}{r_2 + j\omega L_2}.$$

61. Separate the components of the complex expression ϵ^{jy}. Ans. $\epsilon^{jy} = \cos y + j \sin y$.

Suggestion.—Look up the series for ϵ^x. Substitute jy for x in this series. Separate real and imaginary terms into two series, and compare these series with the series for $\cos y$ and $\sin y$.

62. Show that $E(\cos\theta + j\sin\theta)$ is $\theta°$ ahead of E in phase and that its numerical value is equal to the numerical value of E.

CHAPTER VII.

FURTHER PROBLEMS.

APPLICATION OF THE SYMBOLIC METHOD

63. Problem VII. Coils in series.—An alternating electromotive force E acts upon two coils connected in series as shown in Fig. 61. It is required to find E_1 and E_2 each in terms of E, r_1, r_2, x_1, and x_2.

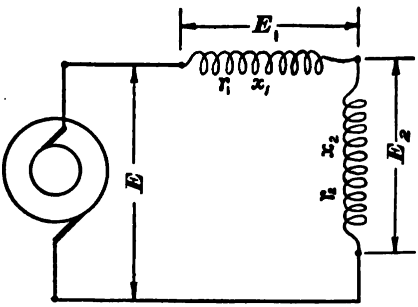


Fig. 61.

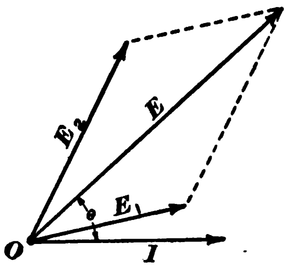


Fig. 62.

Let I be the current in the circuit. The general relation between E, E_1, E_2, and I is shown in Fig. 62. This figure is given in order that the following equations (a), (b), and (c) may be more easily understood. According to equation (51) we have:

$$E_1 = (r_1 + jx_1)I \qquad (a)$$

and

$$E_2 = (r_2 + jx_2)I \qquad (b)$$

Further, the total electromotive force E is the resultant of E_1 and E_2 as shown in Fig. 62, so that

$$E_1 + E_2 = E \qquad (c)$$

This is of course a complex equation. Substituting the values of E_1 and E_2 from (*a*) and (*b*) in (*c*) and solving for I, we have

$$I = \frac{E}{r_1 + r_2 + j(x_1 + x_2)} \qquad (d)$$

Substituting this value in (*a*) and (*b*) we have

$$E_1 = \frac{(r_1 + jx_1)E}{r_1 + r_2 + j(x_1 + x_2)} \qquad (e)$$

and

$$E_2 = \frac{(r_2 + jx_2)\,E}{r_1 + r_2 + j(x_1 + x_2)} \qquad (f)$$

These equations (*e*) and (*f*) express the electromotive forces E_1 and E_2 in terms of the known quantities E, r_1, r_2, x_1 and x_2. For purposes of numerical calculation (*e*) [and likewise (*f*)] must be separated into components, that is, into real and imaginary parts, and the numerical value of E_1 (and likewise of E_2) is then found by taking the square root of the sum of the squares of these components. Thus, multiplying numerator and denominator of (*e*) by $r_1 + r_2 - j(x_1 + x_2)$, we remove j from the denominator, and may then separate the components, namely,

$$E_1 = E\frac{r_1(r_1 + r_2) + x_1(x_1 + x_2)}{(r_1 + r_2)^2 + (x_1 + x_2)^2} + jE\frac{x_1(r_1 + r_2) - r_1(x_1 + x_2)}{(r_1 + r_2)^2 + (x_1 + x_2)^2} \quad (g)$$

The first term of this expression is the component of E_1 parallel to E and the second term, dropping j, is the component of E_1 perpendicular to E, and the numerical value of E_1 is the square root of the sum of the squares of these components.* The final result is of no use whatever in giving a conception of the phenomenon under consideration. It is useful only when it is desired to carry out numerical calculations. It is, indeed, generally the case that the use of the symbolic method in the solution of the alternating current problems is simple and instructive in its initial steps only, while the final solution itself is unin-

$$* E_1 \text{ (numerical value)} = \frac{\sqrt{r_1^2 + x_1^2}}{\sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2}} \times E$$

telligible. The final results will, therefore, be written out in full only when the student is expected to use them in numerical calculations.

The following simple cases of the problem under consideration are particularly interesting.

1. When $x_1 = 0$ and $x_2 = 0$; then E, E_1, E_2, and I are parallel and $E = E_1 + E_2$ (numerically).

2. When $\frac{x_1}{r_1} = \frac{x_2}{r_2}$; then E, E_1, and E_2 are parallel to each other and all are ahead of I in phase, by the angle of which the tangent is $\frac{x_1}{r_1}$. In this case, also, $E = E_1 + E_2$ (numerically).

3. When $\frac{x_1}{r_1}$ is very small and $\frac{x_2}{r_2}$ very large; then E_1 is parallel to I and E_2 is at right angles to I, as shown in Fig. 63. This figure is, of course, a particular case of Fig. 62. In the present case the numerical relation between E, E_1 and E_2 is

$$E = \sqrt{E_1^2 + E_2^2}$$

and when E_1 is small E_2 is sensibly equal to E, numerically.

Example.—A transmission line of large resistance r_1 and small reactance x_1 supplies current to a receiving circuit of large reactance x_2 and small resistance r_2, so that $\frac{x_1}{r_1}$ is very small and $\frac{x_2}{r_2}$ is very large. The electromotive force E_1, Fig. 63, consumed in the line is due almost wholly to resistance and if E_1 is not very large then E_2 is very nearly equal to E. That is the *resistance drop* in a transmission line produces but very little diminution of electromotive force at the terminals of a receiving circuit of large reactance.

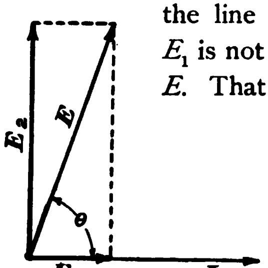


Fig. 63.

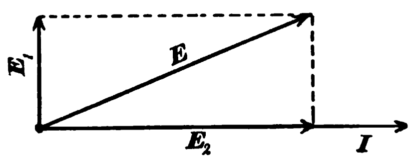


Fig. 64.

4. When $\frac{x_1}{r_1}$ is very large and $\frac{x_2}{r_2}$ very small the state of affairs is shown in Fig. 64, and $E = \sqrt{E_1^2 + E_2^2}$, numerically.

Example.—A transmission line of large reactance x_1 and small resistance r_1 supplies current to a receiving circuit of large resistance r_2 and small reactance x_2. The electromotive force E_1, Fig. 64, consumed in the line is due almost wholly to reactance and if E_1 is not very large, then E_2 is very nearly equal to E. That is, the *reactance drop* in a transmission line affects the electromotive force at the terminals of a non-inductive receiving circuit but little.

5. When $\frac{x_1}{r_1}$ is large and positive and $\frac{x_2}{r_2}$ is large, but negative, then E_1 is nearly 90° ahead of I and E_2 is nearly 90° behind I, as shown in Fig. 65. The figure shows the limiting case for which $\frac{x_1}{r_1} = +\infty$ and $\frac{x_2}{r_2} = -\infty$. In this case the total electromotive force E is numerically equal to the difference between E_1 and E_2.

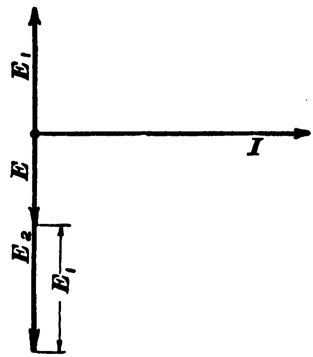


Fig. 65.

Examples.—(*a*) A transmission line of small resistance r_1 and large reactance x_1 supplies current to a condenser. The state of affairs is shown in Fig. 65 and the electromotive force E_2 at the terminals of the condenser exceeds the generator electromotive force E by the amount E_1. Therefore reactance drop in a transmission line increases the electromotive force at the terminals of a receiving circuit of negative reactance.

(*b*) A transmission line of small resistance r_1 and large reactance x_1 supplies current to a synchronous motor running at light load, electromotive force of motor being greater than electromotive force of generator. In this case the electromotive force at the terminals of the receiving circuit is nearly 90° behind the current

in phase, as in case of the condenser, and the electromotive force at the receiving circuit terminals is increased by the reactance drop in the line.

(*c*) An inductance is connected in series with a condenser between alternating current mains. If the resistance of the circuit is comparatively small the electromotive force at the condenser terminals is nearly equal to the sum of the electromotive force between mains plus the electromotive force between the terminals of the inductance. (See Articles 56 and 57.)

64. Problem VIII. Coils in parallel.—An alternating current I divides between two coils connected in parallel as shown in Fig. 66. It is required to find I_1 and I_2 each in terms of I, r_1, r_2, x_1 and x_2. The general relation between I, I_1, I_2 and E is shown in Fig. 67. This figure is given in order that the following equations may be more easily understood. The problem under discussion is greatly simplified if we use conductance g and susceptance b of each circuit instead of resistance and reactance. According to the definitions given in Article 61 we have:

$$\left.\begin{aligned} g_1 &= \frac{r_1}{r_1^2 + x_1^2} \\ b_1 &= \frac{x_1}{r_1^2 + x_1^2} \\ g_2 &= \frac{r_2}{r_2^2 + x_2^2} \\ b_2 &= \frac{x_2}{r_2^2 + x_2^2} \end{aligned}\right\} \tag{a}$$

From equation (52) we have:

$$I_1 = (g_1 - jb_1) E \tag{b}$$

and

$$I_2 = (g_2 - jb_2) E \tag{c}$$

Furthermore the total current I is the vector sum of I_1 and I_2, so that:

$$I = I_1 + I_2 \tag{d}$$

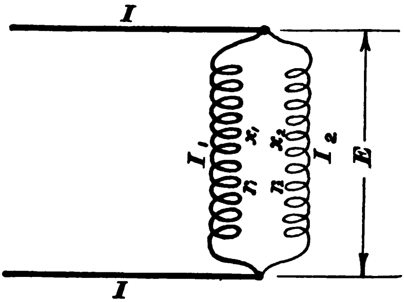


Fig. 66.

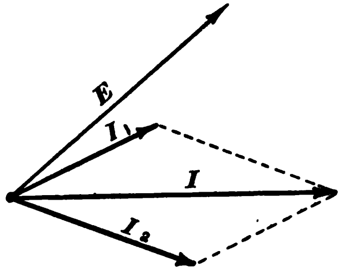


Fig. 67.

This is, of course, a complex equation. Substituting the values of I_1 and I_2 from (b) and (c) in (d) and solving for E we have

$$E = \frac{I}{g_1 + g_2 - j\,(b_1 + b_2)} \qquad (e)$$

Substituting this value of E in (b) and (c) we have:

$$I_1 = \frac{(g_1 - jb_1)\,I}{g_1 + g_2 - j\,(b_1 + b_2)} \qquad (f)$$

and

$$I_2 = \frac{(g_2 - jb_2)\,I}{g_1 + g_2 - j\,(b_1 + b_2)} \qquad (g)$$

These equations (f) and (g) express the currents I_1 and I_2 in terms of the known quantities, I, g_1, g_2, b_1 and b_2. For purposes of numerical calculations the components of each I_1 and I_2 must be separated. The numerical value of each is then found by taking the square root of the sum of the squares of its components.

The following simple cases of the problem under consideration are interesting.

1. When $x_1 = 0$ and $x_2 = 0$; then I, I_1, I_2 and E are parallel and $I = I_1 + I_2$ (numerically).

2. When $\frac{x_1}{r_1} = \frac{x_2}{r_2}$ then I, I_1, and I_2 are parallel to each other and all behind E in phase by the angle of which the tangent is $\frac{x_1}{r_1}$. In this case, also, $I = I_1 + I_2$ (numerically).

3. When $\frac{x_1}{r_1}$ is very small and $\frac{x_2}{r_2}$ is very large (or *vice versa*). In this case I_1 is parallel to E and I_2 is 90° behind E (or *vice versa*) as shown in Figs. 68 and 69. These figures are particular cases of Fig. 67. In the present case the numerical relation between I, I_1 and I_2 is $I = \sqrt{I_1^2 + I_2^2}$ and when either I_1 or I_2 is small the other is sensibly equal (numerically) to I.

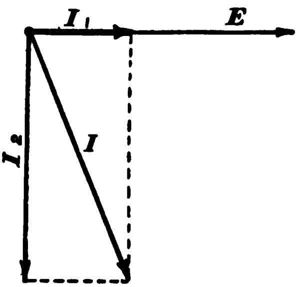


Fig. 68.

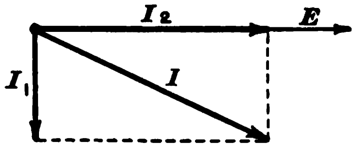


Fig. 69.

4. When $\frac{x_1}{r_1}$ is large and positive and $\frac{x_2}{r_2}$ is large but negative, then I_1 is nearly 90° behind E, and I_2 is nearly 90° ahead of E, as shown in Fig. 70. The figure shows the limiting case for which $\frac{x_1}{r_1} = +\infty$ and $\frac{x_2}{r_2} = -\infty$. In this case $I = I_2 - I_1$, numerically.

Examples of case 4.—A condenser and an inductance are connected in parallel in an alternating current circuit. The current I in the circuit divides in the two branches formed by the inductance and the condenser. The currents I_1 and I_2 in these two branches are nearly opposite to each other in phase and the current I is numerically equal to the difference of I_1 and I_2.

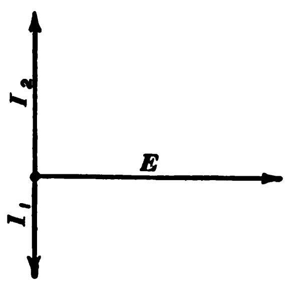


Fig. 70.

If the frequency of the alternating current I is such that ωL and $\frac{1}{\omega C}$ are equal then I_1 and I_2 will be nearly equal; and I_1 and I_2 will each be very much greater than I.

65. Compensation for lagging current in a receiving circuit.— A transmission line delivers a lagging current I_2 to a receiving circuit of resistance r_2 and reactance x_2, as shown in Figs. 71 and 72. A condenser connected in parallel with the receiving circuit takes current I_1, which is 90° ahead in phase of the electromotive force E which acts upon

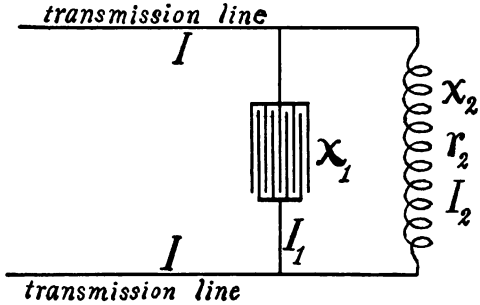


Fig. 71.

the receiving circuit. The total current, I, generated by the distant alternator and delivered by the transmission line may be reduced in value and brought into phase with E, if the reactance of the condenser is so chosen that the current taken by the condenser is equal and opposite to the component of I_2 which is at right angles to E.

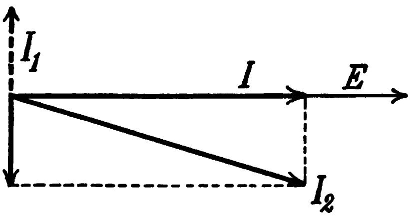


Fig. 72.

Discussion.—The current I_2 is, by equation (51),

$$I_2 = \frac{E}{r_2 + jx_2}$$

or

$$I_2 = \frac{r_2 E}{r_2^2 + x_2^2} - j \cdot \frac{x_2 E}{r_2^2 + x_2^2}$$

so that the component of I_2 perpendicular to E is $-j \cdot \dfrac{x_2 E}{r_2^2 + x_2^2}$, and, to bring the resultant current I into phase with E, as shown in Fig. 72, this component of I_2 must be equal and opposite to the current E/jx_1, which flows into the condenser. Therefore

$$\frac{E}{x_1} = \frac{Ex_2}{r_2^2 + x_2^2}$$

or
$$x_1 = -\frac{r_2^2 + x_2^2}{x_2}$$

which expresses the value of the negative reactance of the condenser, in order that the current delivered by the transmission lines may be in phase with the electromotive force acting on the receiving circuit.

Remark 1.—This arrangement is seldom used in practice, on account of the large and expensive condensers required and on account of the very considerable loss of power in such large condensers. See problem 65.

Remark 2.—It is shown in Chapter XII. that the synchronous motor running at light load has negative reactance, if its electromotive force is greater than the electromotive force of the generating alternator. Such a synchronous motor, called an *over-excited* synchronous motor, may be used to compensate for lagging currents.

66. The transformer without iron.—Two coils of wire A and B, Fig. 73, are placed near together, but not electrically connected. The coil A, called the *primary coil*, receives alternating current from a generator; its resistance is r_1 and its inductance is L_1. The coil B, called the *secondary coil*, is connected to a receiving circuit. The total resistance of coil B and its receiving circuit is r_2, and the total inductance is L_2. It is required to find the electromotive force which must act upon A to maintain in it a given harmonic current I_1. To solve this problem, one must consider the electromotive force induced in each coil by the changing current in the other coil. This electromotive force in one coil is proportional to the rate of change of the current in the other coil, and the proportionality factor M has a definite mutual value for the two coils.

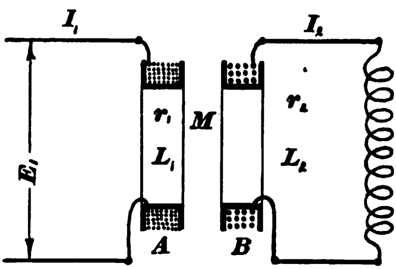


Fig. 73.

Determination of the secondary current I_2.—The given primary current induces in the secondary coil an electromotive force which is at each instant equal to $M\frac{di_1}{dt}$.

This electromotive force is 90° ahead of I_1, its effective value is ωMI_1, and its symbolic expression is $j\omega MI_1$, and according to equation (51) this electromotive force produces in the secondary coil a current

$$I_2 = \frac{j\omega MI_1}{r_2 + j\omega L_2} \tag{a}$$

Reaction of the secondary current upon the primary coil.—The secondary current I_2 induces in the primary coil an electromotive force which is at each instant equal to $M\frac{di_2}{dt}$. This electromotive force is 90° ahead of I_2 in phase, its effective value is ωMI_2, and its complex expression is $j\omega MI_2$. This electromotive force induced in the primary by the secondary current must be overcome by the electromotive force which acts upon the primary. The portion of the acting electromotive force which thus balances the reaction of the secondary current is equal to this reaction and opposite to it in sign and is, therefore, equal to $-j\omega MI_2$.

Determination of total electromotive force acting on primary.—This total electromotive force consists of three parts as follows:

1. The part described above which balances the reaction of the secondary current. This part is equal to $-j\omega MI_2$ or using the value of I_2 from equation (a) we have for this part of the total electromotive force

$$+\frac{\omega^2 M^2 I_1}{r_2 + j\omega L_2}$$

2. The part used to overcome the resistance of the primary coil. This is at each instant equal to $r_1 i_1$, its effective value is $r_1 I_1$, and its complex expression is $r_1 I_1$ since it is in phase with I_1.

3. The part used to overcome the inductance of the primary

coil. This is at each instant equal to $L_1 \frac{di_1}{dt}$, it is 90° ahead of I_1, its effective value is $\omega L_1 I_1$, and its complex expression is $j\omega L_1 I_1$. Therefore the total electromotive force required to maintain the given primary current is

$$E_1 = r_1 I_1 + j\omega L_1 I_1 + \frac{\omega^2 M^2 I_1}{r_2 + j\omega L_2} \qquad (b)$$

or separating components

$$E_1 = \left[\left(r_1 + \frac{r_2 \omega^2 M^2}{r_2^2 + \omega^2 L_2^2}\right) + j\left(\omega L_1 - \frac{\omega^3 L_2 M^2}{r_2^2 + \omega^2 L_2^2}\right)\right] I_1 \qquad (c)$$

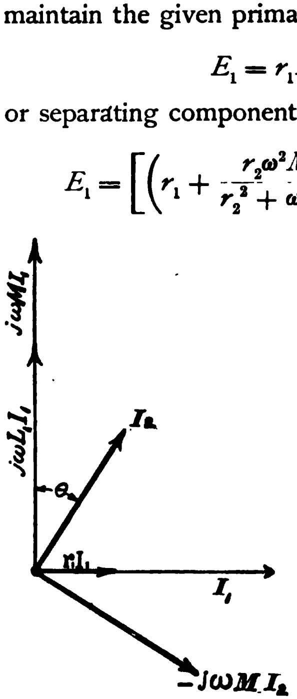


Fig. 74.

Fig. 74 shows the primary current I, the electromotive force $j\omega M I_1$ induced in the secondary coil, the secondary current I_2, the portion of the primary electromotive force $-j\omega M I_2$ used to balance the reaction of the secondary current, the portion of the primary electromotive force $r_1 I_1$ used to overcome primary resistance, and the portion of the primary electromotive force $j\omega L_1 I_1$ used to overcome primary inductance. The total primary electromotive force is the vector sum of $-j\omega M I_2$, $r_1 I_1$ and $j\omega L_1 I_1$.

Equation (c) shows that the effect of the secondary coil is to make the primary coil behave as if its resistance were increased by the amount

$$\frac{r_2 \omega^2 M^2}{r_2^2 + \omega^2 L_2^2}$$

and its inductance decreased by the amount

$$\frac{L_2 \omega^2 M^2}{r_2^2 + \omega^2 L_2^2}$$

67. Inductance error of the wattmeter.—It was shown in Article 41 that a wattmeter which has been calibrated with continuous electromotive force and current indi-

cates power correctly when used with alternating current, provided the inductance of the shunt coil is zero. When the shunt circuit has inductance, however, the wattmeter indicates incorrectly when used with alternating currents. The following discussion refers to Fig. 27, and it is assumed that the wattmeter is provided with a Weston compensating coil so that the current in coil b, Fig. 27, may be assumed to be equal to the current in the receiving circuit CC.

Let R be the resistance of the receiving circuit and X its reactance; and let r be the resistance of the shunt circuit and x its reactance.

Imagine the wattmeter connected to direct current mains as shown in Fig. 27 so that a continuous current C flows through the coil b and the receiving circuit. The electromotive force between the terminals of the receiving circuit is CR, and the current in the shunt circuit is CR/r. The force action between the wattmeter coils is proportional to the product of the two currents C and CR/r, or to C^2R/r; and C^2R, the watts expended in the receiving circuit, is the wattmeter reading W.

Imagine the wattmeter connected to alternating current mains, as shown in Fig. 27, so that an alternating current I flows through the coil b and the receiving circuit. The electromotive force between the terminals of the receiving circuit is $I(R+jX)$, and the current in the shunt circuit is $\dfrac{I(R+jX)}{r+jx}$. The component of this current parallel to I is $\dfrac{I(Rr-Xx)}{r^2+x^2}$ and the product of this component into I gives the average product* of the two currents, viz: $\dfrac{I^2(Rr-Xx)}{r^2+x^2}$. If this alternating current gives the same deflection of the wattmeter as the above-mentioned direct current we have

$$\frac{C^2R}{r}=\frac{I^2(Rr-Xx)}{r^2+x^2} \tag{i}$$

since equal deflections require equal force actions, and equal force actions depend upon equal current-products.

Multiply both members of (i) by R, write W (wattmeter reading) for C^2R, and write P for I^2R, this being the watts actually expended in the receiving circuit by the alternating current, and we have, after solving for P,

$$P=\frac{R(r^2+x^2)}{r(Rr-Xx)}\cdot W$$

That is, the wattmeter reading W must be multiplied by $\dfrac{R(r^2+x^2)}{r(Rr-Xx)}$ to give the power expended in the receiving circuit.

PROBLEMS.

63. A transmission line having an inductance of 0.02 henry and a resistance of 25 ohms supplies current at 60 cycles per

* Compare Article 52.

second to a condenser of which the capacity is 24 microfarads. The generator electromotive force is 20,000 volts. What is the electromotive force at the condenser terminals? Ans. 20,870 volts.

64. A circuit *a* consists of 50 ohms resistance and 2 henrys inductance; another circuit *b* consists of 50 ohms resistance and a 0.75 microfarad condenser. The two circuits *a* and *b* are connected in parallel in a circuit in which a 125-cycle current of 1 ampere is flowing. What is the current in *a* and what is the current in *b*? Ans. Current in $a = 12.02$ amperes, current in $b = 11.13$ amperes.

65. An alternator delivers 100 amperes at 1,100 volts and 60 cycles to a receiving circuit of which the power factor is 0.85. What capacity condenser would be required to compensate for lagging current? What number of leaves of a paraffined paper condenser 20 by 25 centimeters would be required for this condenser, thickness of paraffined paper being 0.08 cm.? Take inductivity of paraffined paper equal to 2. Ans. 127.1 microfarads, 114,875 leaves.

66. A transformer (without iron) consists of two long cylindrical coils each having 10 turns of wire per centimeter of length (one layer). The coils are each 50 cm. long and their radii are 2 cm. and 3 cm. respectively, the smaller coil being inside the larger. Calculate the value and phase of the electromotive force required to maintain a current of 10 amperes at 60 cycles per second in the outer coil; calculate the current in the inner coil, and calculate the true and apparent reactance and apparent resistance of the outer coil. The outer coil has 2 ohms resistance and the inner coil has 1 ½ ohms resistance and its terminals are short-circuited.* Ans. 21.123 volts, 6°21′ phase difference,

* The mutual inductance, in henrys, of two coaxial solenoids is, approximately,

$$M = 4\pi^2 r''^2 z' z'' l \div 10^9$$

in which z' and z'' are the turns of wire per unit length on the respective coils, r'' is the radius of the inside coil, and l is the length of the coils.

1.5384 amperes, 0.66967 true reactance, 0.66896 ohms reactance, 2.0036 ohms resistance.

67. The expression (*c*), Article 66, may be written

$$E_1 = [(r_1 + R) + j(x_1 - X)] I_1$$

where R is the apparent change of resistance of the primary coil due to the presence of the secondary coil and X is the apparent change of reactance of the primary coil due to the presence of the secondary coil. The primary coil has in circuit with it an adjustable non-inductive resistance so that r_1 may be varied at will. Assume that R and X are small compared to r_1 and x_1 so that R^2 and X^2 are negligible. Show that the impedance (numerical value) of the primary circuit is not altered by the presence of the secondary when $r_1/x_1 = R/X$. How is the impedance (numerical value) of the primary circuit affected by the presence of the secondary when r_1 is less than x_1R/X; when r_1 is greater than x_1R/X?

68. A wattmeter is to be used on a circuit of which the power factor is 0.85. The resistance of the shunt circuit is 1,000 ohms. Find the four values of reactance of shunt circuit for which the true power will differ from the reading by $\pm$ 0.005 of the reading. Ans. Reading too great $x = +7.45$ or -631.05, reading too small $x = -8.15$ or -608.75.

CHAPTER VIII.

SINGLE-PHASE AND POLYPHASE ALTERNATORS.

68. The single-phase alternator and its limitations.—The simple alternator described in Chapter II. is called the *single-phase* alternator. It has one pair of collector rings to which the terminals of the armature windings are connected. The current given by the single-phase alternator is entirely satisfactory for the operation of electric glow lamps, fairly satisfactory for the operation of electric arc lamps, and in general for all purposes in which the heating effect, only, of the current is important. For motive purposes the simple alternating current is not satisfactory, as it is difficult to make a single-phase alternating-current motor which will start satisfactorily under load. For electrochemical processes the alternating current cannot be used. The satisfactory use of alternating currents for motive purposes depends mainly upon the use of the *induction motor* described in Chapter XIII. It is the requirements of this motor which have led to the development of the polyphase systems.

69. The two-phase alternator.—Consider two similar and independent single-phase armatures A and B, Fig. 75, mounted rigidly on the same shaft, one beside the other, and revolving inside the same crown of field magnet poles. In the figure, armature B is shown inside of A for the sake of clearness. These armatures are so mounted on the shaft that the slots of A are midway under the poles when the slots of B are midway between the poles as shown. Under these conditions the *electromotive forces* of A and B are so related in their pulsations that the electromotive force of A is at its maximum when the electromotive force of

B' is zero, that is, the electromotive forces are 90° apart in phase, or in quadrature with each other. Two alternators connected (mechanically) in the manner indicated constitute a *two-phase alternator*. The two distinct and independent electromotive forces generated by such a machine are used to supply two distinct and independent currents to two distinct and independent circuits. In practice the two-phase alternator is made by placing the armature windings of A and B upon one and the same armature body. For this purpose the armature body has twice as many slots as A or B, Fig. 75.

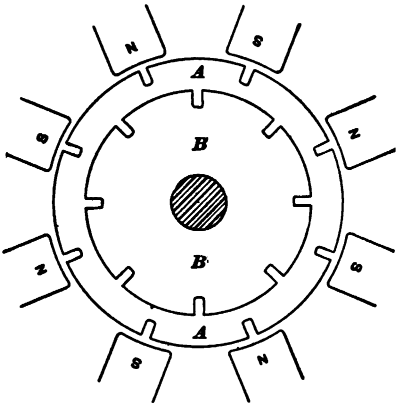


Fig. 75.

Fig. 76 shows such an armature. The slots marked a_1, a_2, a_3, etc., receive the conductors belonging to phase A, and those marked b_1, b_2, b_3, etc., receive those belonging to phase B. The A winding would pass up* slot a_1, down a_2, up a_3 and so on, the terminals of the winding being connected to two collector rings. The B winding would pass up slot b_1, down b_2, up b_3, and so on, its terminals being connected

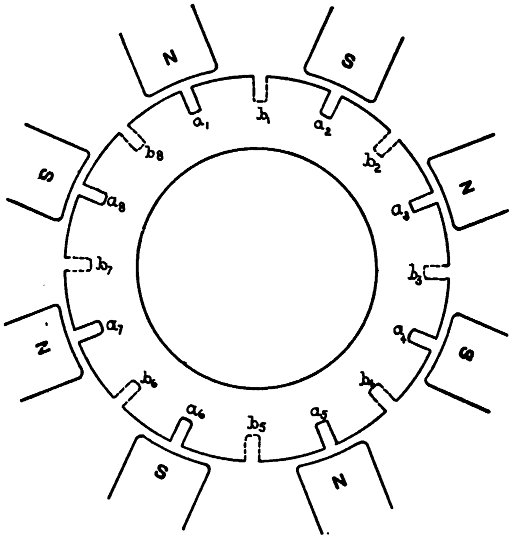


Fig. 76.

* Up and down being parallel to the armature shaft *to* and *from* one end of the armature.

to two collector rings distinct from those to which the *A* winding is connected.

The armature windings *A* and *B* here described are of the concentrated type (see Article 20), having only one slot per pole for each winding. Distributed windings also are frequently used for two-phase alternators. Thus Fig. 77 shows a portion of a two-phase armature with its *A* and *B* windings each distributed in two

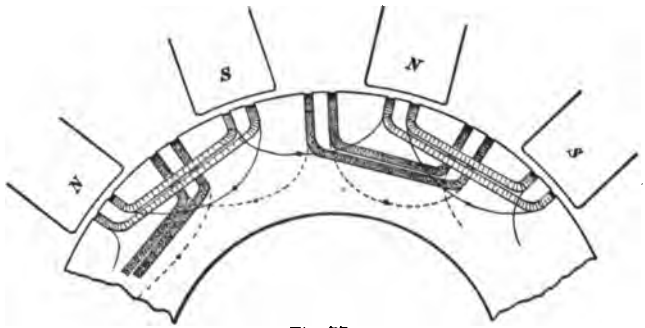


Fig. 77.

slots per pole. The coils belonging to windings *A* and *B* are differently shaded to distinguish them. The coils belonging to winding *A* are connected together in a manner indicated by the dotted lines and the coils belonging to winding *B* are connected together as indicated by the full lines. (See Article 84.)

Two-phase alternators are usually provided with two sets of collector rings; one ring may, however, be made to serve as a common connection for the two armature windings, as shown in

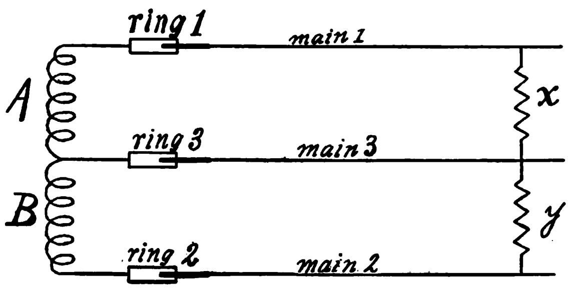


Fig. 78.

Fig. 78. In this case a three-wire transmission line is used and the separate receiving circuits x and y are connected as shown in the figure. The objection to this arrangement is that when the receiving circuits x and y are inductive the electromotive forces at the terminals of x and y respectively are not equal and not 90° apart in phase on account of the electromotive force lost in the common main 3. This disarrangement of the electromotive forces in a polyphase system is called *distortion*.

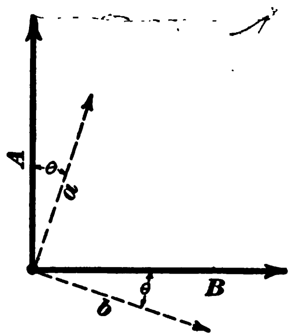


Fig. 79.

70. Two-phase electromotive forces and currents.—The two lines A and B, Fig. 79, represent the electromotive forces of the A and B windings respectively of a two-phase alternator. If the circuit which receives current from A is of the same resistance and reactance as the circuit which receives current from B then the system is said to be *balanced* and each current lags behind its electromotive force by the same amount. In this case the currents are equal and in quadrature with each other and are represented by the two dotted lines a and b in Fig. 79.

71. Electromotive force and current relations in two-phase three-wire system. *Electromotive force.*—The electromotive force between the mains 1 and 2, Fig. 78, is the vector sum* of the electromotive forces A and B, Fig. 79. This electromotive force is therefore represented by the diagonal of the parallelogram constructed on A and B, Fig. 79. It is 45° behind A in phase and its effective value is $\sqrt{2}E$ where E is the common effective value of the electromotive forces A and B.

Current.—The current in main 3 is the vector sum* of the currents in mains 1 and 2, namely a and b, Fig. 79. This current is therefore represented by the diagonal of the parallelogram constructed on a and b, Fig. 79; it is 45° behind a in phase and its effective value is $\sqrt{2}I$ where I is the common effective value of the currents a and b.

Effect of line drop in the common main of a two-phase three-wire system.—A two-phase alternator supplies current over three wires to two similar inductive receiving circuits x and y as shown in

* Or difference according to convention as to signs.

Fig. 78. The lines A and B, Fig. 80, represent the generator electromotive forces, a represents the current in main 1, b represents the current in main 2, and c represents the current in the common main 3. The lines Rc, parallel to c, represent the elec-

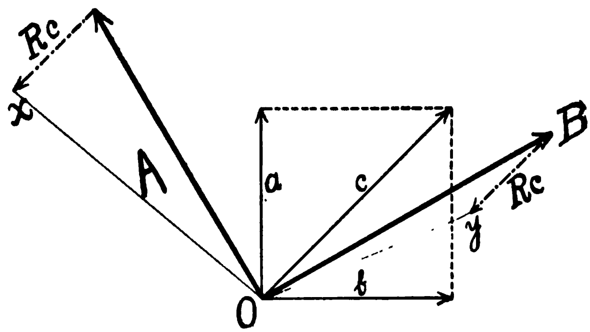


Fig. 80.

tromotive force lost in main 3, the line O to x represents the electromotive force between the terminals of receiving circuit x, and the line O to y represents the electromotive force between the terminals of receiving circuit y. The electromotive forces lost in mains 1 and 2 are not considered in this discussion inasmuch as they do not tend to distort the two-phase electromotive forces.

72. The three-phase alternator.—Consider three similar single-phase armatures, A, B and C, mounted side by side on the same shaft and revolved in the same field, each armature having as many slots as there are field poles. Fix the attention upon a certain armature slot of A and let time be reckoned from the instant that this slot is squarely under an N-pole. Let t be the time which elapses as this armature slot passes from the center of one N-pole to the center of the next N-pole. The armature B is to be so fixed to the shaft that its slots are squarely under the poles at the instant $\frac{1}{3}t$, and the armature C is to be so fixed that its slots are squarely under the poles at the instant $\frac{2}{3}t$. While a slot passes from the center of one N-pole to the center of the

next N-pole the electromotive force passes through one complete cycle. Hence the electromotive forces given by three armatures, arranged as above, will be 120° apart in phase, as shown in Fig. 81, in which the lines A, B and C represent the respective electromotive forces. The currents given by the armatures to three similar receiving circuits lag equally behind the respective electromotive forces and are represented by the dotted lines a, b and c. This combination of three alternators is called a *three-phase alternator*. In practice the three distinct windings A, B and C are

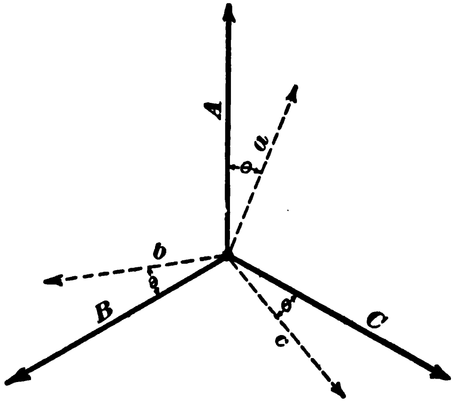


Fig. 81.

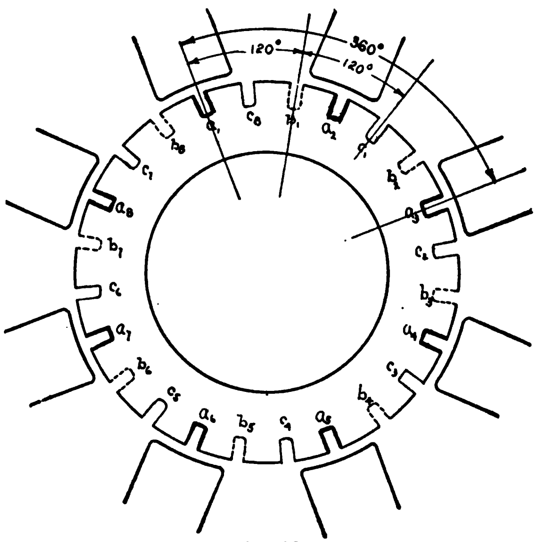


Fig. 82.

placed upon one and the same armature body. For this purpose the armature body has three times as many slots as A, B or C.

Fig. 82 shows the arrangement of the slots for such a winding. The slots belonging to phase A are drawn in heavy lines and are marked a_1, a_2, etc. Those belonging to phase B are shown dotted and those belonging to phase C are shown in light lines. The A winding would pass up slot a_1, down a_2, up a_3, etc.; the B winding, up b_1, down b_2, up b_3, etc.; and similarly for phase C.

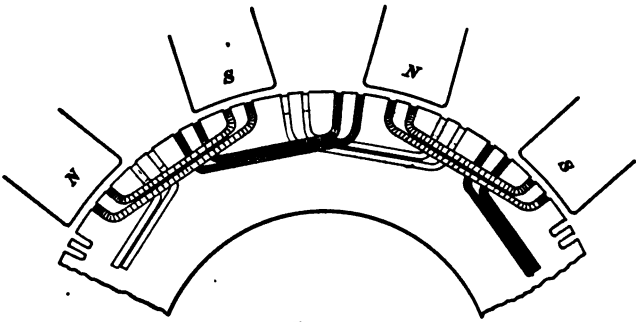


Fig. 83.

The windings A, B and C here described are of the concentrated type, having only one slot per pole for each winding. Distributed windings also are frequently used for three-phase alter-

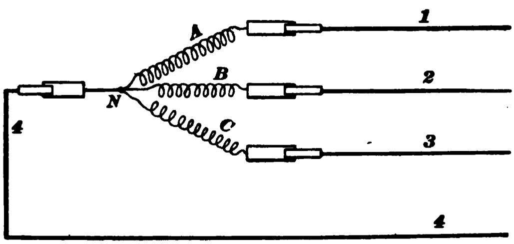


Fig. 84.

nators. Thus Fig. 83 shows a portion of a three-phase armature with its A, B and C windings each distributed in two slots per pole. The coils belonging to windings A, B and C respectively are differently shaded to distinguish them. The manner of connecting the coils of each winding is described in Article 84.

If the three circuits of a three-phase alternator are to be en-

tirely independent six collector rings must be used, two for each winding; however, the circuits may be kept practically independent by using four collector rings and four mains, as shown in Fig. 84. The main 4 serves as a common return wire for the independent currents in mains 1, 2 and 3. When the three receiving circuits are equal in resistance and reactance, that is, when the system is balanced, the three currents are equal and 120° apart in phase (each current lagging behind its electromotive force by the same amount) and their sum is at each instant equal to zero: in which case main 4, Fig. 84, carries no current and this main and the corresponding collector ring may be dispensed with, the three windings being connected together at the point *N*, called the common junction. This arrangement, shown in the symmetrical diagram, Fig. 85, is called the Y, or star, scheme of connecting the three windings *A*, *B* and *C*.

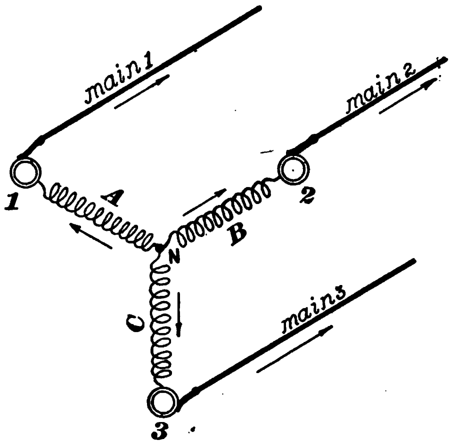


Fig. 85.

Another scheme for connecting the three windings *A*, *B* and *C* (also for balanced loads) called the Δ (delta) or mesh scheme is shown in Fig. 86. Winding *A* is connected between rings 3 and 1, winding *B* between rings 1 and 2 and winding *C* between rings 2 and 3.

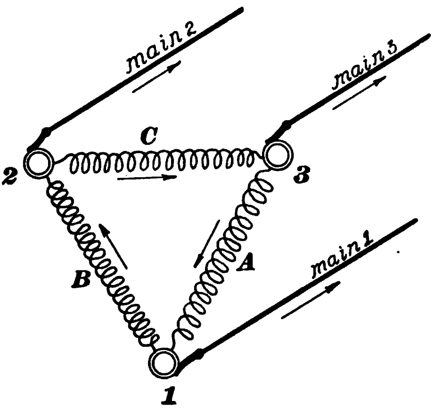


Fig. 86.

The direction in a circuit in which the electromotive force or current is considered as a positive electromotive force or current is called the *positive direction through the circuit.* This direction is chosen arbitrarily. The arrows in Figs. 85 and 86 indicate the positive directions in the mains and through the windings. It must be remembered that these arrows do not represent the actual directions of the electromotive forces or currents at any given instant, but merely the directions of *positive* electromotive forces or currents. Thus in Fig. 85 the currents are considered positive when flowing from the common junction towards the collecting rings and the currents are never all of the same sign.

73. Electromotive force and current relations in Y-connected armatures. *Electromotive force relations.*—Passing through the

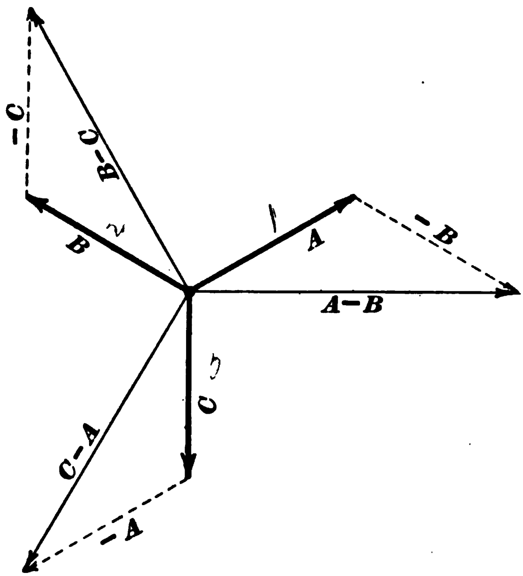


Fig. 87.

windings A and B from ring 2 to ring 1,* in Fig. 85, the winding A is passed through in the positive direction and the

* Which is the direction in which an electromotive force must be generated to give an electromotive force, acting upon a receiving circuit from main 1 to main 2.

winding B in the negative direction. Therefore the electromotive force between mains 1 and 2 is $A - B$. Similarly the electromotive force between mains 2 and 3 is $B - C$ and the electromotive force between mains 3 and 1 is $C - A$. These differences are shown in Fig. 87. The electromotive force between mains 1 and 2, namely, $A - B$, is 30° behind A in phase and its effective value is $2E \cos 30° = \sqrt{3}E$, where E is the common value of each of the electromotive forces A, B and C. Similar statements hold concerning the electromotive forces between mains 2 and 3 and between mains 3 and 1. Hence the electromotive force between any pair of mains leading from a three-phase alternator with a Y-connected armature is equal to the electromotive force generated per phase multiplied by $\sqrt{3}$.

Current relations.—In the Y connection the currents in the mains are equal to the currents in the respective windings, as is evident from Fig. 85.

74. Electromotive force and current relations in Δ-connected armatures. *Electromotive force relations.*—In Δ-connected armatures the electromotive forces between the mains or collector rings are equal to the electromotive forces of the respective windings, as is evident from Fig. 86.

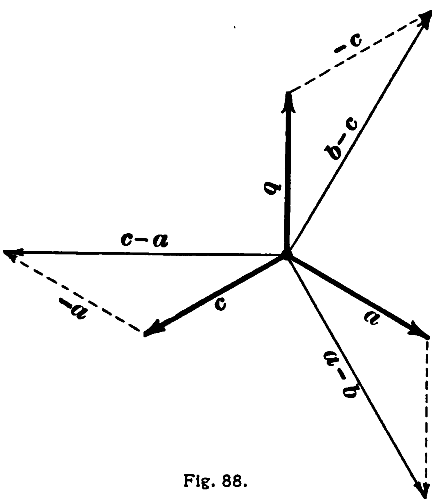


Fig. 88.

Current relations.—Referring to Fig. 86 we see that a positive current in winding A produces a positive current in main 1 and that a negative current in winding B produces a positive current in main 1, therefore the current in main 1 is $a - b$ when a is the current

in A and b is the current in B. Similarly the current in main 2 is $b-c$ and the current in main 3 is $c-a$. These differences are shown in Fig. 88. The current in 1, namely $a-b$, is 30° behind a in phase and its effective value is $\sqrt{3}\,I$ when I is the common effective value of the currents a, b, c in the different phases. Similar statements hold for the currents in mains 2 and 3; so that the current in each main of a Δ-connected armature is $\sqrt{3}$ times the current in each winding.

75. Connection of receiving circuits to three-phase mains. *Dissimilar circuits (unbalanced system).*—When the receiving circuits which take current from three-phase mains are dissimilar, four mains should be employed as indicated in Fig. 84; each receiving circuit being connected from main 4 to one of the other mains. It is, however, desirable to keep the three windings A, B and C of the alternator as nearly equally loaded as possible, and the receiving circuits are so disposed in practice as to satisfy this condition as nearly as possible.

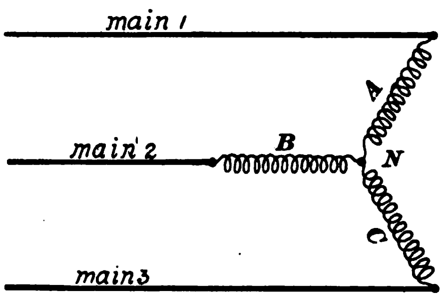


Fig. 89.

Similar circuits (balanced system).—When three-phase currents are used to drive induction motors, synchronous motors or rotary converters, each unit takes current equally from the three mains, and since three-phase currents are utilized mainly in the operation of the machines mentioned, the system is usually balanced. In this case three mains only are employed and each receiving unit has three similar receiving circuits connected

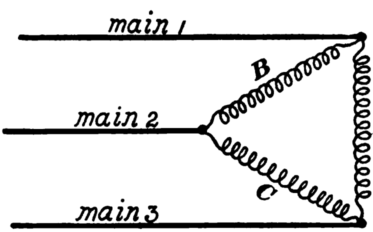


Fig. 90.

to the mains according to either the Y or Δ method. The Y method of connecting receiving circuits is shown in Fig. 89. One terminal of each receiving circuit is connected to a main and the other terminals are connected together at N. In this case the current in each receiving circuit is equal to the current in the main to which it is connected. The electromotive force between the terminals of each receiving circuit is equal to $\frac{E}{\sqrt{3}}$ where E is the electromotive force between any pair of mains.

The Δ method of connecting receiving circuits is shown in Fig. 90. Here the three receiving circuits are connected between the respective pairs of mains, the electromotive force acting on each receiving circuit is the electromotive force between the mains, and the current in each receiving circuit is $\frac{I}{\sqrt{3}}$ where I is the current in each main.

76. Power in polyphase systems.—The several circuits of a polyphase system are in general entirely separate and independent, and the total power delivered to a receiving apparatus is to be found by measuring the power delivered to each separate receiving circuit; the total power delivered is the sum of the amounts delivered to the different receiving circuits.

Balanced systems.—When a polyphase system is balanced the several circuits are entirely similar and the same amount of power is delivered to each receiving circuit of a given piece of receiving apparatus.

Balanced two-phase.—Let E be the electromotive force of each phase, I the current furnished to each of two similar receiving circuits, and $\cos\theta$ the power factor of each receiving circuit. Then $EI\cos\theta$ is the power delivered to each circuit, so that the total power delivered is

$$P = 2EI\cos\theta \tag{55}$$

Balanced three-phase.—Let E be the electromotive force between the terminals of each receiving circuit, I the current in each

receiving circuit, and $\cos\theta$ the power factor of each circuit. Then $EI\cos\theta$ is the power delivered to each receiving circuit, so that the total power delivered is

$$P = 3\,EI\cos\theta \tag{56}$$

in which, as must be remembered, E is the electromotive force at the terminals of each receiving circuit and I is the current in each receiving circuit. On the other hand,

$$P = \sqrt{3}\,EI\cos\theta \tag{57}$$

in which E is the electromotive force between each pair of mains, I is the current in each main, and $\cos\theta$ is the power factor of each receiving circuit. Equation (57) may be derived from (56) by considering that the current I_m in each main, for the case of Δ connection for example, is equal to $\sqrt{3}\,I$, so that, substituting $\frac{I_m}{\sqrt{3}}$ for I in equation (56) we have equation (57).

77. The flow of energy in balanced polyphase systems.—It was pointed out in Article 26 that the power developed by a single-phase alternator pulsates with the alternations of electromotive force and current. The power delivered to a balanced system by a polyphase generator, on the other hand, is not subject to pulsations, but is entirely steady and constant in value.

Discussion for a two-phase alternator.—Consider a single-phase alternator of which the electromotive force is

$$e = \mathbf{E}\sin\omega t \tag{a}$$

and which gives a current

$$i = \mathbf{I}\sin(\omega t - \theta)$$

or

$$i = \mathbf{I}\sin\omega t\cos\theta - \mathbf{I}\cos\omega t\sin\theta \tag{b}$$

The instantaneous power is

$$ei = \mathbf{EI}\cos\theta\sin^2\omega t - \mathbf{EI}\sin\theta\sin\omega t\cos\omega t$$

which pulsates with a frequency twice as great as the frequency of e and i.

Let equations (a) and (b) express the electromotive force and current of one phase of a (balanced) two-phase alternator, then electromotive force and current of the other phase are

$$e' = \mathbf{E}\cos\omega t \tag{c}$$

$$i' = \mathbf{I}\cos(\omega t - \theta) = \mathbf{I}\cos\omega t\cos\theta + \mathbf{I}\sin\omega t\sin\theta \tag{d}$$

The instantaneous power output of this phase is

$$e'i' = \boldsymbol{EI} \cos \theta \cos^2 \omega t + \boldsymbol{EI} \sin \theta \sin \omega t \cos \omega t$$

Therefore the total power output of the two-phase machine is

$$\begin{aligned} ei + e'i' &= \boldsymbol{EI} \cos \theta \, (\sin^2 \omega t + \cos^2 \omega t) \\ &= \boldsymbol{EI} \cos \theta \end{aligned}$$

which is constant.

Remark.—The torque of a single-phase alternator pulsates with the pulsations of the power output. In a balanced polyphase alternator, however, the torque is steady, since the power does not pulsate; also polyphase synchronous motors, rotary converters, and induction motors are driven by a steady torque.

78. Measurement of power in polyphase systems.—In a polyphase system, balanced or unbalanced, the power taken by any unit, such as an induction motor, may be determined by measuring the power taken by each single receiving circuit and adding the results. In order to measure the power taken by a single receiving circuit the current coil of the wattmeter is connected in series with the circuit and the fine wire coil is connected to the terminals of the circuit. The inconvenience of connecting and

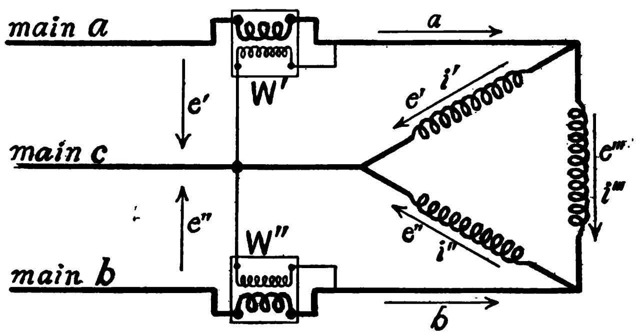


Fig. 91.

disconnecting the wattmeters makes it necessary to use a separate wattmeter for each receiving circuit. Two wattmeters are sufficient for the complete measurements of the power taken by any three-phase receiving unit. The connections are shown in Fig.

91. The receiving circuit may be balanced or unbalanced and connected Y or Δ.

Proof.—Let the positive direction in the mains a and b and in the three receiving circuits be chosen as indicated by the arrows in Fig. 91. These directions are chosen symmetrically with respect to the two wattmeters. Let the instantaneous currents in the receiving circuits be i', i'', and i''' as shown. Let a be the instantaneous current in main a, and let b be the instantaneous current in main b. Then

$$a = i' + i'''$$
$$b = i'' - i'''$$

The reading W' of the upper wattmeter is equal to the average value of the product of the current a which flows through the current coil of the instrument into the electromotive force e' which acts upon the shunt circuit of the instrument. That is

$$W' = \text{average } ae'$$

Similarly

$$W'' = \text{average } be''$$

Substituting the above values of a and b in the expression for $W' + W''$ we have

$$W' + W'' = \text{average } e'i' + \text{average } e''i'' + \text{average } (e' - e'')\, i'''$$

But $e' - e'' = e'''$ so that

$$W' + W'' = \text{average } e'i' + \text{average } e''i'' + \text{average } e'''i'''$$

Q. E. D.

In a balanced polyphase system, a condition which is seldom strictly realized, the power taken by one only of the receiving circuits need be measured.

PROBLEMS.

69. A common return wire is used for the two currents of a two-phase system. The system is balanced and each current is equal to 100 amperes. What is the current in the common return wire? Ans. 141.4 amperes.

70. The electromotive force of each phase, problem 1, is 500 volts. What is the electromotive force between the outside wires? Ans. 707 volts.

71. Three similar receiving circuits are Δ-connected to 3-phase mains, the electromotive force between each pair of mains being 110 volts. The power delivered to the three circuits is 150

kilowatts and the power factor of each circuit is .90. What is the current in each circuit and in each main? Ans. 505 amperes, 885 amperes.

72. Three similar receiving circuits are Y-connected to the 3-phase mains, problem 71; the total power delivered is 150 kilowatts and the power factor of each circuit is .90. What is the current in each circuit and in each main; and what is the electromotive force between the terminals of each circuit? Ans. 885 amperes, 885 amperes, 63.5 volts.

73. A three-phase alternator is provided with 4 collecting rings and 4 mains as shown in Fig. 84. Three similar receiving circuits are connected as follows: one from main 1 to main 4, one from main 2 to main 4, and one from main 3 to main 4. Each receiving circuit takes 150 amperes. When the armature windings are properly connected to the collector rings the current in main 4 is zero. What is the current in main 4 when one armature winding has its connections reversed? Draw a diagram showing the phase relations between the currents in mains 1, 2, 3, and 4 when the armature winding *A*, Fig. 84, is reversed. In constructing this diagram consider directions out from machine as the positive direction in each main. Ans. 300 amperes.

74. The three windings of a three-phase alternator are Y-connected to three mains as shown in Fig. 85. The electromotive force of each winding is 125 volts. The connections of winding *A* are reversed. Draw a diagram representing electromotive forces from main 1 to main 2, from main 2 to main 3, and from main 3 to main 1.

75. The distance from center to center of adjacent poles of an alternator measured on the surface of the armature is 10.4 inches. How far apart must two armature conductors be placed so that there may be a phase difference of 55° in the electromotive forces induced in the respective conductors? Ans. 3.18 inches.

CHAPTER IX.

ALTERNATORS.

(*Continued.*)

79. Armature reaction.—The amount of magnetic flux entering the armature core from the field poles and the manner of its distribution over the pole faces, depend upon the combined magnetic action of the field current and of the armature current.

Distortion of field.—The armature current in an alternator tends to concentrate the magnetic flux under the trailing horns of the pole pieces very much as in the direct-current dynamo. The effect of this concentration of flux is to slightly increase the magnetic reluctance of the saturated portions of the pole pieces and armature core. This increase of magnetic reluctance causes a decrease of flux and a consequent decrease of the electromotive force of the alternator, other things being equal. This effect may be appreciable, but it cannot be predetermined accurately by calculation.

Magnetizing and demagnetizing action of the armature current.—When the current given by an alternator is in phase with its electromotive force the only effect of the armature current upon the field is the distorting effect described above. When the current is not in phase with the electromotive force the distorting effect is decreased * and in addition there is a magnetizing action or demagnetizing action upon the field according as the current is *ahead of* or *behind* the electromotive force in phase.

* The distorting effect is due to the component of the current parallel to the electromotive force and the magnetizing effect is due to the component at right angles to the electromotive force.

Consider a bundle of N armature wires, grouped in a slot, for example. Let

$$e = \boldsymbol{E} \sin \omega t$$

be the alternating electromotive force induced in this bundle of conductors. This electromotive force is a maximum when the slot is at a, Fig. 92. It is zero when the slot is at b and it is a minimum (negative maximum) at c. Therefore the value of ωt is 90° at a, 180° at b and 270° at c. Let i be a given current flowing in the bundle of wires. The *ampere-turns* of the bundle is then Ni. If the bundle of wires is at a its ampere turns will be without appreciable effect on the magnetic circuit m m m shown by the dotted line; at b the ampere-turns will have their full demagnetizing effect (negative) * upon the magnetic circuit, and at c the effect will again be zero.

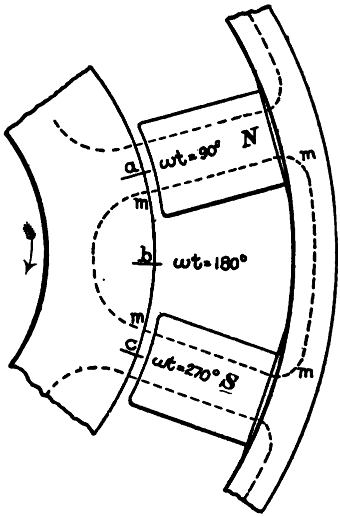


Fig. 92.

Now, $\cos \omega t$ is zero at a, negative unity at b and zero at c. Therefore $Ni \cos \omega t$ is an expression which gives the true magnetic effect of the bundle of wires at a, b and c with the given current. *We assume this expression to hold for all points between a and c.* The actual current in the bundle of wires is

$$i = \boldsymbol{I} \sin (\omega t - \theta)$$

in which θ is the angle of lag of the current behind the electromotive force. Therefore substituting this value of i in the expression $Ni \cos \omega t$ we have

$$m = N\boldsymbol{I} \cos \omega t \sin (\omega t - \theta)$$

in which m is the effective ampere-turns of the bundle of wires at the instant t. To find the average value of m expand $\sin (\omega t - \theta)$, whence

$$m = N\boldsymbol{I} \sin \omega t \cos \omega t \cos \theta - N\boldsymbol{I} \cos^2 \omega t \sin \theta \qquad \text{(i)}$$

Now the average value of m is the sum of the average values of the two terms of the right-hand member; but the average value of $\sin \omega t \cos \omega t$ is zero and the average value of $\cos^2 \omega t$ is ½. Therefore

$$\text{average } m = -\tfrac{1}{2} N\boldsymbol{I} \sin \theta \qquad \text{(ii)}$$

or putting $\sqrt{2}\, I$ for $\boldsymbol{I}$ we have

$$\text{average magnetizing ampere-turns} = -0.707\, NI \sin \theta \qquad (58)$$

When this formula is applied to an alternator the windings of which are not too widely distributed, N is to be taken as the total number of armature conductors divided by the number of poles. This equation shows that when the current lags behind the

* The current i is considered positive when it is in the direction of the electromotive force which is induced under the N pole. A current in this direction has a demagnetizing effect for all positions of the slot between a and c.

electromotive force (angle θ positive) the armature current weakens the field and *vice versa*.

Remark.—The distorting action and the magnetizing or demagnetizing action of the armature currents of a polyphase alternator supplying currents to a balanced receiving system are steady in value, that is, they do not pulsate as in the single-phase alternator.

In the case of the two-phase alternator the constancy of the magnetizing action of the armature currents is easily shown. Let m, equation (i), be the instantaneous magnetizing action of the armature currents in the A winding of a two-phase machine. Since the A and B windings supply current to entirely similar receiving circuits, I and θ have the same values for both phases, and the instantaneous value of the magnetizing action of the armature currents in the B winding is obtained by substituting $\omega t \pm 90°$ for ωt in equation (i). This gives:

$$m' = -NI \sin \omega t \cos \omega t \cos \theta - NI \sin^2 \omega t \sin \theta \qquad \text{(iii)}$$

The combined magnetizing action of the currents in A and B is

$$m + m' = -NI(\sin^2 \omega t + \cos^2 \omega t) \sin \theta$$

or

$$m + m' = -NI \sin \theta \qquad \text{(iv)}$$

Remark.—The pulsating magnetizing action of the armature currents of an alternator (together with the varying reluctance of the magnetic circuit in the case of armatures with large teeth) causes the field flux to pulsate, and this pulsation of field flux induces double frequency alternating electromotive forces in the field windings. These alternating electromotive forces cause in their turn a pulsation of the field current. A direct current ammeter connected in the field circuit indicates the *average* current, which is slightly less than the *square-root-of-mean-square* which is indicated by an alternating current ammeter. A direct-current voltmeter connected to the terminals of a field coil indicates the *average* value of the electromotive force, and this is sometimes much less than the *square-root-of-mean-square* value as indicated by an alternating-current voltmeter.

80. Armature inductance.—The value of the inductance of an alternator armature varies with the position of the armature coils with respect to the field magnet poles, so that the inductance of

an armature pulsates at a frequency twice* as great as the frequency of the electromotive force of the alternator. The armature of the alternator shown in Fig. 10, for example, has about two times as great inductance when the armature teeth are squarely under the field poles as it has when the armature teeth are midway between field poles. That is, the flux produced through the armature teeth by a given current is three or four times as great in the first case as in the second case. This fluctuation of armature inductance makes it very difficult to carry out accurate calculations upon the action of the machine. In the following discussion the armature inductance is assumed to be constant.

The inductance of an alternator armature is proportional to the linear dimensions of the armature, other things being equal; and the inductance of an armature of given size is much greater when the winding is concentrated than it is when the winding is distributed.

Armature inductance is advantageous in an alternator which is especially liable to be short circuited. The armature inductance keeps the current from becoming excessive. Armature inductance is more or less objectionable in an alternator which is to be used to supply current at constant electromotive force on account of the electromotive force lost in the armature as explained in the next article.

The inductance of an armature is best determined by sending a measured alternating current I through it at standstill from an outside source, and measuring the electromotive force E between the collector rings. Then

$$E = I\sqrt{R^2 + \omega^2 L^2}$$

from which L may be calculated when the armature resistance R and the frequency ($\omega/2\pi$) are known. The value of L depends

*The electromotive force of an alternator passes through a cycle as an armature coil passes from a north pole of the field to the next north pole. The inductance passes through a cycle of values as an armature coil passes from one field pole to the next field pole.

greatly upon the position in which the armature is held, as explained above.

For further information concerning armature inductance, see Parshall and Hobart, "Electric Generators," pages 160 to 175, London, 1900, and London Engineering, Vol. 70, pages 141 to 145, August 3, 1900.

81. The electromotive force lost in the armature. *Armature drop.*—The electromotive force at the collecting rings of an alternator is less than the total electromotive force induced in the armature, for the reason that a portion of the induced electromotive force is used to overcome the resistance, and a portion is also used to overcome the inductance of the armature windings. The numerical difference between the electromotive force induced in the armature and the electromotive force between the brushes is called the armature drop.

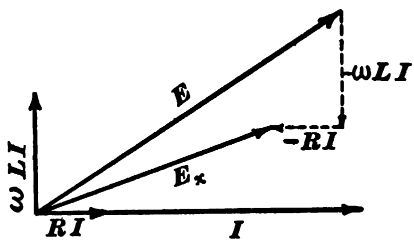


Fig. 93.

General case.—Let I, Fig. 93, be the current given by the alternator and E the total induced electromotive force. Then ωLI is the portion of E used to overcome the inductance L of the armature, and RI is the portion of E used to overcome the resistance R of the armature. Subtracting ωLI and RI from E gives the electromotive force at the collecting rings, or "external electromotive force," E_x.

Armature drop, non-inductive load.—In this case E is nearly in phase with I, and the subtraction of ωLI from E scarcely reduces its value, ωLI being nearly at right angles to E. Therefore, with a non-inductive receiving circuit, the armature drop depends almost wholly upon the armature resistance.

Armature drop, inductive load.—When the phase difference between E and I is nearly 90°, then the subtraction of RI from E scarcely reduces its value. Therefore, with a highly inductive

receiving circuit, the armature drop depends almost wholly upon the armature inductance.

Algebraic expression for armature drop.—Let I, Fig. 94, represent the current delivered by the alternator. Let E represent the total electromotive force induced in the armature, let E_a be the electromotive force lost in the armature, and let E_x be the electromotive force between the collecting rings. The electromotive force E is the vector sum of E_a and E_x. The angle θ is the phase difference between E_x and I, and $\cos\theta$ is the power factor of the receiving circuit. The angle θ' is the phase difference between E_a and I. The electromotive forces E and E_x are

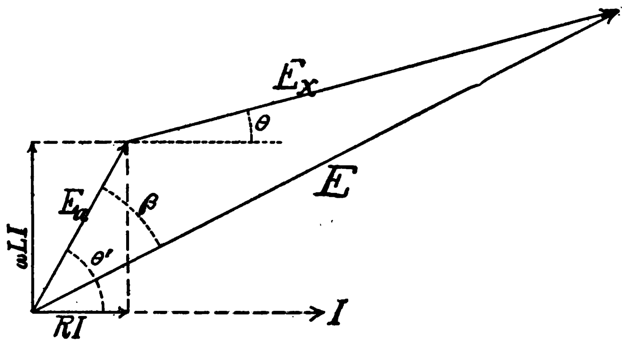


Fig. 94.

usually large in comparison with E_a, E and E_x are therefore approximately parallel to each other, and the angle β is approximately equal to $\theta' - \theta$. The numerical difference between E and E_x, the *armature drop*, is approximately equal to

$$E_a \cos\beta = I\sqrt{R^2 + \omega^2L^2}\cdot\cos\beta = I\sqrt{R^2 + \omega^2L^2}\cos(\theta' - \theta).$$

We shall have occasion in the discussion of the compensated alternator of Mr. E. W. Rice to remember that the armature drop is proportional to $I\cos(\theta' - \theta)$, the factor $\sqrt{R^2 + \omega^2L^2}$ being a constant for a given alternator.

Remark.—If one considers the *actual flux* through a circuit, due to the combined action of all causes which tend to produce

flux, then the rate of change of this flux is an electromotive force, no portion of which can be lost in overcoming inductance, inasmuch as the inductance flux will have already been considered. Thus the electromotive force lost in an armature because of inductance may be allowed for, as above, by subtracting ωLI, as a vector, of course, from the electromotive force E, which would be produced by the given field excitation with no armature current, or the demagnetizing ampere-turns on the armature may be considered with the ampere-turns on the field spools, the actual flux due to both found, and the electromotive force induced in the armature by this net flux will all be available as external electromotive force, except that a portion of it is lost in overcoming the resistance of the armature.

82. The characteristic curve of the alternator.—The curve obtained by plotting observed values of the external electromotive force for various currents taken from an alternator is called the characteristic curve of the alternator. Such characteristic curves are shown in Fig. 95. Curve A is for a separately excited alternator having but small armature inductance, and curve B is for a separately excited alternator having large armature inductance. The shape of the characteristic curve of a given alternator depends to a greater or less extent upon the inductance of the receiving circuit. The falling off of electromotive force with increase of current is due in part to the demagnetizing action of the armature current, which weakens the field, and in part to the increased armature drop with increase of current.

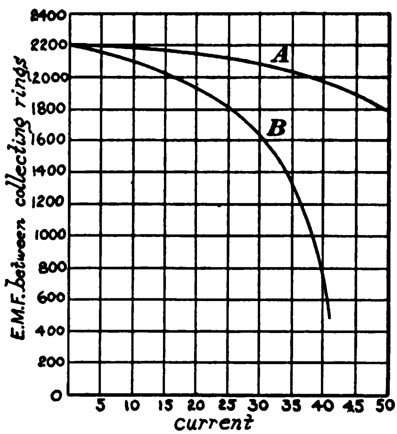


Fig. 95.

83. The constant current alternator.—An alternator of which the armature has an excessive inductance, or an ordinary alternator in circuit with which a large inductance is connected, gives a current which does not vary greatly with the resistance * of the receiving circuit.

This may be shown as follows: Let E, Fig. 96, be the total induced electromotive force of an alternator sending current through a circuit of which the reactance ωL is constant and large, compared with the resistance R. Then ωLI will be large, compared with RI. Further, RI and ωLI are at right angles to each other and their vector sum is E, so that the point P, Fig. 96, lies on a semicircle constructed on E as a diameter. Now, when RI is small, compared with E, then ωLI is very nearly equal to E, that is, ωLI is approximately constant and, therefore, I is approximately constant.

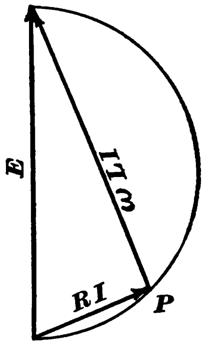


Fig. 96.

84. Effect of distributed winding upon the electromotive force of an alternator.†—Consider an armature winding A concentrated in a set of slots, one slot per pole. The effective electromotive force of this winding is

$$A = \frac{4.44\Phi Tf}{10^8}$$

according to equation (21), Chapter II. Suppose another similar concentrated winding, B, is placed upon the same armature in slots distant s from the first set of slots, s being the angle shown in Fig. 97. This figure shows one slot only of the first set and one slot only of the second set. The phase difference

* Unless the resistance becomes very large.

† This question is discussed in a slightly different manner in the chapter on the rotary converter. The discussion in this article is based upon the assumption that the magnetic flux passing into (and out of) the armature from the field magnet is so distributed that a harmonic electromotive force is induced in each armature conductor. The discussion for any other type of flux distribution would lead into the discussion of non-harmonic electromotive forces, which is beyond the scope of this text.

between the electromotive forces in these two windings is the angle $\frac{s}{q} \times 360°$, inasmuch as the angle q from N to N is equivalent to 360° of phase difference. These two electromotive forces are represented by the lines A and B, Fig. 98. Similarly the lines C and D represent the electromotive forces in two additional similar windings concentrated in two additional sets of slots c and d, Fig. 97. If all these windings are connected in series the effective electromotive force produced will be the vector sum E of A, B, C and D. If we were to calculate the effective electromotive force produced by A, B, C and D in series on the assumption that all the windings are concentrated in one

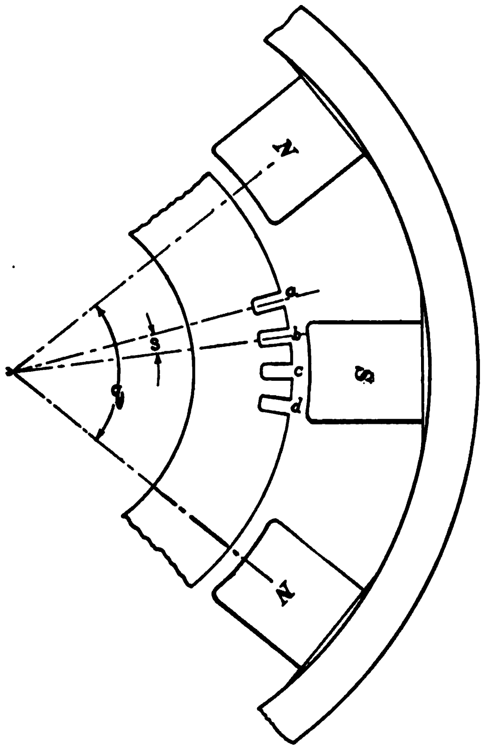


Fig. 97.

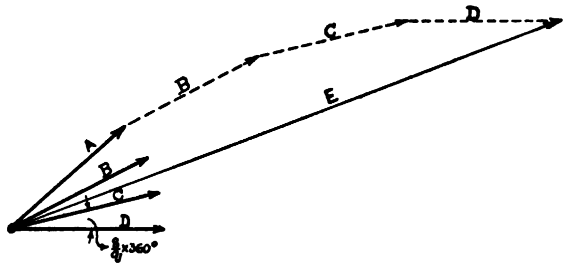


Fig. 98.

set of slots, that is, if we were to calculate the total electromotive force by means of equation (21), using for T the total number of turns in all the windings, we would get a result greater than E in the ratio of the sum of the sides A, B, C and D of the polygon to the chord E, Fig. 98. This ratio may be called the *phase constant* of the distributed winding. By introducing the phase constant k in equation (21) this equation becomes

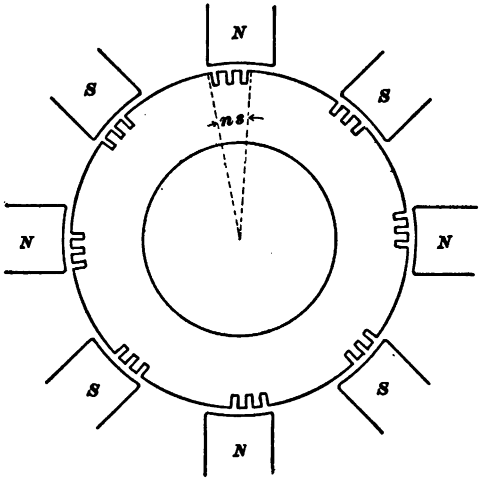


Fig. 99.

$$E = \frac{4.44 k \Phi T f}{10^8} \qquad (59)$$

This form of the fundamental equation of the alternator is applicable to armatures with distributed windings. The following table gives the values of k for various degrees of distribution. The slots for a given winding are always grouped so many per pole and a group of slots may cover 1/4, 1/3, 1/2, etc., of the space

VALUES OF PHASE CONSTANT k FOR DISTRIBUTED WINDINGS.

Number of slots in each group.	Widths of groups of slots in fractional parts of N to S.				
	1/4	1/3	1/2	3/4	Whole.
1	1.000	1.000	1.000	1.000	1.000
2	.980	.966	.924	.831	.707
3	.977	.960	.912	.805	.666
4	.976	.958	.908	.795	.653
Infinity.	.975	.955	.901	.784	.637

Note.—Column headed 1/3 applies to 3-phase alternators. Column headed 1/2 applies to 2-phase alternators. Width of group $= ns$, where n is number of slots in a group and s is distance from center to center of adjacent slots.

from the center of an N pole to the center of an S pole. Thus in Fig. 99 is shown an 8-pole machine, of which the armature is slotted for a distributed winding, there being three slots per pole, these slots being grouped so as to cover ⅓ of the space from an N to an S pole. In the table the width of a group of slots is ns where n is the number of slots in a group and s is the distance from center to center of adjacent slots.

85. Practical and ultimate limits of output.—The dotted curve, Fig. 100, is the characteristic curve of a given alternator. This curve shows the relation between the current output and the electromotive force between the collecting rings, the field excitation being kept constant. The ordinates of the full-line curve represent the power outputs corresponding to the different currents (receiving circuit non-inductive). The maximum output of the alternator is thus 68 kilowatts when the current output is 38 amperes.

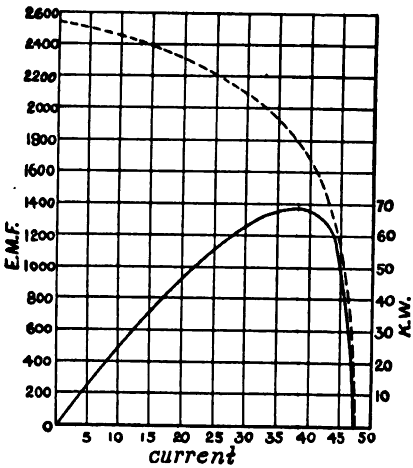


Fig. 100.

In practice the allowable power output of an alternator is limited to a smaller value than this maximum output by one or the other of the following considerations:

(*a*) Electric lighting and power service usually demands an approximately constant electromotive force and it is not permissible to take from an alternator so large a current as to greatly reduce its electromotive force. This difficulty may be largely overcome by providing for an increase of field excitation of the alternator with increase of load as is done in the alternator with a compound field winding. (See Article 94.)

(*b*) The current delivered by an alternator generates heat * in the armature of the alternator and the temperature of the armature rises until it radiates heat as fast as heat is generated in it by the current. Excessive heating of the armature endangers the insulation of the windings and it is not permissible to take from an alternator so large a current as to heat its armature more than 40° or 50° C. above the temperature of the surrounding air. This heating effect of the armature currents usually fixes the allowable output of an alternator, except in those rare cases where extreme steadiness of electromotive force is required, or where the alternator is not compounded.

Influence of inductance upon output.—An alternator is rated according to the power it can deliver steadily to a non-inductive receiving circuit without overheating. The amount of power which an alternator can satisfactorily deliver to an inductive receiving circuit is less than that which it can deliver to a non-inductive receiving circuit, because of the phase difference of electromotive force and current. The cosine of the angle of phase difference (cos θ) is called the power factor of the receiving circuit as before pointed out. The power factor of lighting circuits is very nearly unity.

The power factor of induction motors, synchronous motors and rotary converters is often as low as .75 and sometimes even less.

86. Frequencies.—The frequencies employed in practice range from 20 or 25 to 150 cycles per second. Very low frequencies are not suitable for lighting on account of the tendency to produce flickering of the lights; on the other hand high frequencies, which tend to make transformers cheaper for a given output, are entirely satisfactory and are often employed for lighting.

High frequencies are not well adapted for the operation of induction motors, synchronous motors and rotary converters because high frequencies necessitate either great speed or a great

* Additional heat is generated in the armature by the hysteresis and eddy current losses in the armature core.

number of poles. For such purposes frequencies as low as 25 per second are often employed.

A frequency of 60 has been quite generally adopted for machines used to operate both lights and motors.

87. Speeds. Number of poles.—A machine which is to be belt-driven may be driven as fast as is compatible with the strength and rigidity of the rotating part. The allowable speed of rotation in ordinary dynamos and alternators is such as will give a peripheral velocity of from 4,000 to 6,000 feet per minute. When a machine is direct-connected to an engine or water-wheel its speed is fixed by that of the prime mover.

The number of poles depends upon the speed of an alternator and the frequency it is to give, according to equation (17). Large machines as a rule must run slower than small ones, and they, therefore, have a greater number of poles. The accompanying table gives data as to speed, frequency, and number of poles of a few recent American machines. Machines 7 and 8 are of the direct-connected type.

TABLE.

125 CYCLES.				60 CYCLES.			
No.	No. of poles.	Output K. W.	Speed r. p. m.	No	No. of poles.	Output K. W.	Speed r. p. m.
1	10	60	1500	4	8	75	900
2	14	125	1070	5	12	150	600
3	16	200	937	6	16	250	450
				7	36	250	200
				8	40	750	180

88. Armatures.—Alternator armatures are usually of the drum type or disc type. The former type is almost universal in America, while the disc type is frequently used in England; for example, the Ferranti and Mordey machines have disc armatures. Drum armatures have laminated iron cores similar to the armature cores used for direct-current dynamos, while disc armatures are usually made up without iron. Ring armatures have been used only to a very limited extent.

Drum armatures are, in nearly all modern machines, of the toothed or ironclad type. The conductors are bedded in slots. This has the double advantage of shortening the gap space from pole face to armature core and of protecting the armature conductors from injury. One type of such an armature has already been shown in Fig. 10, the heavy coils being first wound on forms and then pressed into position on the armature core. When distributed windings are used straight slots as shown in Fig. 101 are

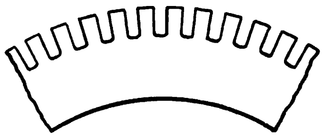


Fig. 101.

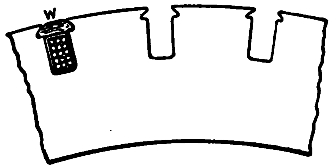


Fig. 102.

often employed. Fig. 102 shows a style of slot commonly used in which the coils are held in position by the wooden wedge *W*. Armature core discs should be varnished, japanned or in some manner insulated from each other to prevent eddy currents. This is especially necessary in the case of alternator armature cores because the frequency is comparatively high..

89. Armature windings.—Any direct-current dynamo * may be converted into a single-phase or polyphase alternator by providing it with collecting rings as explained in the chapter on the rotary converter. Ordinarily, however, the armature windings of alternators are very different from the armature windings of direct-current dynamos. In the type of winding most frequently employed a number of distinct coils are arranged on the armature ; in these coils alternating electromotive forces are induced as they pass the field magnet poles, and these coils are connected in series between the collecting rings if high electromotive force is desired, or in parallel † between the collecting rings if low electromotive force is desired.

* Except the so-called unipolar dynamo.

† The coils of a distributed winding cannot all be connected in parallel between

Single-phase winding.—Fig. 10 shows a common type of single-phase winding having one coil per pole. Fig. 103 shows another type of concentrated single-phase winding having one coil to each pair of poles or one slot per pole. In the diagram, Fig. 103, the heavy sector-shaped figures represent the coils and the light lines represent the connections between the terminals of the coils. The radial parts of the sector-shaped figures repre-

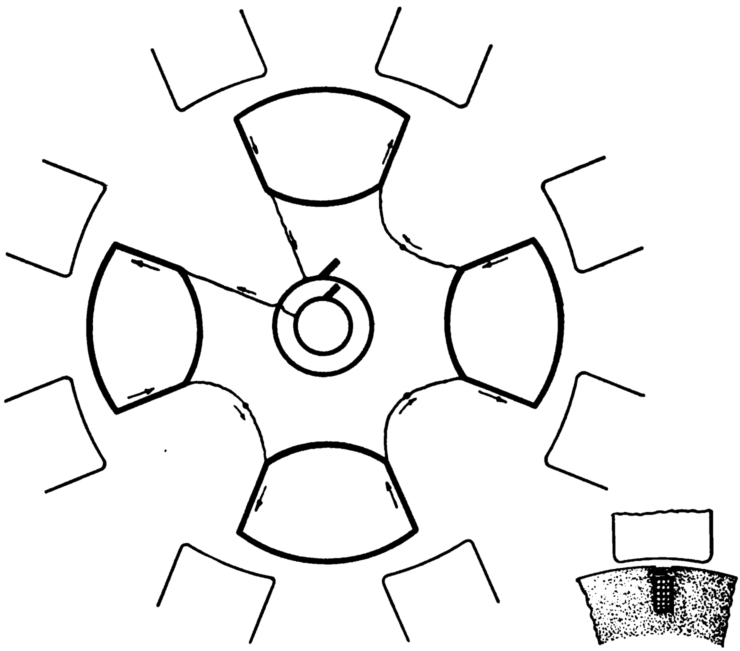


Fig. 103.

sent the portions of the coils which lie in the slots, and the curved parts represent the ends of the coils. The circles at the center of the figure represent the collecting rings, one being shown inside the other for clearness. The arrows represent the direction of the current at a given instant. All electromotive

the collecting rings for the reason that the induced electromotive forces in the various coils are not exactly in phase and local currents would circulate in the coils if connected in parallel.

forces under N poles are in one direction and all electromotive forces induced under S poles are in the opposite direction. These remarks apply to Figs. 103 to 110 inclusive. Fig. 104 represents a single-phase winding distributed in two slots per pole, all the coils being connected in series. Fig. 104 is a type of winding which, for the same number of conductors, has a smaller inductance than the type shown in Fig. 103 and the armature

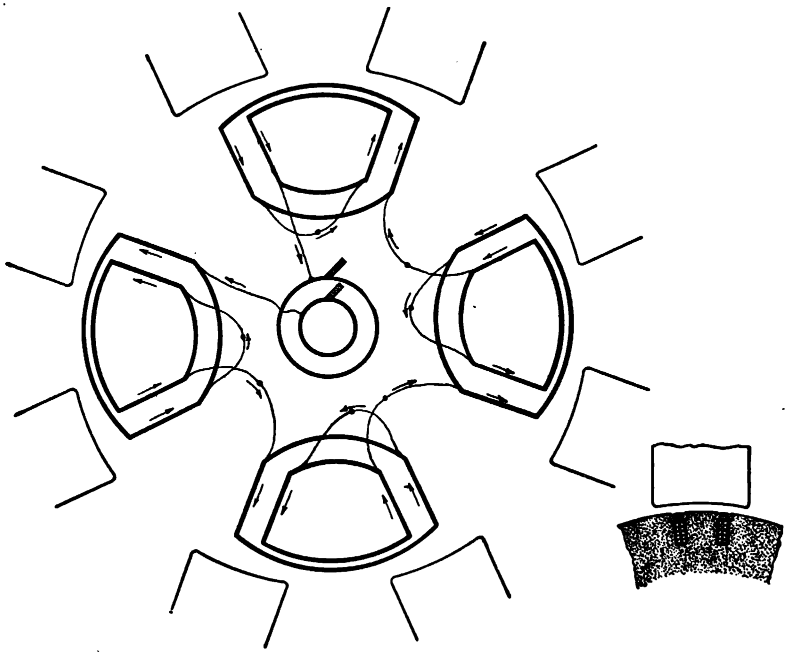


Fig. 104.

shown in Fig. 104, for the same number of conductors, gives a smaller electromotive force than the armature shown in Fig. 103.

Two-phase windings.—The two-phase winding is two independent single-phase windings on the same armature, each being connected to a separate pair of collecting rings, as shown in Figs. 105 and 106. Fig. 105 shows a two-phase concentrated winding, one slot per pole for each phase. Fig. 106 shows a two-phase winding distributed in two slots per pole for each phase.

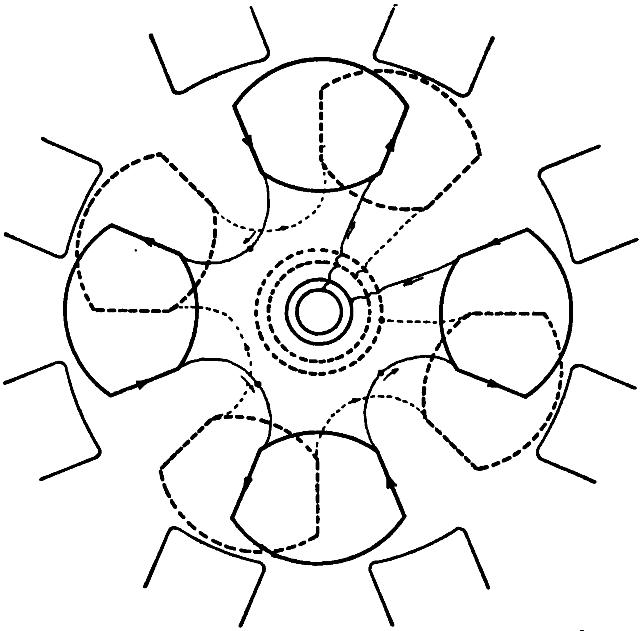


Fig. 105.

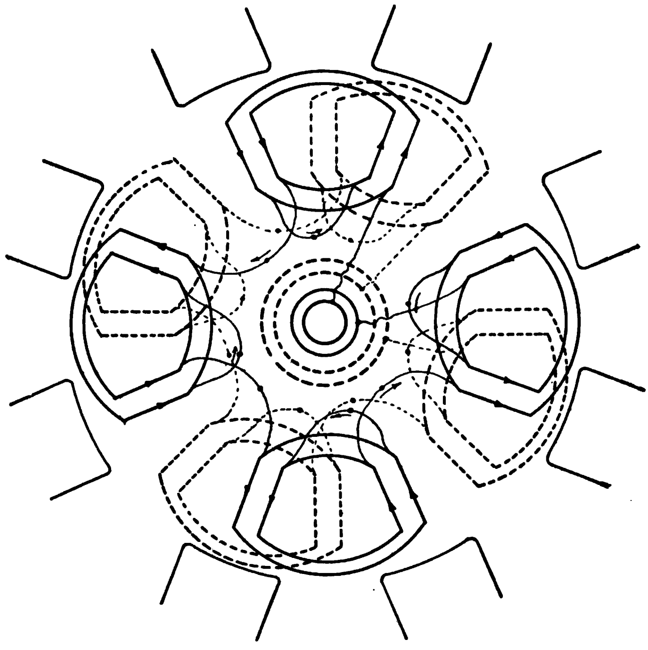


Fig. 106.

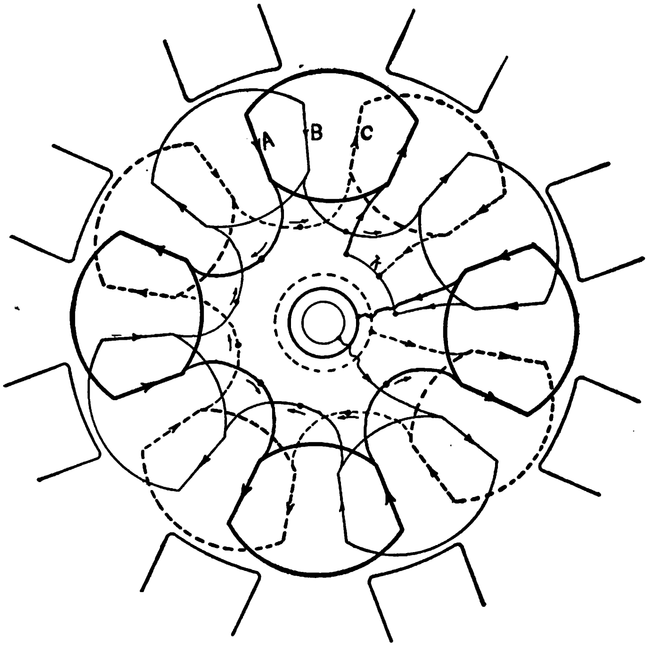


Fig. 107.

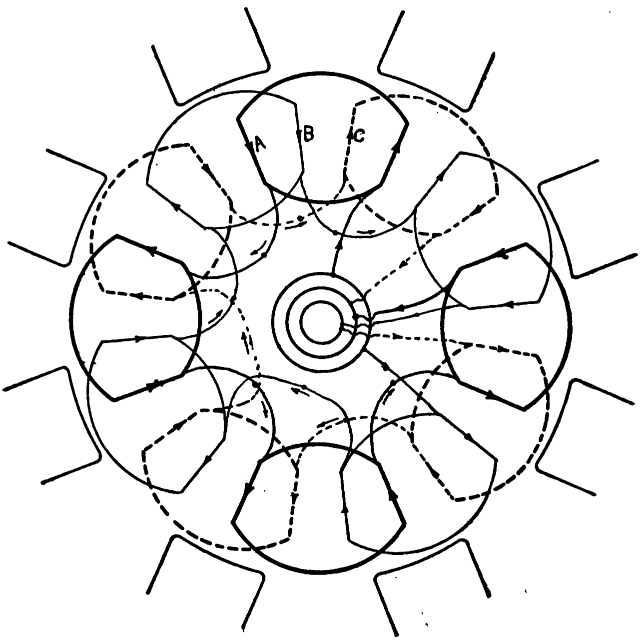


Fig. 108.

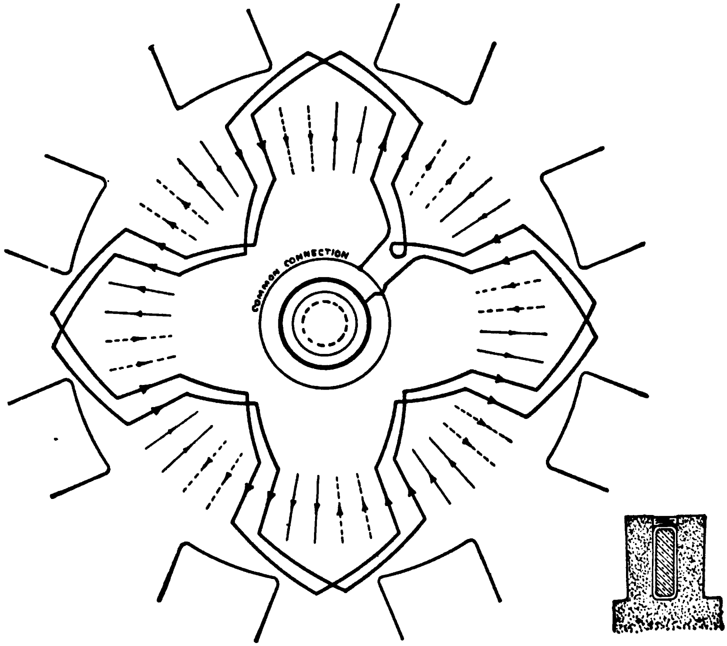


Fig. 109.

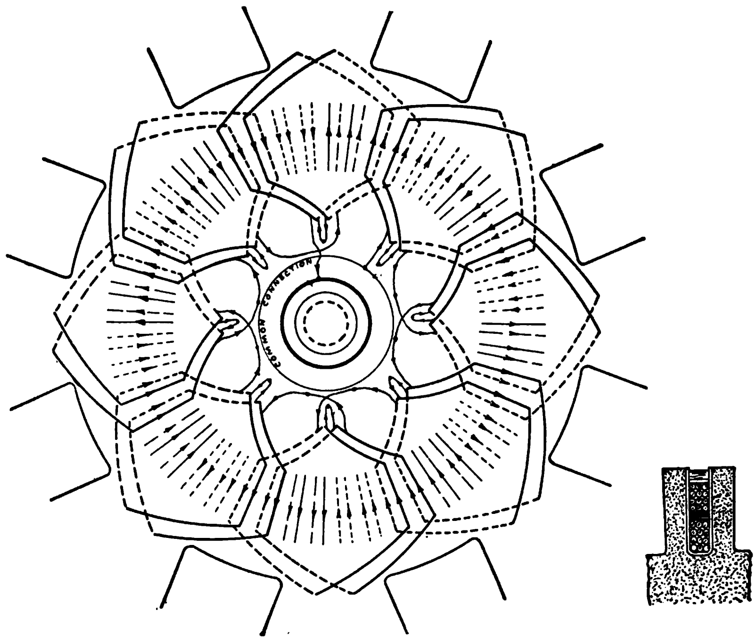


Fig. 110.

Three-phase windings.—The three-phase winding is three independent single-phase windings on the same armature, the terminals of the individual windings being connected according to the Y scheme or Δ scheme, as explained in Article 72. Fig. 107 shows a three-phase concentrated winding, one slot per pole for each phase; Y-connected. Fig. 108 shows the same winding Δ-connected. The Y connection gives $\sqrt{3}$ times as much electromotive force between collecting rings as the Δ connection for the same winding. The Y connection is more suitable for high electromotive force machines and Δ connection for machines for large current output. The line current is $\sqrt{3}$ times as great as the current in each winding in a Δ-connected armature. Fig. 109 shows a three-phase bar * winding distributed in two slots per pole for each phase. Fig. 110 shows a three-phase coil winding distributed in two slots per pole for each phase and arranged in two layers, there being as many coils on the armature as there are slots, so that portions of two coils lie in each slot, one above the other. The portions of the coils represented by full lines lie in the upper parts of the slots and the adjacent dotted portions lie in the bottoms of the same slots.

The Y-connection.—The terminals of the individual windings which are to be connected to the common junction and to the collecting rings may be determined as follows: Consider the instant when winding A is squarely under the pole as shown in Fig. 107; the electromotive force in this winding (and current also if the circuit is non-inductive) is a maximum and the currents in the other two phases B and C are half as great. If winding A is connected so that its current is flowing away from K, windings B and C must be connected so that their currents flow towards K.

The Δ-connection.—The three windings form a closed circuit when Δ-connected. The total electromotive force around this circuit at any instant must be zero. Therefore the electromotive force in winding A when it is directly under the poles must oppose the electromotive forces of windings B and C.

90. Insulation of armatures.—Armatures for alternators must be well insulated in cases where the electromotive force generated is high, as the electromotive force tending to break down the

* One conductor in each slot. This conductor is usually in the form of a copper bar of rectangular cross section.

insulation is the maximum value of the electromotive force generated, and this is considerably greater than the rated or effective electromotive force. Concentrated or partially distributed windings admit of quite a high degree of insulation, inasmuch as the slots may be made quite large and there are comparatively few crossings of the coils at the ends of the armature. Distributed windings can not be so highly insulated because there are many crossings of the coils and the slots are necessarily small; such windings are, therefore, not suitable for the generation of high electromotive forces. Alternators having this type of winding should therefore be used in connection with step-up transformers if a high electromotive force is desired. When it is desired to generate a high pressure directly, it is best to use a machine with a stationary armature. Such armatures have been built for electromotive forces of 8,000 or 10,000 volts, thus doing away with the necessity of step-up transformers for power transmission lines of moderate length. There is usually more room for thorough insulation on such armatures and the insulation is less liable to deteriorate as it is not disturbed in any way by motion of the armature. Moreover, the use of the stationary armature does away with collector rings and brushes (for the armature) and the consequent necessity of their insulation for high potentials.

The individual coils of an alternator armature are generally heavily taped and treated with insulating oil or varnish, the slots are lined with heavy tubes built up of paper and mica and all parts of the core which are near the coils are also covered with a heavy layer of insulating material.

91. Magnetic densities in armature and air gap.—The armature core is usually made of sufficient cross section to insure a fairly low magnetic density. This is done in order to keep down the hysteresis and eddy current losses which would otherwise be high on account of the comparatively high frequencies employed. The allowable magnetic density in the armature core depends largely upon the frequency, since the density for a given loss may

be higher the lower the frequency. The following table from Kolben gives values of the density suitable for various frequencies:

	B. Lines per cm².
40 cycles	6,500 to 5,500
50 "	6,000 " 5,000
60 "	5,000 " 4,500
80 "	4,500 " 4,000
100 "	4,000 " 3,500
120 "	3,500 " 3,000

The allowable magnetic density in the air gap will depend to some extent upon the material used for the pole pieces. With cast-iron poles this density should not exceed 4,000 to 4,500 lines per sq. cm.; with wrought-iron pole pieces it may be as high as 6,000 to 7,000 lines per sq. cm.

92. Current densities.—The current density in early alternator armatures was often very high, not more than 300 circular mils per ampere being allowed in many cases. Such armatures usually ran very hot at full load. The current densities used in modern machines are much lower, from 500 to 700 circular mils per ampere being allowed, as in the case of direct current machines. The armature conductor is usually of ordinary cotton-covered magnet wire in the smaller machines, and when a conductor of considerable cross section is required a number of wires are grouped in multiple. In larger machines copper bars are frequently used, as these admit of a large cross section being put in a minimum space. Wire of rectangular cross section and copper ribbon are also used in some cases.

93. Outline of alternator design.—An alternator is usually designed to give an electromotive force of prescribed value and frequency, and to be capable of delivering a prescribed current without undue heating. To design an alternator is to so proportion the parts as to satisfy the following conditions:

(*a*) The product of revolutions per second into the number of

pairs of field magnet poles must equal the prescribed frequency according to equation (17).

(*b*) Equation (59) namely

$$E = \frac{4.44\, k\Phi Tf}{10^8}$$

must be satisfied, to give the prescribed electromotive force.

(*c*) Peripheral speed of armature must not exceed allowable limits.

(*d*) The armature must have sufficient surface to radiate the total watts lost in the armature (including eddy current and hysteresis losses) without excessive rise of temperature.

There are two distinct cases in the designing of an alternator, as follows:

Case I.—Where the speed is fixed by independent considerations, as, for example, in direct-connected machines. In this case the number of poles is determined by the given speed and prescribed frequency. The diameter of the armature follows from the allowable peripheral speed.* Assuming from 2% to 5% † of total rated output as armature loss, the approximate length of the armature is then determined by the radiating surface required. A well-ventilated armature should radiate from .05 to .06 watt per square inch (of cylindrical surface) per degree Centigrade rise of temperature. This constant of radiation varies greatly with the style of construction of the armature and with peripheral speed. The length may be slightly modified when condition (*b*) comes to be considered. The flux Φ is determined from the flux density in the air gap and the area of each pole face. The combined area of the pole faces is usually about equal to half the cylindrical surface of the armature, or, in other words, the distance between tips of adjacent poles is equal to the breadth of the pole face. The number of armature turns T is then determined from equation (59). The armature turns thus deter-

* Direct-connected dynamos are scarcely ever run up to the allowable peripheral speed. Speeds from 2,200 to 2,600 feet per minute are usual.

† According to size of machine, low percentage being for large machines.

mined may come out an odd or a fractional number and must be adjusted to suit the type of winding employed, that is, to give the required number of coils each having the same number of turns. The length of the armature may then be changed slightly to adjust Φ so that the required electromotive force will be produced with the adopted number of armature turns. The area of cross section of the armature conductors is fixed by the allowable current density and rated current output.

Case II.—When speed is not fixed by independent considerations. In this case a trial combination of poles and speed is adopted, giving a speed suitable for the size of the armature. The remainder of the design is then worked out as above.

Remark.—When a machine is provisionally designed the details of its behavior may be approximately calculated without difficulty, and refinement of design is attained by working out a number of provisional designs and calculating the details of their action; then the most satisfactory design may be recognized and adopted.

The proportioning of the magnetic circuits and the calculation of field windings of an alternator is carried out in the same general way as in the case of a direct-current dynamo.

Remark.—In designing a two-phase alternator each winding is allowed to cover half of the armature surface. In designing a three-phase alternator each winding is allowed to cover one-third of the armature surface.

94. Field excitation of alternators.—The use of an auxiliary direct-current dynamo for exciting the field of an alternator has been pointed out in Article 20. The electromotive force of an alternator excited in this way falls off greatly with increasing current output, and to counteract this tendency an auxiliary field excitation is frequently provided which increases with the current output of the machine. For this purpose the whole or a portion of the current given out by the machine is rectified * and sent through the auxiliary field coils.

* Connections to field coils are reversed with every reversal of main current so that, in the field coils, the current is unidirectional.

Fig. 111 shows an alternator A with its field coils F separately excited from a direct-current dynamo E. The two rheostats R

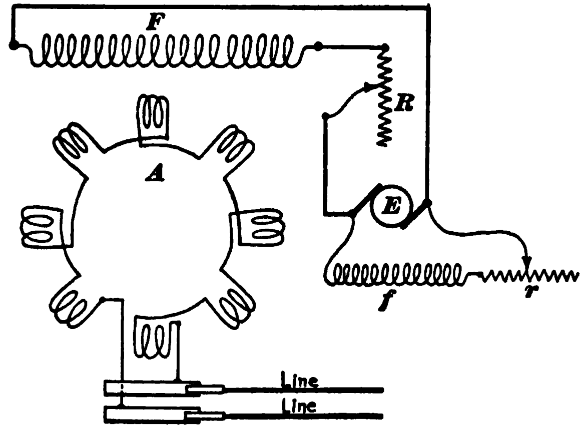


Fig. 111.

and r, in series with the alternator and exciter fields respectively, are used to regulate the field current. Fig. 112 shows an alternator with two sets of field coils F and C. The coils F are separately excited as before. The coils C, known as the series or compound coils, are excited by the main current from the alternator. One terminal of the armature winding is connected directly to a collecting ring. The other armature terminal connects to one set of bars in the rectifying commutator B. From the rectifier the current is led through the winding C, thence back to the rectifier, and

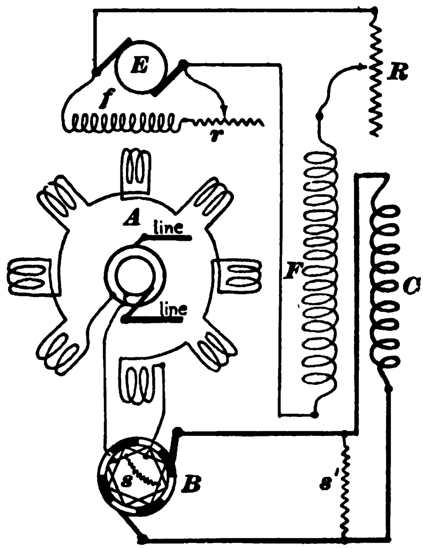


Fig. 112.

thence to the second collecting ring. The rectifying commutator *B* is provided with as many segments as there are poles on the machine. The commutator reverses the connections of the terminals of the coils *C* at every pulsation of the alternating current so that the current flows in *C* always in the same direction. The commutator *B* is fixed to the armature shaft. A shunt *s* moving with the commutator is sometimes used when it is desired to rectify only a portion of the current. A stationary shunt *s'* is also frequently used to regulate the amount of current flowing around the coils *C*, thus giving a method of adjusting the compounding.

Fig. 113 shows an alternator *A* with two sets of field coils *F* and *C* as before. One armature terminal is connected to a collecting ring, and the other armature terminal connects to the primary of a transformer *T* and thence to the other collecting ring. The terminals of the secondary coil of *T* connect to the bars of the rectifying commutator *B*, from which the compound field winding *C* is supplied. The transformer *T* is usually placed inside the armature. All three of the methods, shown in Figs. 111, 112 and 113, are in common use for field excitation of alternators. Compounding is necessary only with alternators which have fairly high armature inductance, and which, with constant field excitation, would give poor regulation. For low

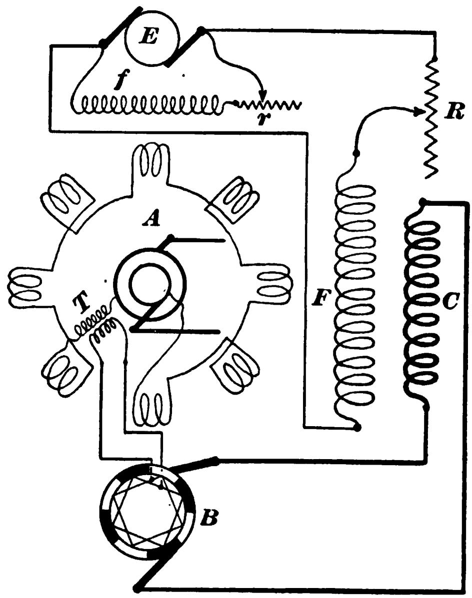


Fig. 113.

inductance machines the separate excitation alone is usually sufficient.

The compensated alternator of the General Electric Company.— An alternator with a compound field winding can be adjusted so that the armature drop is compensated by the compound winding, provided the armature drop is proportional to the current and varies only with the current. In Article 81 it is shown that the armature drop is proportional to $I \cos (\theta' - \theta)$. See discussion of algebraic expression in Article 81 for explanation of I, θ' and θ. A variation of the power factor of the receiving circuit ($\cos \theta$) produces, therefore, variations of armature drop which cannot be compensated by a compound field winding. However, an auxiliary field excitation proportional to $I \cos (\theta' - \theta)$ could be adjusted so as to completely compensate for armature drop. This condition is realized in the *compensated alternator* of Mr. E. W. Rice.

The action of this compensated alternator depends, in the first place, upon the fact that a direct-current dynamo (the exciter) is an alternator, as well as a direct-current dynamo, as explained in Chapter XIII.; in the second place, upon the fact that the current or currents delivered by the alternator may be passed through the exciter armature, so as to exert upon the exciter field a magnetizing action proportional to $I \cos (\theta' - \theta)$ if the exciter is driven at exactly the same frequency as the main alternator and if the phase relation between the alternating electromotive force of the exciter and of the main alternator is properly adjusted; and in the third place upon the fact that the variation thus produced in the field magnetization of the exciter produces a corresponding variation of the direct electromotive force of the exciter, a corresponding variation of its direct-current output and a corresponding variation of field excitation of the main alternator.

The commercial form of Rice's alternator is usually a polyphase alternator with the exciter armature rigidly connected to the shaft of the main alternator. The following description applies to the case in which the exciter armature is mounted upon the

main shaft and rotates with the main armature. The polyphase currents from the main armature pass by direct connection through the exciter armature and thence by way of collecting rings to the polyphase receiving system.

Imagine the main armature to be *standing* in the position in which the polyphase electromotive forces of the main generator are at their maximum values. Then the phase angle ϕ, that the electromotive forces of the exciter, considered as an alternator (its electromotive forces being the electromotive forces between the points at which the main alternating currents enter the exciter armature), lag behind the electromotive forces of the main alternator, depends upon the position of the exciter field magnet poles; and this angle can be changed at will by turning the exciter field magnet about the axis of the machine.

Now the polyphase currents from the main alternator lag $\theta°$ behind the external electromotive forces of the main alternator, so that the alternating currents which pass through the exciter armature are $(\phi - \theta)°$ ahead of the exciter electromotive forces in phase. Therefore these currents have upon the exciter field a magnetizing action proportional to $I \sin (\phi - \theta)$, according to Article 79. If the exciter field be turned until $\phi = 90° - \theta'$, then $\sin (\phi - \theta)$ equals $\cos (\theta' - \theta)$ and the above expression for magnetizing action becomes $I \cos (\theta' - \theta)$. But this is proportional to the armature drop, according to Article 81. Therefore the magnetizing action of the main alternating currents upon the exciter field is proportional to the armature drop in the main alternator.

PROBLEMS.

76. A ten-pole alternator having 720 armature conductors supplies 30 amperes to a receiving circuit of which the power factor is 0.866. Calculate the average demagnetizing ampere turns due to the armature current. Ans. 763 ampere turns.

77. An alternator with a toothed armature like Fig. 10, has

its armature fixed with armature teeth squarely under field poles. 26.7 amperes of 60-cycle current are passed through the armature from an auxiliary source, and the electromotive force between the collector rings is 65 volts.

When the armature is held so that the armature teeth are midway between the field poles, the electromotive force between the collector rings is 43 volts with the same current as before.

When 34 amperes of direct current are sent through the armature, the electromotive force between the collector rings is observed to be 16 volts. What is the inductance of the armature in each position? Ans. 0.006337 henry, 0.004083 henry.

78. Calculate for the above alternator at a frequency of 133 cycles per second, the armature drop for various power factors of receiving circuit and for a fixed current of 30 amperes. Assume the total electromotive force to be so great that it may be assumed to be in phase with the external electromotive force. Plot a curve showing the results, representing values of power factor by abscissas and armature drops by ordinates.

79. A so-called constant current alternator generates a total electromotive force of 1,100 volts and has an armature inductance of 0.12 henry or, at its rated frequency, an armature reactance of 94 ohms. Calculate the current delivered by the alternator to a non-inductive receiving circuit of which the resistance is zero, 10 ohms, 20 ohms, and 30 ohms respectively, neglecting the resistance of the armature. Ans. 11.7, 11.63, 11.45, and 11.15 amperes.

80. A three-phase alternator has its windings connected in series as a single-phase winding. What is its phase constant? Ans. zero or $\frac{2}{3}$.

CHAPTER X.

THE TRANSFORMER.

95. The transformer consists of a laminated iron core upon which two separate and distinct coils of wire are wound. Alternating current is supplied to one of these coils from an alternator or other source. This alternating current produces rapid reversals of magnetization of the iron; and these magnetic reversals induce an alternating electromotive force in the other coil, which delivers alternating current to a receiving circuit. The coil to which the alternating current is supplied is called the *primary coil*, and the coil which delivers current to a receiving circuit is called the *secondary coil*.

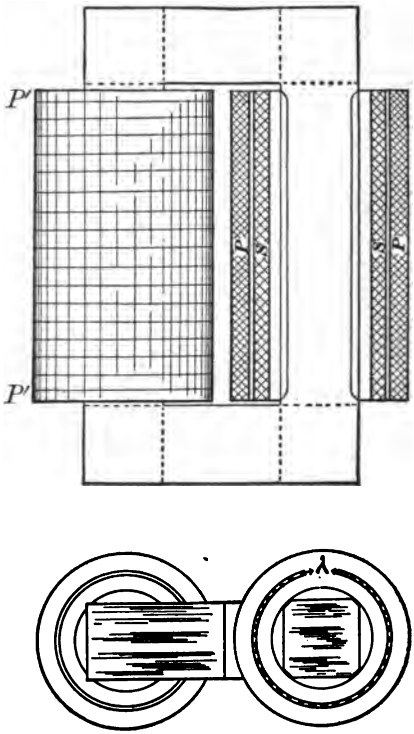


Fig. 114.

Fig. 114 shows the essential features of a commercial type of transformer. The iron core forms a closed magnetic circuit. The primary coil is in two parts, PP and $P'P'$, one of which, PP, is shown in section. The secondary also consists of two parts wound, in

this instance, underneath the primary. One part SS of the secondary is shown in section.

Step-up and step-down transformation.—Usually one coil of a transformer has many more turns of wire than the other. When the coil of few turns is the primary coil the transformer takes large current at low electromotive force and delivers small current at high electromotive force. This is called *step-up* transformation.

When the coil of many turns is the primary the transformer takes small current at high electromotive force and delivers large current at low electromotive force. This is called *step-down* transformation. The object of step-up and step-down transformation is explained in Article 21.

96. The action of the transformer.—In the following discussion N' represents the number of turns of wire in the primary coil and N'' represents the number of turns of wire in the secondary coil. The effect of the resistance of the coils is usually quite small and it is ignored in the present discussion.

Ratio of primary current to secondary current.—Aside from resistance, the only thing which opposes the flow of current through the primary coil is the *reacting* electromotive force induced in the primary coil by the reversals of magnetization of the core. The greater the range of this magnetization the greater the value of the reacting electromotive force. *The combined magnetizing action of the primary and secondary coils is always such as to magnetize the core to that degree which will make the reacting electromotive force in the primary coil equal to the electromotive force of the alternator which is forcing current through the primary coil.* Action is equal to reaction.

When the secondary coil is on open circuit, just enough current flows through the primary coil to produce the degree of magnetization above specified. Let this value of the primary current, which is called the *magnetizing current*, be represented by m. When current I'' is taken from the secondary coil, addi-

tional current I', over and above m, flows through the primary coil. The current m still suffices to magnetize the core, and the magnetizing action of I'' is exactly neutralized by the equal and opposite magnetizing action of I'. The magnetizing action of I'' is measured by the product $N''I''$, and the magnetizing action of I' is measured by the product $N'I'$, so that, ignoring algebraic signs, we have

$$N'I' = N''I''$$

or

$$\frac{I'}{I''} = \frac{N''}{N'} \tag{60}$$

Ratio of primary electromotive force to secondary electromotive force.—The rapid reversals of magnetization of the iron core induce a certain electromotive force a in each turn of wire surrounding the core. Therefore the total electromotive force induced in the primary coil is $N'a$. This is the reacting electromotive force in the primary coil and it is equal and opposite, as pointed out above, to the electromotive force E' which is pushing current through the primary coil; so that, ignoring signs, we have

$$E' = N'a$$

Similarly, the total electromotive force, E'', induced in the secondary coil is

$$E'' = N''a$$

Therefore

$$\frac{E'}{E''} = \frac{N'}{N''} \tag{61}$$

Remark.—The above discussion should in strictness refer primarily to *instantaneous* values of I' and I'' and to *instantaneous* values of E' and E''. Thus i' and i'' are at each instant opposite to each other and in the ratio N''/N'; and e' and e'' are at each instant opposite to each other and in the ratio N'/N''. Therefore I' and I'' are opposite or 180° apart in phase and E' and E'' are opposite or 180° apart in phase.

Approximate equality of input and output of power.—If the

secondary coil of a transformer delivers current to an inductive receiving circuit, then E'' and I'' will differ in phase by a certain angle θ; and since E' and I' are opposite to E'' and I'' respectively, therefore the phase difference between E' and I' is also θ. The power output of the transformer is $E''I''\cos\theta$, the power intake is $E'I'\cos\theta$, and these are equal since $E'I' = E''I''$ according to equations (60) and (61). The resistance of the coils is here ignored and the energy taken from the supply dynamo by the magnetizing current m is not considered.

97. Particular cases (*for harmonic electromotive force and current*). 1. *Non-inductive receiving circuit.*—In this case E'' and I'' are in phase and therefore E' and I' are in phase also. The state of affairs is represented in Fig. 115. The line $O\Phi$ repre-

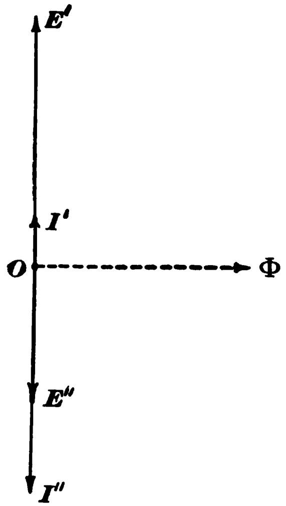


Fig. 115.

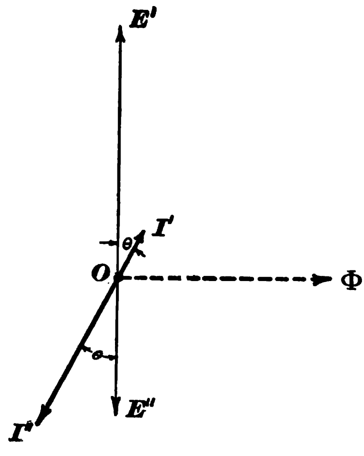


Fig. 116.

sents the harmonically varying core flux, OE' represents the electromotive force acting on the primary and OE'' represents the electromotive force induced in the secondary.

2. *Inductive secondary receiving circuit.*—In this case I'' lags behind E'' by the angle whose tangent is $\frac{\omega L}{R}$, where L is the in-

ductance and R the resistance of the receiving circuit. Also I' lags behind E' by the same angle. The state of affairs is shown in Fig. 116.

3. *Receiving circuit containing a condenser.*—In this case I'' is ahead of E'' by the angle whose tangent is $\left(\frac{1}{\omega C} - \omega L\right) \div R$, where C is the capacity of the condenser, L is the inductance of the connecting wires and R the irresistance. Also, I' is ahead of E' by the same angle. The state of affairs is shown in Fig. 117.

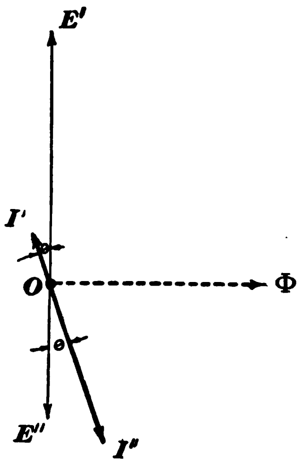


Fig. 117.

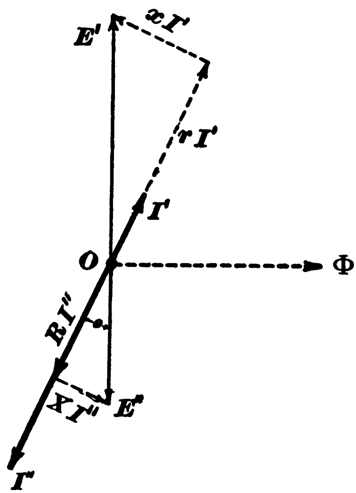


Fig. 118.

98. Equivalent resistance and reactance of a transformer feeding a given receiving circuit.—The primary of a transformer takes from the mains a definite current at a definite phase lag when the secondary is supplying current to a given circuit. Consider a simple circuit of resistance r and reactance x which, connected to the mains, *takes the same current as the primary of the transformer and at the same phase lag.* The circuit is said to be *equivalent* to the transformer and its secondary receiving circuit, and r and x are called the *equivalent primary resistance and reactance* respectively of the secondary receiving circuit.

Resolve the primary electromotive force E', Fig. 118, into components parallel to I' and perpendicular to I' as shown. The component parallel to I' is rI' and the component perpendicular to I' is xI'. The triangle whose sides are E', rI' and xI' is similar to the triangle whose sides are E'', RI'' and XI'', R and X being the given resistance and reactance of the secondary receiving circuit. Therefore

$$\frac{xI'}{XI''} = \frac{E'}{E''}$$

and

$$\frac{rI'}{RI''} = \frac{E'}{E''}$$

But

$$\frac{E'}{E''} = \frac{N'}{N''}$$

and

$$\frac{I'}{I''} = \frac{N''}{N'}$$

so that

$$\left.\begin{aligned} r &= \left(\frac{N'}{N''}\right)^2 R \\ x &= \left(\frac{N'}{N''}\right)^2 X \end{aligned}\right\} \qquad (62)$$

That is, a transformer supplying current from its secondary to a circuit of resistance R and reactance X, takes from the mains the same current at the same phase lag as would be taken by a circuit of resistance $\left(\frac{N'}{N''}\right)^2 R$ and of reactance $\left(\frac{N'}{N''}\right)^2 X$ connected directly to the mains.

99. Maximum core flux.—When the electromotive force acting on the primary of a transformer is harmonic there is a simple and important relation between E', ω, N', and the maximum value of the core flux Φ. Let e' be the instantaneous value of

the primary electromotive force. Since e' is assumed to be harmonic we have

$$e' = E' \sin \omega t \tag{i}$$

Let ϕ be the instantaneous flux through the core. Then $d\phi/dt$ is the instantaneous value of the electromotive force induced in each turn of wire, so that

$$e' = N' \frac{d\phi}{dt}$$

or

$$\frac{d\phi}{dt} = \frac{E'}{N'} \sin \omega t \tag{ii}$$

Therefore

$$\phi = \frac{E'}{\omega N'} \cos \omega t + \text{a constant} \tag{iii}$$

The constant of integration is known to be zero inasmuch as during the reversals of magnetization of the core the flux passes through positive and negative values alike. Therefore equation (iii) becomes

$$\phi = \frac{E'}{\omega N'} \cos \omega t \tag{iv}$$

The coefficient $E'/\omega N'$ is the maximum value reached by ϕ since the maximum value of $\cos \omega t$ is unity. Therefore, representing the maximum value of the core flux by Φ, we have

$$\Phi = \frac{E'}{\omega N'}$$

or, since the maximum value E' of the primary electromotive force is equal to $\sqrt{2}$ times its effective value E', we have

$$\Phi = \frac{\sqrt{2}E'}{\omega N'} \tag{63}$$

100. Transformer losses.—The power output of a transformer is less than its power intake because of the losses in the transformer. These losses are: (*a*) The iron or core losses due to

eddy currents and hysteresis; and (*b*) the copper losses due to the resistances of the primary and secondary coils.

The iron losses are practically the same in amount at all loads, and they depend upon the frequency and range of the flux density B, upon the quality and volume of the iron, and upon the thickness of the laminations.

The hysteresis loss in watts is

$$W_h = aVfB^{1.6} \tag{64}$$

where f is the frequency in cycles per second, B is the maximum flux density in lines per square centimeter, V is the volume of the iron in cubic centimeters, and a is a constant depending upon the magnetic quality of the iron. For annealed refined wrought iron the value of a is about 3×10^{-10}.

The eddy current loss in watts is:

$$W_e = bVf^2l^2B^2 \tag{65}$$

where l is the thickness of the laminations in centimeters, and b is a constant depending upon the specific electrical resistance of the iron. For ordinary iron the value of b is about 1.6×10^{-11}. Insufficient insulation of laminations causes excessive eddy current loss.

Remark.—Equations (64) and (65) may be used for calculating the hysteresis and eddy current losses in any mass of laminated iron subjected to periodic magnetization, such as alternator armatures and the rotor and stator iron in an induction motor.

The copper loss is:

$$W_c = R'I'^2 + R''I''^2 \tag{66}$$

This loss is nearly zero when the transformer is not loaded; it increases with the square of the current, and becomes excessive when the transformer is greatly overloaded.

101. Efficiency of transformers.—The ratio *power output* ÷ *power intake* is called the efficiency of a transformer. The ac-

companying table shows the full-load efficiencies of various sized transformers of a recent type.

TABLE OF TRANSFORMER EFFICIENCIES.

OUTPUT KILOWATTS.	PER CENT EFFICIENCY FULL LOAD.
1	94.8
2	95.75
3	96.2
4	96.45
5	96.65
6	96.73
7	96.8
8	96.85
9	96.9
10	96.95
15	97.2

The efficiency of a given transformer is very low when the output is small; it increases as the output increases, reaches a maximum, and falls off again when the output is very great. This falling off of efficiency when the output is great is due to the great increase of copper losses. Fig. 119 shows the efficiency of a transformer at various loads.

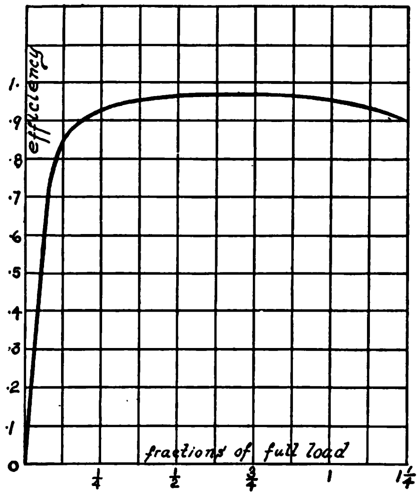


Fig. 119.

Calculation of efficiency.—The transformer output (non-inductive receiving circuit) is $E''I''$. The internal loss is $W_h + W_e + W_c$, so that the intake is $E''I'' + W_h + W_e + W_c$, and the efficiency is:

$$\eta = \frac{E''I''}{E''I'' + W_h + W_e + W_c} \tag{67}$$

All-day efficiency.—Usually a transformer is connected to the

mains continuously, and current is taken from the secondary for a few hours only, each day. In this case the iron loss is incessant and the copper loss is intermittent. The total work given to the transformer during the day may greatly exceed the total work given out by it, especially if the incessant iron losses are not reduced to as low a value as possible. The ratio *total work given out by the transformer* $\div$ *total work received by the transformer during the day* is called the all-day efficiency of the transformer.

102. Practical and ultimate limits of output of a transformer.—When the secondary current of a transformer is increased the secondary electromotive force generally drops off, and the power output increases with the current and reaches a maximum as in the case of the alternator. This maximum power output is the ultimate limit of output of the transformer. Practically the output of a transformer is limited to a much smaller value than this maximum output because of the necessity of cool running, because in most cases it is necessary that the secondary electromotive force be nearly constant, and because the efficiency of a transformer is low at excessive outputs.

Small transformers have relatively large radiating surfaces and in such transformers the requirements of close regulation, as a rule, determine the allowable output.

Large transformers have relatively small radiating surfaces and their allowable output is limited by the permissible rise in temperature. Very large transformers are usually provided with air passages through which air is made to circulate by a fan. Sometimes transformers are submerged in oil, which, by convection, carries heat from the transformer to the containing case, where it is radiated.

Large transformers are much more efficient, under full load, than small ones, and give closer regulation.

103. Rating of transformers.—A transformer is rated according to the power it can deliver steadily to a non-inductive receiv-

ing circuit without undue heating; and the ratio of transformation, together with a specification of the frequency and effective value of the primary electromotive force to which the transformer is adapted, are given.

The rating of a transformer is by no means rigid. Thus, if a transformer is used to give more than its rated output it will become somewhat more heated by the internal losses and its regulation will not be so close. If a transformer is used for a primary electromotive force greater than its rated electromotive force or for a frequency lower than its rated frequency, the range of flux density B in the core will be increased, which will increase the core losses. Some manufacturers rate their transformers generously, so that they may be greatly overloaded or used with greatly increased primary electromotive force or decreased frequency without difficulty.

104. Outline of transformer design.—A transformer is usually designed to take current from mains at a prescribed electromotive force and frequency, and to deliver current at a prescribed electromotive force to a receiving circuit. The transformer must be so proportioned and of such size as to deliver the prescribed amount of current steadily without undue heating and without any great variation of its secondary electromotive force from zero to full load.

In the designing of a transformer there is but one condition which must be precisely met, namely, the ratio of primary to secondary turns must be equal to the ratio of the prescribed primary and secondary electromotive forces. All other points in design are to a great extent matters of choice guided in a general way by experience.

The accompanying table gives magnetic flux densities which are usually employed in transformer cores.

The allowable temperature rise varies greatly with different makers, the extent of radiating surface required per watt of loss per degree rise of temperature varies between extremely wide limits, and no simple rule can be given covering this matter.

MAGNETIC DENSITIES B FOR TRANSFORMER CORES.

FREQUENCY.	SMALL TRANSFORMERS.	MEDIUM SIZE TRANSFORMERS.	LARGE TRANSFORMERS.
25	7500	6750	6000
40	6500	5750	5000
60	5000	4750	4500
80	4500	4250	4000
100	4000	3750	3500
120	3500	3250	3000

Given the required power output* of a transformer, the value and frequency of the primary electromotive force and the value of the secondary electromotive force, the design of the transformer is conveniently determined as follows:

Find from the table the efficiency which can probably be attained, and calculate the total transformer loss at full load. Of this total loss about half should be iron loss and half copper loss.† The total iron loss is:

$$W_i = aVfB^{1.6} + bVf^2l^2B^2 \tag{68}$$

according to equations (64) and (65).

Having decided upon maximum flux density B (see accompanying table) and upon thickness‡ of laminations l, equation (68) gives the volume V of iron to be used in the transformer core. The core may be made of the type shown in Fig. 120 or of the type shown in Fig. 121. The proportions (relative dimensions) indicated in Figs. 120 and 121 will be found to give satisfactory results although the form of the core may be considerably modified without greatly affecting the action of the transformer. In fact, it is usually necessary to modify the core slightly after the coils have been designed.

* Rated output is the output which the transformer can deliver satisfactorily to a *non-inductive* circuit.

† If the transformer is to be connected to the mains all day, but is to deliver current only four hours per day, for example, then the iron loss during 24 hours should be about equal to the copper loss during four hours, or under the full load the copper loss should be several times as great as the iron loss.

‡ 12 to 16 thousandths of an inch is the thickness usually employed.

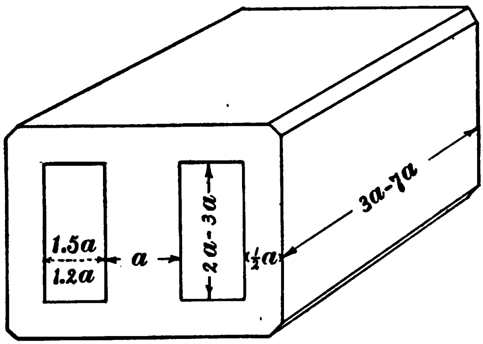


Fig. 120.

The maximum core flux Φ is equal to the product of the sectional area of the magnetic circuit (where it passes through the

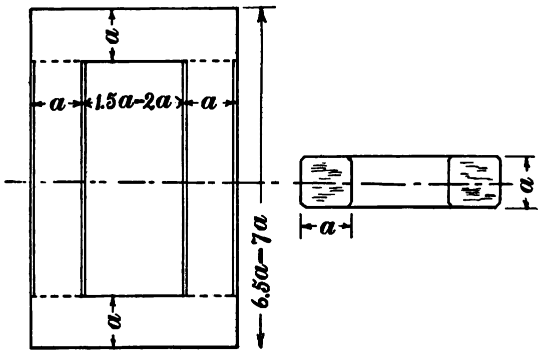


Fig. 121.

coils) into the maximum flux density B. Then equation (63) determines the primary turns, namely,

$$N' = \frac{\sqrt{2}E' \, 10^8}{\omega\Phi}.$$

or

$$N' = \frac{E' \, 10^8}{4.44\Phi f} \tag{69}$$

The number of secondary turns is then determined by equation (61) namely,

$$N'' = \frac{E''}{E'} N' \tag{70}$$

From the provisionally designed core the mean length of a turn of primary and of secondary coils may be determined which together with N' and N'' gives the total lengths of wire in primary and secondary. The size of this wire is then easily chosen so that $R'I'^2$ and $R''I''^2$ may be each equal to half the full-load copper loss.

The size of wires being thus determined the space necessary for the coils and insulation can be estimated. If the provisionally designed core gives more or less space than is required for the coils its dimensions may be altered to suit.

TRANSFORMER CONNECTIONS.

105. Simple connection. In parallel. In series.—When used to supply current to lamps or motors from constant potential

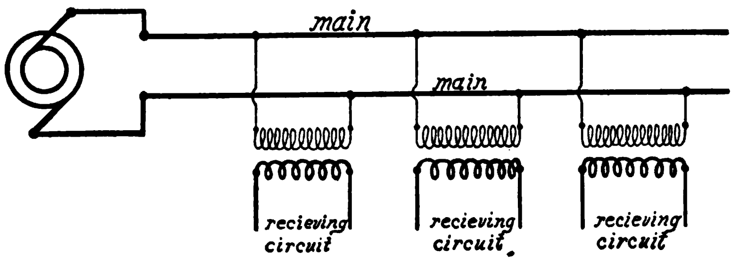


Fig. 122.

mains the primary of the transformer is connected to the mains and the secondary of the transformer is connected to the terminals of the receiving circuit. When a number of receiving circuits

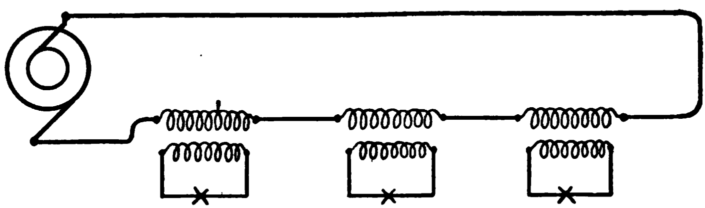


Fig. 123.

are supplied through separate transformers the primaries of the transformers are connected in parallel, as shown in Fig. 122.

When current is supplied through transformers to a number of arc lamps from a constant current alternator the transformer primaries are connected in series and the lamps are connected to the respective secondaries, as shown in Fig. 123. This arrangement is seldom employed.

106. Transformers with divided coils.—Alternators for isolated lighting plants give usually 1,000 or 2,000 volts electromotive

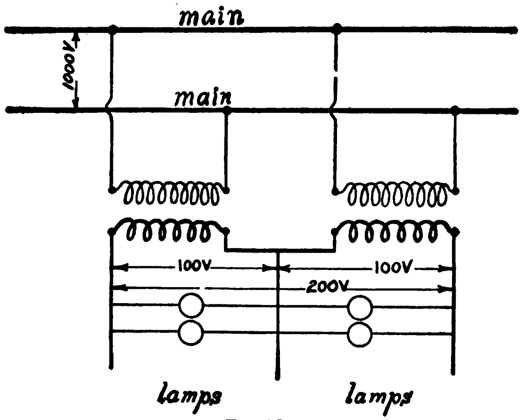


Fig. 124.

force and the standard electromotive forces for incandescent lamps are 55 and 110 volts. Transformers are frequently made with two primary coils, which may be connected in series for 2,000 volts or in parallel for 1,000 volts, and with two secondary coils, which may be connected in series to give 110 volts or in parallel to give 55 volts.

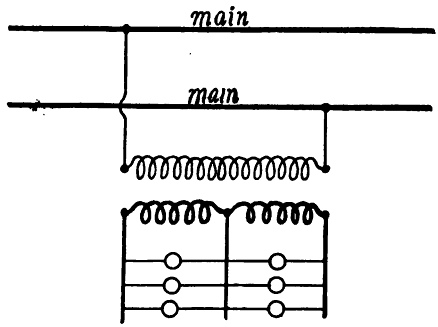


Fig. 125.

Transformers for supplying current for testing purposes are frequently made with a number of secondary coils, which may be connected to give high or low electromotive forces as desired.

Transformers for supplying current to the Edison three-wire system.—For this purpose two similar transformers may be used as shown in Fig. 124, or a single transformer with two secondary coils may be used as shown in Fig. 125.

107. The autotransformer.—The two coils of a transformer may be connected in series between supply mains and the receiving circuit connected to the terminals of either coil as shown in Fig. 126; or either coil of the transformer may be connected to the supply mains and current delivered to the receiving circuit through the two coils in series as shown in Fig. 127. A trans-

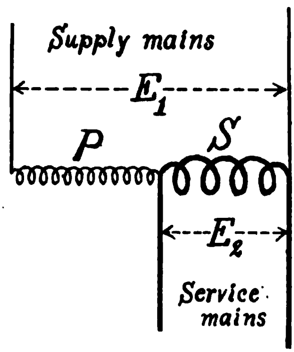


Fig. 126.

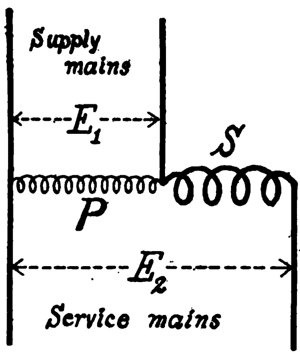


Fig. 127.

former arranged in this way is called an *autotransformer*. This arrangement of a transformer is especially advantageous when the ratio of supply electromotive force to receiver electromotive force is nearly unity.

Electromotive force relations.—Let P and S be the electromotive forces induced in the respective coils by the alternations of the core flux. These electromotive forces are proportional to the number of turns of wire in the respective coils.

From Fig. 126 we have

$$E_1 = P \pm S$$

$$E_2 = S$$

so that

$$\frac{E_1}{E_2} = \frac{P \pm S}{S} \qquad (a)$$

The sign is + or — according as the coils are so connected that the induced electromotive forces are in the same direction or in opposite directions respectively between *a* and *b*. If the receiving circuit is connected to P in Fig. 126 we have

$$\frac{E_1}{E_2} = \frac{P \pm S}{P} \qquad (a')$$

From Fig. 127 we have

$$E_1 = P$$

$$E_2 = P \pm S$$

so that

$$\frac{E_1}{E_2} = \frac{P}{P \pm S} \qquad (b)$$

If the supply mains are connected to S in Fig. 127 we have

$$\frac{E_1}{E_2} = \frac{S}{P \pm S} \qquad (b')$$

In equations (*a*) and (*b*) P and S may be simply the numbers of turns of wire in the respective coils.

Current relations.—The magnetizing actions of the currents in the coils P and S are at each instant equal and opposite (core reluctance zero). Therefore, the currents in coils P and S are inversely proportional to the number of turns, so that we may represent by P the current in the coil S, and by S the current in the coil P. These currents may be made to flow in the same or in opposite directions through the circuit PS by reversing the connections of one coil, and the current in the main which connects to the middle terminal of the two coils is $P \pm S$, according as the currents flow at a given instant in opposite directions, or in the same direction with reference to the circuit PS.

Let I_1 be the current supplied to the autotransformer and I_2

the current delivered to the receiving circuit. Then, on the basis of the above considerations, we have from Fig. 126

$$I_1 = S$$

and

$$I_2 = P \pm S$$

so that

$$\frac{I_1}{I_2} = \frac{S}{P \pm S} \qquad (c)$$

If the receiving circuit is connected to P in Fig. 126, we have

$$\frac{I_1}{I_2} = \frac{P}{P \pm S} \qquad (c')$$

From Fig 127 we have

$$I_1 = P \pm S$$

$$I_2 = P$$

so that

$$\frac{I_1}{I_2} = \frac{P \pm S}{P} \qquad (d)$$

If the supply mains are connected to S in Fig. 127, we have

$$\frac{I_1}{I_2} = \frac{P \pm S}{S} \qquad (d')$$

High duty of autotransformer.—Only a portion of the power delivered by an autotransformer is transformed through the medium of the iron core from one coil to the other. The remainder is supplied to the receiving circuit direct by virtue of the series connection. Thus, to take an extreme case, if an autotransformer takes 11 amperes from 100-volt mains and delivers 10 amperes at 110 volts, of the total 1,100 watts only 100 watts are actually transformed, as may readily be shown by a careful scrutiny of the above discussion. That is, the transformer, so far as the size of its wires and its induced electromotive forces are concerned, is a transformer which would be rated as a 100-watt transformer.

Remark.—It is not allowable in practice to connect transformers as autotransformers since this manner of connection involves

the connection of low electromotive force service mains to the high electromotive force transmission mains, and a ground connection on the transmission mains becomes dangerous.

108. Arrangements of transformers in polyphase systems (*without changing the number of phases*).—In general, step-up and step-down transformation of polyphase currents is accomplished by using an independent transformer for each phase. In case of a three-wire three-phase system, the primaries of the three separate transformers may be Δ-connected or Y-connected to the supply mains, and the three secondaries may be Δ-connected or Y-connected to the service mains.

In a three-wire three-phase system, transformation may be accomplished by connecting *two transformers* exactly as they would be connected for a three-wire *two-phase* system, as shown in Fig.

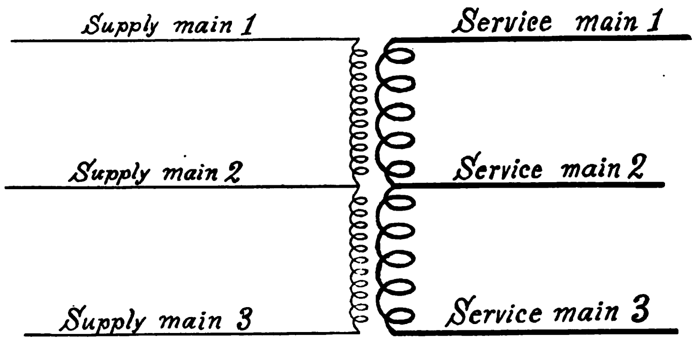


Fig. 128.

128. This arrangement is frequently used in practice. When the secondaries are properly connected to the service mains the electromotive forces *ab*, *bc*, and *ca* are equal in value and 120° apart in phase. The reversing of the connections of the secondary of the one or the other transformer gives the following electromotive forces between the service mains, namely, *ab* and *bc* are equal in value and 60° apart in phase, and *ca* is $\sqrt{3}$ times as large as *ab* and *bc* and midway between them in phase.

109. Two-phase three-phase transformers.—Transformers are frequently used to transform from two-phase supply mains to three-phase service mains or *vice versa*. The principle involved in this kind of transformation is best brought out by considering the following preliminary problem, namely:

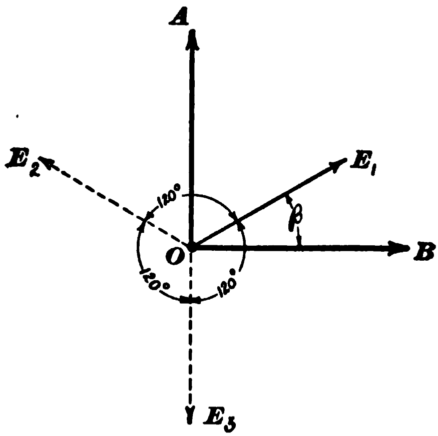


Fig. 129.

To produce an electromotive force of any specified value and phase.—Let A and B, Fig. 129, be the two electromotive forces of a two-phase dynamo and let it be required to produce an electromotive force E_1 of any given value and phase. The component of E_1 parallel to A is $E_1 \sin \beta$ and the component of E_1 parallel to B is $E_1 \cos \beta$. Fig. 130 shows two distinct transformers with similar primary coils, one connected to phase A, the other to phase B. A secondary a may be wound upon the one transformer to give the component $E_1 \sin \beta$; and a secondary b may be wound upon the other transformer to give the other component $E_1 \cos \beta$. These two secondaries when connected in series give the desired electromotive force E_1. Similarly any other electromotive force, such as E_2 or E_3, Fig. 129, may be produced by a pair of properly proportioned secondary coils.

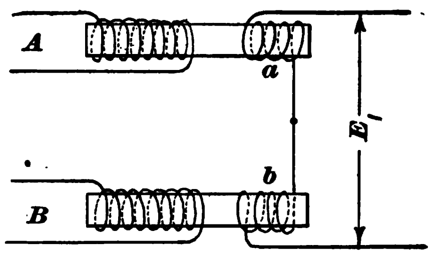


Fig. 130.

The two-phase three-phase transformer consists of two distinct transformers A and B, Fig. 130, wound with similar primary coils to which the two-phase electromotive forces are connected.

Each of the three-phase electromotive forces is (in general) generated in a pair of secondary coils, one on each transformer. Such a pair of coils constitutes a three-phase *unit.* The three units may be connected according to the Δ scheme or Y scheme to the service mains. In the first case the electromotive forces between the three-phase mains *are* the electromotive forces produced in the respective pairs of coils. In the second case the electromotive forces between the mains are related to the electromotive forces generated by the respective pairs of coils as explained in Article 73. Such a transformer transforms equally well from three-phase to two-phase or from two-phase to three-phase.

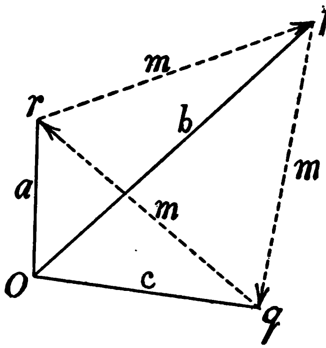


Fig. 131.

The Scott transformer.—To understand the Scott transformer, which is the simplest kind of two-phase three-phase transformer, it is helpful to consider the most general possible type of two-phase three-phase transformer as follows: Consider any point O, Fig. 131, from which are drawn three lines, a, b and c, terminating at the corners of an equilateral triangle pqr. If three pairs of coils are arranged on the cores A and B, Fig. 130, one pair to give the electromotive force a, another pair to give the electromotive force b, and the third pair to give the electromotive force c, Fig. 131, then these three pairs of coils Y-connected to three mains would give the symmetrical three-phase electromotive forces mmm, Fig. 131. The three-phase units of this general type of two-phase three-phase transformer cannot be Δ-connected.

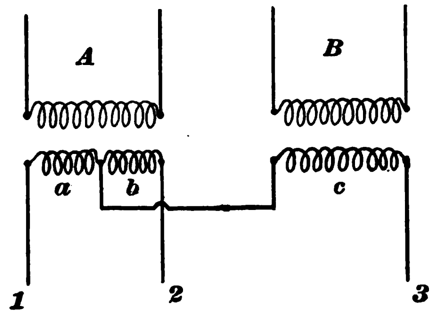


Fig. 132.

Scott's transformer consists of two cores with similar primaries A and B, Fig. 132. These two primaries are connected to the two-phase mains. One core has two similar secondaries a and b, and the other core has a single secondary c having $\sqrt{3}$ times as many turns as a or b. The coils a, b and c are Y-connected to the three-phase mains 1, 2 and 3, as shown. The point O, Fig. 131, lies, for Scott's transformer, midway between the points p and r as shown in Fig. 133. In this figure a, b and c represent the respective electromotive forces induced in the secondary coils a, b and c, Fig. 132. The two-phase electromotive forces A and B are parallel to a and to c respectively, as shown. Scott's transformer cannot be Δ-connected.

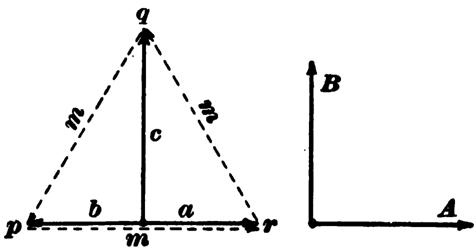


Fig. 133.

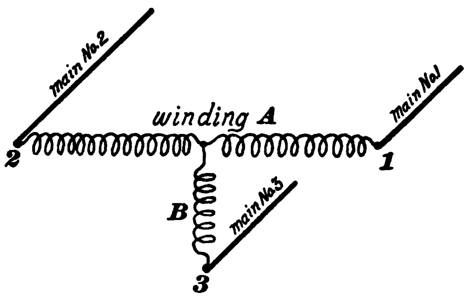


Fig. 134.

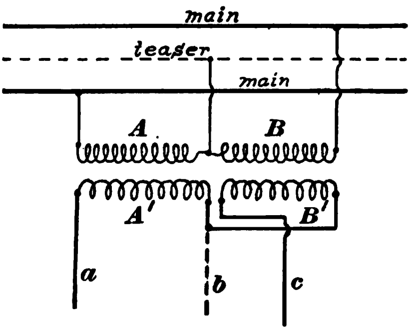


Fig. 135.

110. The monocyclic system.—The monocyclic generator of the General Electric Company is a polyphase dynamo not strictly to be called two-phase or three-phase. It is employed in stations where a small portion of the output is used for motors and a large portion for lighting. The armature winding of the monocyclic generator is essentially a two-phase winding. The A winding has four times as many conductors as the B winding,

and one end of the B winding is connected to the middle point of the A winding, as shown in Fig. 134. The three collecting rings are indicated by 1, 2 and 3. Main 3 is called the *teaser*.

Lamps, or transformers feeding lamps, are connected to mains 1 and 2, and two similar transformers connected, as shown in Fig. 135, are used to supply three-phase currents to induction motors.

PROBLEMS.

81. The primary coil of a transformer has 60 turns of wire, it takes 750 amperes from an alternator at 110 volts and steps up to 20,000 volts. How many turns of wire are there in the secondary coil? A usual allowance in transformers is 500 circular mils of sectional area of wire for each ampere of current. Find size of primary wire and of secondary wire of the above transformer. Ans. 10,909 turns, 375,000 circular mils, 2,062.5 circular mils.

82. A transformer has 800 turns of wire on the primary coil, 40 mils in diameter, and 80 turns on the secondary, secondary wire being ten times as large in sectional area. The sectional area of the magnetic circuit of the iron core is 12 square inches. Allowing 500 circular mils of conductor per ampere, and allowing a maximum flux density of 4,000 lines per square centimeter, calculate the electromotive force, current and power ratings of the transformer at 60 cycles per second. Ans. 656 volts, 32 amperes, 2.099 kilowatts.

83. A given transformer is rated at 5 kilowatts and is designed to take current from 1,100-volt mains at a frequency of 60 cycles per second. Under these conditions W_h, W_e and W_c will be called normal.

(*a*) The transformer is used at 6 kilowatts output at the rated electromotive force and frequency. Find W_c in terms of normal.

(*b*) The transformer is used at rated electromotive force but at a frequency of 75 cycles per second. Find W_h and W_e each in terms of normal.

(*c*) The transformer is used at rated frequency but with primary electromotive force of 1,500 volts. Find W_h and W_e each in terms of normal.

(*d*) The transformer is used on primary electromotive force of 1,500 volts. Find f for which W_h is normal. Show that with given E, W_e is independent of frequency.

(*e*) With primary electromotive force of 1,500 volts what load would give normal W_c?

Ans. (*a*) 1.44 × normal W_c; (*b*) 0.87 × normal W_h, no change in W_e; (*c*) 164 × normal W_h, 186 × normal W_e; (*d*) 36.5 cycles per second.

84. A transformer has 1,300 turns of wire in its primary coil and 130 turns in its secondary coil. The primary coil takes current from 1,100-volt mains and the secondary coil delivers 200 amperes to a receiving circuit of which the power factor is 0.85. What is the equivalent primary resistance and reactance of the secondary receiving circuit? Ans. $r = 21.67$ ohms, $x = 13.44$ ohms.

85. The sectional area of the core of the transformer in problem 81 is 161 square centimeters. Find the maximum core flux and maximum flux density at a frequency of 60 cycles per second. Ans. $\Phi = 691{,}000$, $B = 4{,}250$.

86. A bundle of iron wires of which the total sectional area is 12.6 square centimeters is to be magnetized by alternating current taken from 133-cycle 110-volt mains so that the maximum flux density in the core may be 4,500 lines per square centimeter. How many turns of wire are required? How many turns of wire would be required at 60 cycles per second, other things remaining the same? Ans. 330 turns, 728 turns.

87. An electromotive force of 110 volts, from a battery for example, is applied during $\frac{1}{100}$ second intervals in reversed direc-

tions to a coil of 100 turns of wire wound on an iron core. Plot the curve representing the above electromotive force and, neglecting the resistance of the coil, plot the curve representing the core flux produced. The iron core is supposed to have a constant magnetic reluctance of 0.00014. Plot the curve representing the magnetizing current.

A secondary coil of 50 turns is wound on the above core. This secondary coil supplies current to a non-inductive receiving circuit having 200 ohms resistance. Plot curves representing (*a*) secondary electromotive force, (*b*) secondary current, and (*c*) total primary current. Resistance of secondary coil is to be neglected.

88. A transformer has its primary in two sections which can be thrown in series or in parallel at will. This primary takes current through a rheostat of resistance R', and the secondary supplies current to a non-inductive receiving circuit of resistance R''. Find the relation between R', R'', N' (total), and N'' for which the secondary current is the same whether the primary coils are in series or in parallel. Ans. $R' = \frac{1}{2}\left(\frac{N'}{N''}\right)^2 R''$.

89. An ordinary transformer, rated at one kilowatt, 100 volts primary, and 10 volts secondary, is connected up as an auto-transformer to transform from 100 volts to 110 volts. What is the full load rating of the transformer so used, what is the full load output of current, and what is the full load intake of current? What is the full load current in the 100-volt coil, and what is the full load current in the secondary coil? Ans. (*a*) 11 kilowatts, (*b*) 100 amperes, (*c*) 110 amperes, (*d*) 10 amperes, (*e*) 100 amperes.

Assuming the coils to be as right-handed helices in one layer on an iron core, give a diagram of the connections.

90. The primary coils of two transformers have each 560 turns of wire and they are connected to two-phase mains, the electromotive force of each phase being 800 volts. Calculate

the turns of wire in each of two secondary coils (one on each transformer) so that these coils when connected in series give an electromotive force of 400 volts, 30° ahead of one of the two-phase electromotive forces. Ans. 140 turns, 242.5 turns.

91. A Scott transformer is to transform from 1,000 volts, 60 cycles, two-phase, to 100 volts three-phase. The cross section of each iron core is 75 square centimeters. Find the number of turns of wire in each primary, and in each secondary coil, allowing a maximum flux density of 3,500 lines per square centimeter. Ans. $N' = 1{,}430$, $a = 71.5$, $b = 71.5$, $c = 124$.

92. Three similar 1,000 to 100-volt transformers have their 1,000-volt coils Δ-connected to three-phase 1,000-volt mains. The secondaries are Y-connected to service mains. Give a diagram of the connections and find the electromotive force between the pairs of service mains. Ans. 173 volts.

CHAPTER XI.

THE TRANSFORMER.

(Continued.)

111. The actual transformer and the ideal transformer.—The discussion given in Articles 95 to 99 ignores the resistances R' and R'' of the transformer coils, it takes but little account of the magnetizing current m, which depends upon eddy currents and magnetic hysteresis in the iron core, and it assumes that all the lines of magnetic flux which pass through one coil pass through the other also. A transformer which would meet these conditions would be an ideal transformer. A well-designed transformer operating on moderate load does approximate quite closely to the ideal transformer in its action, and equations (60) and (63) are much used in practical calculations. For some purposes, however, it is desirable to consider the action of the transformer, taking account of coil resistances, of eddy currents and hysteresis, and of the fact that some lines of magnetic flux pass through one coil without passing through the other (*magnetic leakage*). The present chapter is devoted to this discussion.

The effects of coil resistances, of eddy currents and hysteresis, and of magnetic leakage are small. Their influence on each other is very much smaller and is ignored in the following discussion. That is, the effects of coil resistances, the effects of eddy currents and hysteresis, and the effects of magnetic leakage are considered separately.

We shall first discuss these various effects with the help of vector diagrams for the sake of clearness, giving afterwards the formulation of the symbolic equations and Steinmetz's solution.

112. The magnetizing current of a transformer is not harmonic, as is shown below, but since it is usually small, it may, for the purpose of calculation, be replaced by an equivalent harmonic current M, of which the component parallel to E' (the power component) is M_p; and the component at right angles to E' (the wattless component) is M_w. The power taken from the mains by the magnetizing current is $E'M_p$ and this, ignoring coil resistances, is equal to the total core loss, $W_e + W_h$ [see equations (64) and (65)]. Therefore

$$M_p = \frac{W_e + W_h}{E'} \tag{a}$$

The wattless component M_w of the magnetizing current reaches its maximum value $\sqrt{2}\, M_w$ when the core flux is at its maximum value Φ, and inasmuch as this component of the magnetizing current overcomes the magnetic reluctance of the core we have

$$\Phi = \frac{\frac{4\pi}{10} \cdot N' \cdot \sqrt{2} M_w}{G}$$

or

$$M_w = \frac{10 \Phi G}{4\pi\sqrt{2} N'} \tag{b}$$

in which G is the magnetic reluctance of the transformer core corresponding to the maximum flux Φ.

Admittance of the transformer at zero output.—The magnetizing current is the current that flows through the primary coil when the secondary current is zero, and the actual primary current always exceeds the ideal primary current, $\frac{N''}{N'} \cdot I''$, by the amount of the magnetizing current. Therefore the effect of the magnetizing current may be represented in an ideal transformer by connecting in parallel with the primary coil a circuit through which flows a current equal in value and phase to the magnetizing current of the actual transformer. The admittance to this shunt circuit, $g_1 - jb_1$, is used in the symbolic solution of the transformer problem. The values of g_1 and b_1 are determined from the mag-

netizing current as follows: The component of M parallel to E' is $g_1E' (= M_p)$, and the component of M perpendicular to E' is $b_1E' (= M_w)$. See equations (52) and (53). Therefore, using the values of M_p and M_w from equations (a) and (b), we have

$$g_1 = \frac{W_e + W_h}{E'^2} \tag{71}$$

and

$$b_1 = \frac{10\Phi G}{4\pi\sqrt{2}N'E'}$$

or using the value of Φ from equation (63) we have

$$b_1 = \frac{10\,G}{4\pi\omega N'^2} \tag{72}$$

118. Actual value of the part m of the primary current.

(a) *When the core is assumed to be without hysteresis.*—Let the ordinates of the curve $m\Phi$, Fig. 136, represent values of the core flux Φ produced by various given current strengths m in the primary coil, these current strengths being represented by the abscissas of the curve $m\Phi$.

When the primary electromotive force is harmonic then the core flux Φ is harmonic also, and 90° behind E' in phase, according to equations (ii) and (iii), Article 99. Let the sine curve Φ,

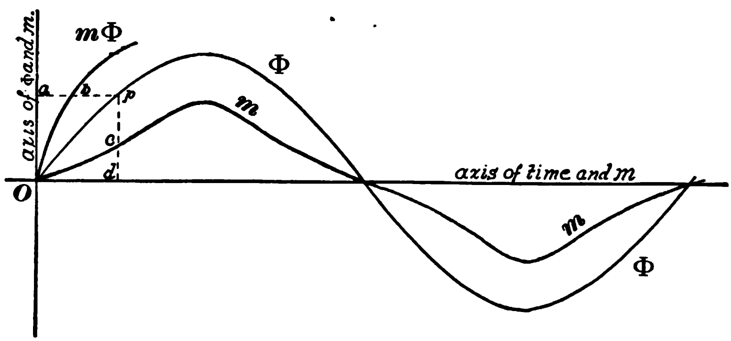


Fig. 136.

Fig. 136, represent the value of Φ as time passes; time as abscissas and Φ as ordinates.

Then the curve m of which the ordinates represent successive instantaneous values of the current m is constructed as follows: Draw the ordinate dp and the abscissa ap. Lay off dc equal to ab which is the magnetizing current required to force through

the core the flux dp. The locus of the point c is the required curve. The figure shows that the magnetizing current is not harmonic although it is wattless.

(b) *When the hysteresis is taken into account.*—Let the ordinates of the curve $m\Phi$, Fig. 137, represent values of the core flux produced by various given current strengths in the primary coil, these current strengths being represented by the abscissas of the curve $m\Phi$. The curve m of which the ordinates represent the successive instantaneous

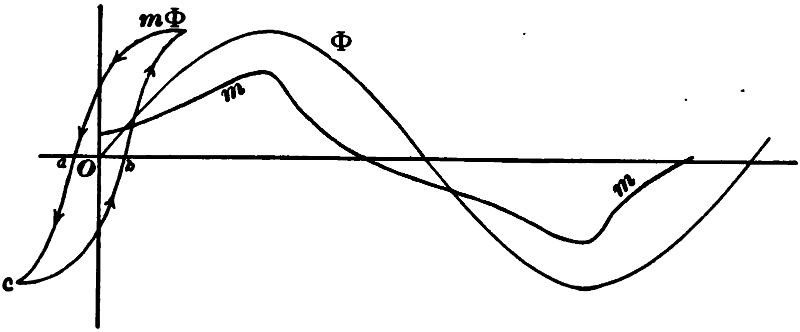


Fig. 137.

values of the current m is constructed as before; the ascending branch of the hysteresis loop $m\Phi$ being used for increasing values of Φ and the descending branch for decreasing values of Φ.

114. Transformer regulation. *Preliminary statement concerning the effects of magnetic leakage and of resistances of primary and secondary coils on the action of a transformer.*—In the ideal transformer the whole of the primary electromotive force is balanced by the opposite electromotive force induced in the primary coil by the varying magnetic flux which passes through both coils, and the whole of the electromotive force induced in the secondary coil by this varying flux is available at the terminals of the secondary coil.

In the actual transformer a portion of the primary electromotive force is lost in overcoming the resistance of the primary coil and a portion is lost in balancing the electromotive force which is induced in the primary coil by the flux which passes through the primary coil, but does not pass through the secondary coil (leakage flux). These lost portions of the primary electromotive force are proportional to the primary current, so

that the useful part* of the primary electromotive force falls short † of the total primary electromotive force by an amount which is proportional to the current.

The total electromotive force induced in the secondary coil is proportional to the *useful part* of the primary electromotive force and a portion of the total secondary electromotive force is lost in overcoming the resistance of the secondary coil. This lost portion of the secondary electromotive force is proportional to the secondary current (or to primary current, since the ratio of the currents is constant).

Therefore the effect of magnetic leakage and of coil resistances is to make the electromotive force between the terminals of the secondary coil fall short † of its ideal value $\frac{N''}{N'} \cdot E'$ by an amount which is proportional to the current.

This falling off of secondary electromotive force with increasing current is of practical importance, inasmuch as most receiving apparatus must be supplied with current at approximately constant electromotive force. A transformer of which the secondary electromotive force falls off but little with increase of current is said to have good regulation. A transformer to regulate well must have low resistance coils and little magnetic leakage. Large transformers as a rule regulate more closely than small ones.

115. Effect of resistance of coils upon the action of a transformer.—Fig. 138 shows the general effect of the resistances of the coils upon the action of a transformer. The line $O\Phi$ represents the harmonically varying flux in the core. Oa represents the useful part of the primary electromotive force and Ob the total electromotive force induced in the secondary coil. The line OI'' represents the secondary current and the line OI'

* The part, namely, which balances the electromotive force induced in the primary coil by the magnetic flux which passes through both coils.

† The lost portions of primary and secondary electromotive forces are, in general, not in phase with total primary and total secondary electromotive forces. These losses are therefore to be subtracted as vectors as explained in the following articles.

represents the primary current. The total primary electromotive force E' exceeds Oa by the amount $R'I'$ (parallel to I'), and the electromotive force E'' at the terminals of the secondary coil fall short of Ob by the amount $R''I''$ (parallel to I'').

Remark.—When the angle θ, Fig. 138, is nearly zero (secondary receiving circuit non-inductive) then $R'I'$ and $R''I''$ are nearly parallel to Oa and Ob respectively, so that Oa is much less than E' in value and E'' is much less than Ob in value. On the other hand, when the angle θ is nearly $\pm 90°$ (secondary receiving circuit containing a large inductance or a condenser) then $R'I'$ and $R''I''$ are nearly perpendicular to Oa and Ob respectively, so that Oa is nearly equal to E' in value and E'' is nearly equal to Ob in value. Therefore the regulation of a transformer is largely affected by coil resistance when the secondary receiving circuit is non-inductive, but scarcely at all affected by the coil resistance when the secondary receiving circuit contains a large inductance or a condenser.

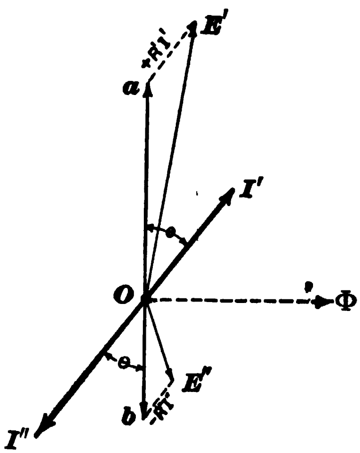


Fig. 138.

116. Effect of magnetic leakage upon the action of a transformer.—It is shown in the next article that magnetic leakage is in its effects equivalent to an auxiliary outside inductance P through which the primary current passes on its way to the primary of the transformer. The part of the primary electromotive force E' which is lost in this inductance is equal to $\omega PI'$ and it is $90°$ ahead of the primary current I' in phase.

When the secondary receiving circuit is inductive I'' lags behind E'' (Ob, Fig. 139) by the angle θ as shown in Fig. 139, and the useful part Oa of the primary electromotive force is *less* than the total primary electromotive force. In this case the secondary

electromotive force, which is equal to $\frac{N''}{N'} \times Oa$, falls off in value as I' (and also $\omega PI'$) increases.

When the secondary receiving circuit contains a condenser, I'' is ahead of E'' (Ob, Fig. 140) as shown in Fig. 140, and the use-

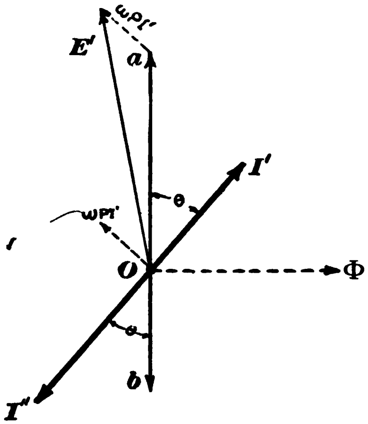


Fig. 139.

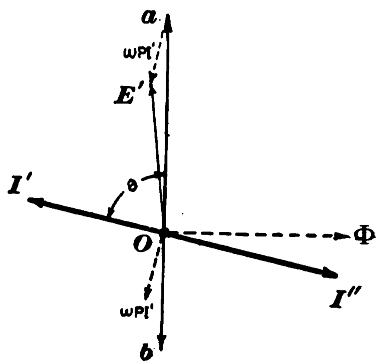


Fig. 140.

ful part, Oa, of the primary electromotive force is *greater* than the total primary electromotive force in value. In this case the secondary electromotive force, which is equal to $\frac{N''}{N'} \times Oa$, increases in value as I' (and also $\omega PI'$) increases.

When the secondary receiving circuit is non-inductive the angle θ is zero and $\omega PI'$ is at right angles to Oa, so that Oa is sensibly equal to E' in value, and therefore sensibly constant. In this case the secondary electromotive force remains sensibly constant as I' (and also as $\omega PI'$) increases.

117. Proposition.—*The effect of magnetic leakage in a transformer is equivalent to a certain outside inductance P, connected in series with the primary coil.*

Discussion.—Let A, Fig. 141, be the primary coil, B the secondary coil and C the iron core of a transformer. As the (harmonic) alternating currents in A and B pulsate, harmonically

varying fluxes are produced through the core and around the coils. Let OC, Fig. 142, represent the harmonically varying flux through the core, Op the harmonically varying flux which encircles coil A only, and Os the harmonically varying flux which encircles coil B only. The fluxes Op and Os are proportional to and in phase with I' and I'' respectively, so that the total flux Op + Os (represented by the lines sp or ba, Fig. 142) which passes between A and B is proportional to and in phase with I'.*

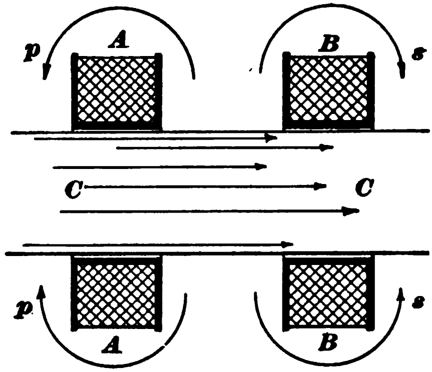


Fig. 141.

The total harmonically varying flux through coil A is $OC +$ Op [$= Oa$], and the total harmonically varying flux through coil B is $OC + Os$ [$= Ob$]. Now, $Oa = Ob + ba$, so that we may look upon the action of the transformer as due to the flux Ob passing through *both coils* and the flux ba passing through the *primary coil only*. This latter flux being proportional to the primary current is equivalent in its effects to an inductance P, connected in series with the primary coil. Let Φ' be the value of the leakage flux ab, which, for a given value i' of the primary current, encircles the primary coil, then, according to equations (5) and (6), we have

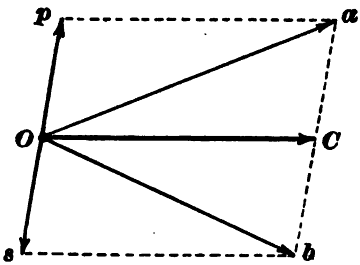


Fig. 142.

$$P = \frac{\Phi' N'}{i'} \tag{73}$$

118. The constant current transformer.—A transformer of which

* Since I' is proportional to I'' and opposite to it in phase.

the leakage inductance P is very large is sometimes called a constant current transformer for the reason that the current delivered by such a transformer varies but little with the resistance of the receiving circuit, so long as this resistance is comparatively small, the primary of the transformer being connected to constant electromotive force mains. The action of the inductance P in controlling the current is explained in the article on the constant current alternator. (Article 83.)

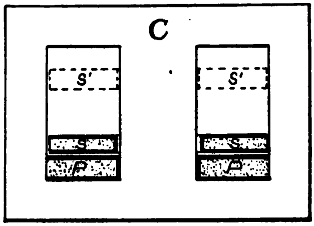


Fig. 143.

Fig. 143 is a sketch of the General Electric Company's type of constant current transformer; C is the iron core, PP the primary coil, and SS the secondary coil. The secondary coil is movable and nearly counterbalanced, and the increased repulsion between PP and SS due to a slight increase of current lifts the secondary coil to $S'S'$.

When the primary and secondary coils are near together the leakage inductance is very small and a decrease in the resistance of the receiving circuit would be accompanied by a great increase of current were it not for the movement of the secondary coil and the consequent increase of leakage inductance.

119. Calculation of leakage inductance P.—The leakage flux Φ' equation (73) [$= ba$, Fig. 142], and therefore the value of P also depends upon the size and shape of the primary and secondary coils and upon their proximity to the core. In considering the flux between the coils (leakage flux) we need not consider whether a given portion of this flux is a part of Op or a part of Os, Fig. 142, inasmuch as these two fluxes are added together to give ba or Φ'.

Figs. 144 and 145 show side and end views of a shell type transformer. The trend of the leakage flux is shown in the upper part of Fig. 145 (omitted from lower part for the sake of clearness), and the dimensions X, Y, g, λ and l are shown. Fig. 146

is an enlargement of the upper part of Fig. 145. Consider the flux across between the dotted lines aa, Fig. 146. The magnet-

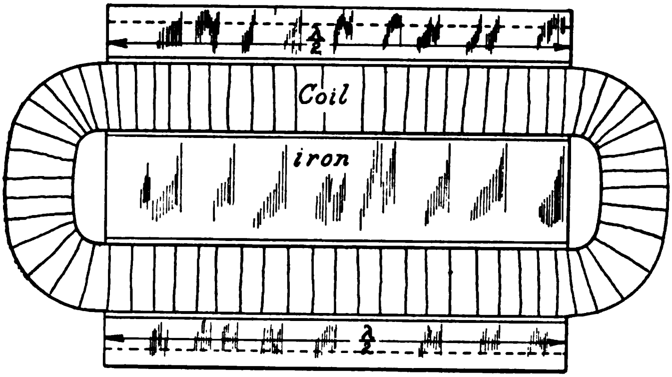


Fig. 144.

omotive force pushing this flux across is $4\pi i' \frac{x}{X} N'$.* The length of the air portion of the magnetic circuit through which the leakage flux flows is l and its sectional area is λdx (counting both limbs of the coils). Therefore, the magnetic reluctance of this leakage circuit is $\frac{l}{\lambda dx}$ and the flux across between aa is

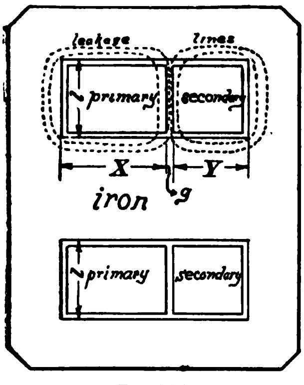


Fig. 145.

$$\frac{m.m.f.}{m.r.} = 4\pi i' N' \frac{\lambda x dx}{lX}.$$

This flux encircles the fractional part $\frac{x}{X}$ of the primary turns and, therefore, the fractional part $\frac{x}{X}$ of the flux is to be counted as encircling the entire primary coil so that

$$d\Phi' = \frac{4\pi N' i' \lambda}{lX^2} \cdot x^2 dx$$

* All quantities in this article are expressed in c.g.s. units.

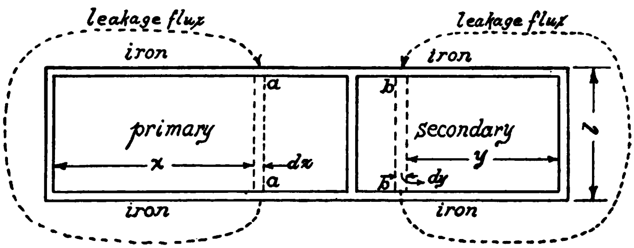


Fig. 146.

The part of Φ' which flows across the primary coil is the integral of this expression from $x = 0$ to $x = X$. This part of Φ' is therefore

$$\frac{4\pi N'i'\lambda X}{3l}$$

Similarly, the part of Φ' which flows across the secondary coil is

$$\frac{4\pi N'i'\lambda Y}{3l}*$$

The flux across the gap g between the primary and secondary coils is all counted as a part of Φ', and is equal to

$$\frac{4\pi N'i'\lambda g}{l}$$

Therefore

$$\Phi' = \frac{4\pi N'i'\lambda}{l}\left[\frac{X}{3} + \frac{Y}{3} + g\right] \tag{74}$$

There is some leakage flux passing between the primary and secondary coils where they project beyond the iron core. This part of the leakage flux has a longer air-path than the leakage flux which flows from iron to iron, say three times as long. Therefore, for λ we may take the total length of the coils lessened, by say $2/3$ the length which is surrounded by air only.

Substituting the value of Φ' from (74) in (73), we have

$$P = \frac{4\pi N'^2\lambda}{l}\left[\frac{X}{3} + \frac{Y}{3} + g\right] \tag{75}$$

* In this expression $N'i'$ being equal to $N''i''$ is written therefor.

This equation gives the value of P is centimeters, all dimensions being expressed in centimeters.

The equivalent inductance P may be reduced in value by lessening λ, X, Y or g or by increasing l. Fig. 147 shows the proportions of a recent type of transformer for which the leakage inductance P is very small. The value of P may be further reduced by winding the primary and secondary coils in alternate sections.

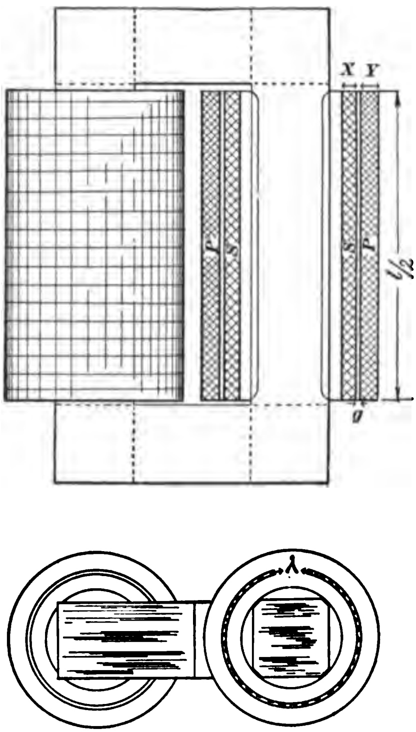


Fig. 147.

120. Formulation of the complex equations of the transformer.—The following discussion is taken from Steinmetz,* with the following changes. A circuit which takes a lagging current has its impedance written $r + jx$ and its admittance written $g - jb$. Magnetic leakage is here represented as equivalent to a primary inductance only, while Steinmetz uses both primary and secondary inductance. The notation is slightly altered.

Let N' = number of primary turns.

N'' = number of secondary turns.

$$a = \frac{N'}{N''}$$

r_1 = resistance of primary coil.

r_2 = resistance of secondary coil.

* "Alternating Current Phenomena," third edition, page 204.

$x =$ reactance value, ωP, of the primary leakage inductance P.

$Z = r_1 + jx$.

$Y = g - jb =$ admittance of entire secondary circuit including resistance of secondary coil.

$E' =$ primary impressed electromotive force.

$A =$ that part of E' which is used to balance the electromotive force induced in the primary coil by the magnetic flux which passes through both coils.

$B =$ secondary induced electromotive force.

$E'' =$ secondary terminal voltage.

$I' =$ total primary current.

$M =$ magnetizing current.

$I'' =$ secondary current.

$Y_1 = g_1 - jb_1 = \frac{M}{A} =$ admittance of transformer at zero output.

Remark.—In Article 112, Y_1 is defined as $\frac{M}{E'}$. The mutual influences of coil resistances, eddy currents and hysteresis, and magnetic leakage are partly taken account of by using $\frac{M}{A}$ instead of $\frac{M}{E'}$, because the extent of magnetization of the core depends upon A.

The ratio of A and B is a and they are opposite in phase, so that

$$A = -aB \tag{i}$$

The secondary current is

$$I'' = YB \tag{ii}$$

The part of the primary current corresponding to I'' is opposite to it in phase and $1/a$ as large. It is therefore equal to $-\frac{YB}{a}$, which added to the magnetizing current $M\ (= Y_1A)$, gives

$$I' = Y_1A - \frac{YB}{a} \tag{iii}$$

The electromotive force lost in the secondary coil is $r_2 I''$, so that the secondary terminal voltage is

$$E'' = B - r_2 I'' \tag{iv}$$

The electromotive force used to overcome the impedance Z of the primary coil is ZI', which added to A gives the primary impressed electromotive force. That is,

$$E' = A + ZI' \tag{v}$$

With the help of equations (i) and (ii), I'', I', E'' and E', may be expressed in terms of B, giving

$$I'' = BY \tag{76}$$

$$I' = -B\left(aY_1 + \frac{Y}{a}\right) \tag{77}$$

$$E'' = B\,(1 - r_2 Y) \tag{78}$$

$$E' = -B\left(a + aZY_1 + \frac{ZY}{a}\right) \tag{79}$$

It is somewhat more desirable to express I'', I' and E'', in terms of E'. Eliminating B from equations (78) and (79), we have

$$\frac{E''}{E'} = -\frac{1 - r_2 Y}{a + aZY_1 + \dfrac{ZY}{a}}$$

or

$$E'' = -\frac{a - ar_2 Y}{a^2 + a^2 ZY_1 + ZY} \cdot E' \tag{80}$$

Similarly we find

$$I' = +\frac{a^2 Y_1 + Y}{a^2 + a^2 ZY_1 + ZY} \cdot E' \tag{81}$$

and

$$I'' = -\frac{aY}{a^2 + a^2 ZY_1 + ZY} \cdot E' \tag{82}$$

For purposes of numerical calculation E' may be taken as the reference axis, so that E' becomes a simple quantity and it remains only to separate the components of the various factors by which E' is multiplied. Take for example the expression for

I''. Using $g-jb$ for Y, r_1+jx for Z, and g_1-jb_1 for Y_1 and collecting terms we have

$$\text{factor in equation (82)} = -\frac{ag-jab}{u+jv}$$

where

$$u = a^2 + a^2 g_1 r_1 + g r_1 + a^2 b_1 x + bx$$
$$v = a^2 g_1 x + gx - a^2 b_1 r_1 - b r_1$$

Multiplying numerator and denominator by $u-jv$ we have

$$\textit{aforesaid factor} = -\frac{agu-abv}{u^2+v^2} + j\frac{abu+agv}{u^2+v^2}$$

So that we have the *numerical* relation

$$\textit{numerical value of } I'' = \sqrt{s^2+t^2} \cdot E'$$

where

$$s = \frac{agu-abv}{u^2+v^2}$$

$$t = \frac{abu+agv}{u^2+v^2}$$

The numerical values of I' and I'' may be calculated in a similar manner from the known values of a, r_1, x, g_1, b_1, and r_2, and given values of E', g, and b. If r_3 and x_3 are the resistance and reactance respectively of the external secondary circuit, then

$$g = \frac{r_2+r_3}{(r_2+r_3)^2+x_3^2}$$

and

$$b = \frac{x_3}{(r_2+r_3)^2+x_3^2}$$

Calculation of regulation.—The falling of the secondary terminal voltage of a well-designed transformer depends almost wholly upon magnetic leakage and coil resistances, and is to a very close degree of approximation independent of the magnetizing current. Therefore for the purpose of the calculation of transformer regulation Y_1 may be assumed to be equal to zero. Under these conditions equation (80) becomes

$$E'' = -\frac{a - ar_2 Y}{a^2 + ZY} \cdot E' \qquad (80\,a)$$

from which, by separating the components of the factor

$$\frac{a - ar_2 Y}{a^2 + ZY}$$

the numerical value of E'' may be calculated for any assigned value of Y; a, r_2, and Z being known.

PROBLEMS.

93. Following are the data for a shell type transformer (see Figs. 144, 145 and 146):

The fine wire coil consists of 560 turns of number 17 B. & S. copper wire. Mean length of turn $29\frac{1}{2}$ inches.

The coarse wire coil consists of 28 turns of two number sevens (B. & S.). Mean length of turn $29\frac{1}{2}$ inches.

$l = 1\frac{3}{4}$ inches, $g = \frac{1}{8}$ inch, $Y = 1\frac{1}{4}$ inches (coarse wire coil), $X = 1\frac{1}{2}$ inches (fine wire coil), and $\lambda/2 = 10\frac{1}{2}$ inches.

At each end of core $4\frac{1}{2}$ inches of length of coils are exposed. Section of magnetic circuit $= 10\frac{1}{2} \times 1\frac{18}{16}$ inches and $\frac{8}{10}$ of this is iron. Volume of iron $= 24\frac{1}{2}$ square inches $\times$ $10\frac{1}{2}$ inches $\times \frac{8}{10}$. Thickness of laminations 14 mils.

When the fine wire coil takes current at 1,100 volts at 133 cycles per second find the secondary terminal voltage at zero load.

Find the secondary terminal voltage when the transformer is delivering current to a non-inductive receiving circuit of 1.8 ohms resistance.

Find the secondary terminal voltage when the transformer is delivering current to a circuit of which the resistance is 1.25 ohms and the reactance is 1.3 ohms.

Find the secondary terminal voltage when the transformer is delivering current to a resistanceless circuit of which the reactance is 1.8 ohms.

94. Following are the data for a core type transformer (see Fig. 147):

The fine wire coil consists of 2,000 turns of number 17 B. & S. copper wire. Length of mean turn $= 9\frac{1}{2}$ inches. This coil is wound next the core.

The coarse wire coil consists of 100 turns of two number sevens (B. & S.). Length of mean turn $11\frac{3}{4}$ inches. This coil is wound outside of the fine wire coil.

$l/2 = 5$ inches, $g = \frac{1}{8}$ inch, $Y = \frac{5}{8}$ inch (coarse wire coil), $X = \frac{3}{4}$ inch (fine wire coil), and $\lambda = 11\frac{3}{4}$ inches.

The net sectional area of the iron core is 3.7 square inches, the mean length of the magnetic circuit is 22 inches, and the volume of the iron is 81.4 cubic inches. Thickness of laminations 14 mils.

When the fine wire coil takes current at 1,100 volts at 133 cycles per second find the secondary terminal voltage at zero load.

Find the secondary terminal voltage when the transformer is delivering current to a non-inductive receiving circuit of which the resistance is 1.8 ohms.

Find the secondary terminal voltage when the transformer is delivering current to a circuit of which the resistance is 1.25 ohms and the reactance is 1.3 ohms.

Find the secondary terminal voltage when the transformer is delivering current to a resistanceless circuit of which the reactance is 1.8 ohms.

CHAPTER XII.

THE SYNCHRONOUS MOTOR.

121. Alternators in series.—Two alternators A and B are connected in series and driven by separate engines to give precisely the same frequency. The lines A and B, Fig. 148, represent the electromotive forces of machines A and B respectively, ϕ is the angular lag of the electromotive force B behind the electromotive force A, and the line E represents the resultant electromotive force of A and B. This resultant electromotive force produces in the circuit a current, of which the value is

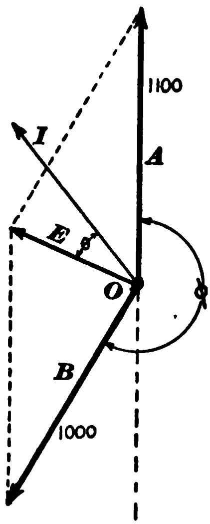


Fig. 148.

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \tag{83}$$

and which lags $\theta°$ behind E in phase, where

$$\tan\theta = \frac{\omega L}{R} \tag{84}$$

in which R is the resistance of the circuit, L is the total inductance of the circuit including the armatures of both machines, and ω ($= 2\pi f$) is the frequency in radians per second.

The power P' put into the circuit by machine A is

$$P' = AI\cos(AI) \tag{85}$$

where (AI) is the angle between A and I. The power put into the circuit by machine B is

$$P'' = BI\cos(BI) \tag{86}$$

The angle AI in Fig. 148 is less than 90°, so that cos (AI) is positive; therefore P' is positive, that is, the machine A is acting as a dynamo. The angle (BI) in the figure is greater than 90°, so that cos (BI) is negative; therefore P'' is negative, that is, the machine B is acting as a motor.

The alternator B used in this way is called a *synchronous motor*, the alternator A being driven by an engine or water-wheel.

122. Variation of P' and P'' with the phase angle ϕ.—Draw a line OC, Fig. 149, representing B to scale. Describe about C a

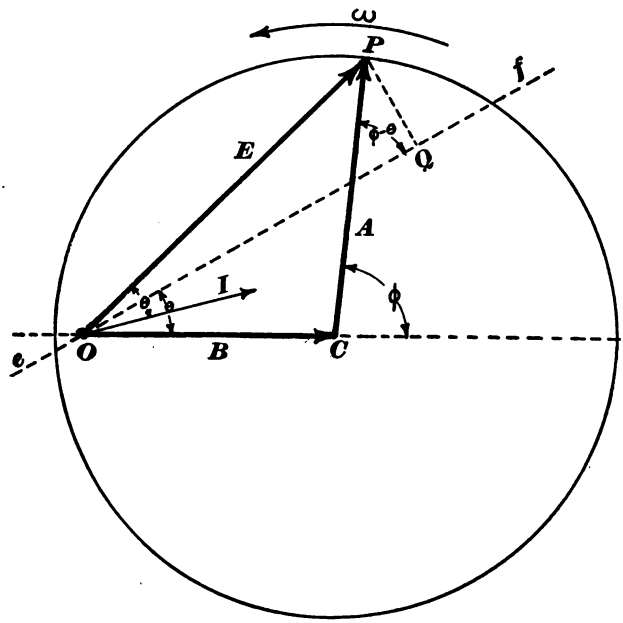


Fig. 149.

circle of which the radius represents A. Then a line OP, from O to any point in the circle represents a possible value of the resultant electromotive force E, ϕ being the corresponding phase difference between A and B. Draw the line ef through O, making with OC the angle θ. Then the angle POf is equal to the angle (BI). Therefore

$$OQ = OP \cos(BI) = E \cos(BI) = \sqrt{R^2 + \omega^2 L^2}\, I \cos(BI)$$

or

$$I \cos (BI) = \frac{\overline{OQ}}{\sqrt{R^2 + \omega^2 L^2}}.$$

Substituting this value of $I \cos (BI)$ in equation (86) we have

$$P'' = \frac{B \cdot \overline{OQ}}{\sqrt{R^2 + \omega^2 L^2}} \tag{87}$$

That is, the output of the machine B is proportional to the projection OQ of the line OP on the line ef, B and $\sqrt{R^2 + \omega^2 L^2}$ being constant.

When Q is towards f from O, $\cos (BI)$ is positive so that P'' is positive and machine B acts as a dynamo. When Q is towards e from O, $\cos (BI)$ is negative so that P'' is negative and machine B acts as a motor.

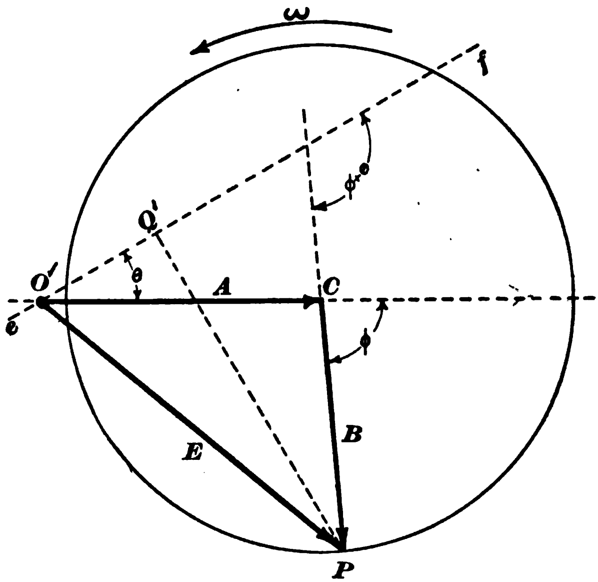


Fig. 150.

Fig. 150 is a construction, for the same value of ϕ, in which $O'Q'$ represents the power P' put into the circuit by machine A. From this diagram we have

$$P' = \frac{A \cdot \overline{O'Q'}}{\sqrt{R^2 + \omega^2 L^2}} \tag{88}$$

The projection of A, Fig. 149, on ef is $A\cos(\phi-\theta)$; the projection of B on ef is $B\cos\theta$; and OQ is the sum of these projections so that

$$OQ = A\cos(\phi-\theta) + B\cos\theta$$

Substituting this value of OQ in equation (87) we have

$$P'' = \frac{AB}{\sqrt{R^2+\omega^2L^2}}\cos(\phi-\theta) + \frac{B^2}{\sqrt{R^2+\omega^2L^2}}\cos\theta \qquad (89)$$

Similarly from Fig. 150 we have

$$P' = \frac{AB}{\sqrt{R^2+\omega^2L^2}}\cos(\phi+\theta) + \frac{A^2}{\sqrt{R^2+\omega^2L^2}}\cos\theta \qquad (90)$$

These equations, (89) and (90), are the fundamental equations of the synchronous motor. The algebraic sum of the outputs of machines A and B is, of course, equal to RI^2, so that

$$P' + P'' = RI^2 \qquad (91)$$

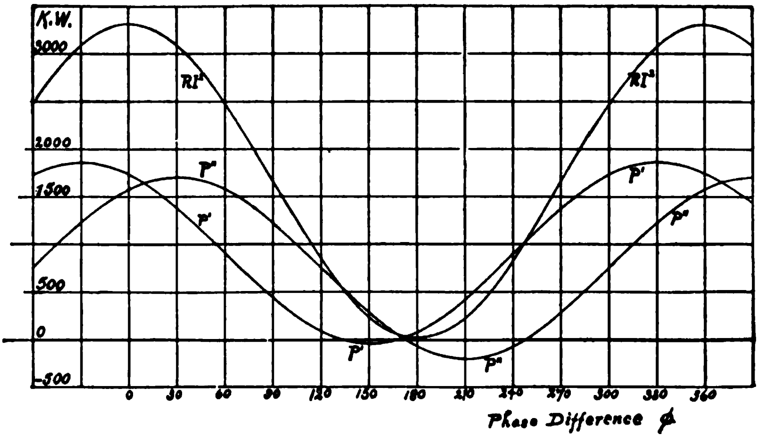
This relation may be derived from equations (89) and (90), remembering that

$$\sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} \qquad \cos\theta = \frac{R}{\sqrt{R^2+\omega^2L^2}}$$

and

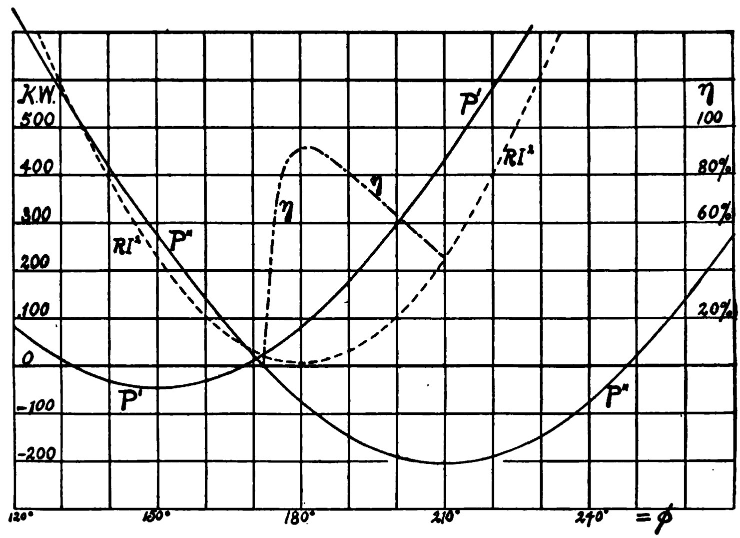
$$I^2 = \frac{E^2}{R^2+\omega^2L^2} = \frac{A^2+B^2+2AB\cos\phi}{R^2+\omega^2L^2}$$

The ordinates of the curves P', P'' and RI^2, Fig. 151, show the values of P', of P'' and of RI^2 for values of ϕ from zero to 360°. Positive ordinates represent positive power (dynamo action), negative ordinates represent negative power (motor action). Each ordinate of the curve RI^2 is the algebraic sum of the ordinates of the curves P' and P''; Fig. 152 shows portions of the curves P', P'' and RI^2 to a larger scale. The ordinates of the curve η represent the efficiency $\left(\frac{P''}{P'}\right)$ of transmissions for various values of ϕ when machine B is a motor.



$A = 1,100$ volts. $R = 1.00$ ohm.
$B = 1,000$ volts. $\omega L = 0.58$ ohm.

Fig. 151.



$A = 1,100$ volts. $R = 1.00$ ohm.
$B = 1,100$ volts. $\omega L = 0.58$ ohm.

Fig. 152.

Steinmetz's derivation of equations (89) *and* (90).—Let B be the electromotive force of machine B, and let this electromotive force be taken as the x-axis of refer-

ence. Let ϕ be the angle that the electromotive force of machine A is ahead of B, and let A be the numerical value of the electromotive force of machine A. Then $A \cos \phi$ is the x-component of the electromotive force of machine A, and $A \sin \phi$ is its y-component. Therefore the complex expression for the electromotive force of machine A is $A \cos \phi + jA \sin \phi$, and the resultant electromotive force of the two machines is:

$$E = B + A \cos \phi + jA \sin \phi$$

The current is therefore

$$I = \frac{E}{R + j\omega L} = \frac{B + A \cos \phi + jA \sin \phi}{R + j\omega L}$$

The real part of I is the component of I parallel to B, and the product of this component of I into B is the power output of machine B, therefore

$$P'' = \frac{B^2 R + ABR \cos \phi + AB\omega L \sin \phi}{R^2 + \omega^2 L^2} \tag{92}$$

123. The necessity of synchronism for the operation of machine B as a motor.—In Fig. 148 the relative phase of the electromotive forces A and B has been so chosen that machine B is a motor. We shall now consider whether synchronism of machines A and B is a necessary condition for the steady * intake of power by the machine B. Suppose that machine B is running steadily (engine driven) at a frequency slightly above or below the frequency of A. Then the phase angle ϕ will change continuously and the point P, Fig. 153, will move slowly around the circle, making one revolution while machine B gains or loses one cycle with reference to A, and the power intake P'' of machine B will pass through a set of positive and negative values during each revolution of the point P. Now the positive values of P'' are larger than the negative values, and also of longer duration, so that the average value of P'' is positive. That is, machine B gives out power on the average if it is either above or below synchronism with A. Therefore synchronism is a necessary condition for the intake of power by machine B.

124. Behavior of a synchronous motor. Stoppage due to overload.—When machine B is running as a motor, unloaded, its

* That is, steady except for the extremely rapid pulsations due to the alternations of electromotive force and current.

intake P'' is approximately zero; point P, Fig. 153, is at s*; the resultant electromotive force is Os; the current is in quadrature with the electromotive force B; and the output of machine A is equal to RI^2.

As the motor B is loaded, its intake P'' increases; the point P, Fig. 153, moves from s towards M; and the resultant electro-

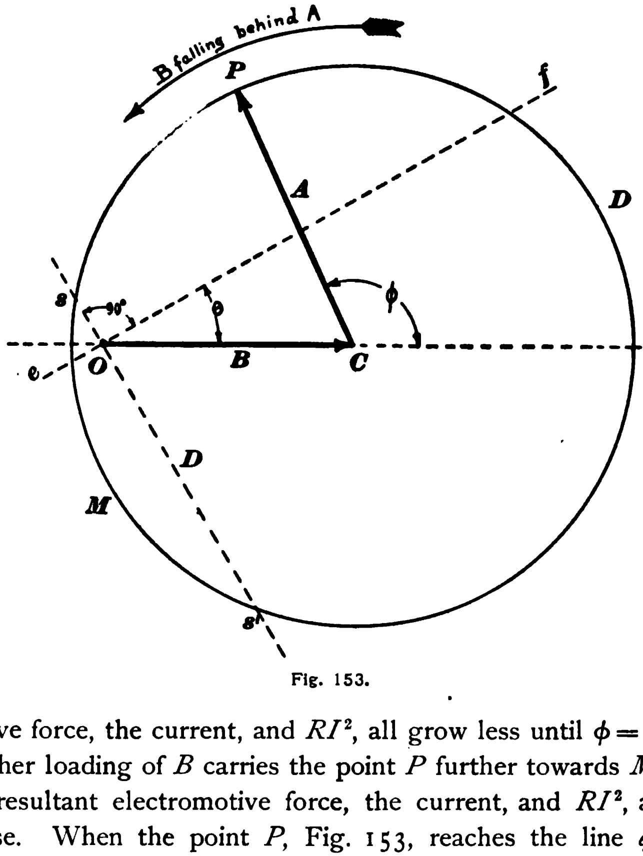


Fig. 153.

motive force, the current, and RI^2, all grow less until $\phi = 180°$. Further loading of B carries the point P further towards M, and the resultant electromotive force, the current, and RI^2, all increase. When the point P, Fig. 153, reaches the line ef, the current is opposite to the electromotive force B in phase. As the motor B is still further loaded the point P moves on towards M, and when the point P reaches M the intake of B has reached

*Running of B is unstable when the point P is at s'.

its maximum value for the given values of A, B, ωL and R. Further loading of B *decreases* its intake. This decrease of intake causes B to fall further and further behind A in phase until the point P moves beyond the point s'; machine B then acts as a dynamo and gives out power to the line, which action, together with the belt load, causes machine B to fall out of synchronism and stop. While machine B is stopping the action is as follows: Every time machine B loses one cycle as compared with the steadily driven machine A, the point P, Fig. 153, moves once around the circle. While P is moving from s' through D to s machine B acts as a dynamo, which action, together with the belt load, slows the machine up greatly. Then as P moves from s through M to s', machine B takes in power from A, but by no means enough to enable it to regain synchronous speed. Therefore the point P moves on past s' through D to s, which slows up machine B still more, and so on.

125. Behavior of a synchronous motor of which the electromotive force is greater than the electromotive force of the generator which drives it.—In this article we shall speak of machine A as the motor and of machine B as the generator, machine A having the greater electromotive force. It is evident from Figs. 151 and 152 that machine A can act as a motor. From Fig. 152 it is also evident that the maximum intake P' of A as a motor is very much less than the maximum intake of B as a motor and it is also evident that when A is acting as a motor its intake P' is much less than the output P'' of B, so that the efficiency of transmission is lower when A is the motor than when B is the motor. When machine A is running as a motor, unloaded, its intake P' is approximately zero; the point P, Fig. 154, is at s; the resultant electromotive force is Os; the current is in quadrature with the electromotive force A; and the output of the generator B is equal to RI^2.

As the motor A is loaded, its intake P' increases; the point P, Fig. 154, moves from s towards M; and the resultant electro-

motive force, the current, and RI^2 all increase. When the point P reaches M the intake of A has reached its maximum value for

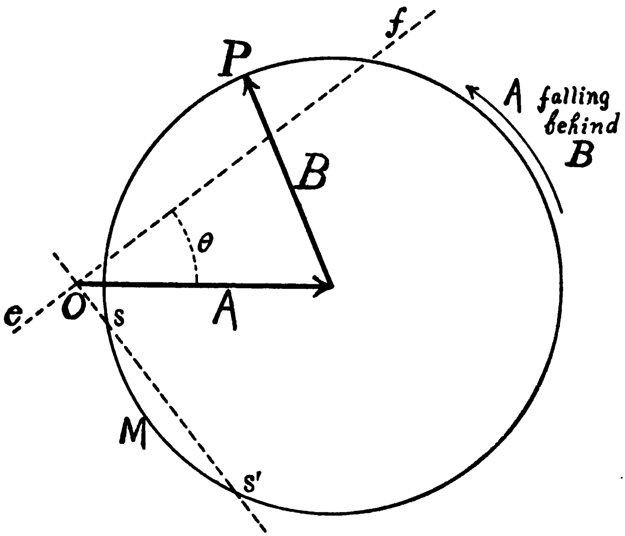


Fig. 154.

the given values of A, B, ωL and R, and further loading causes machine A to fall out of synchronism and stop.

Negative reactance of machine A.—Any receiving circuit which takes current which is ahead in phase of the electromotive force of the generator which supplies the current acts more or less like a condenser and has negative reactance. A careful scrutiny of Fig. 154 will show that I is ahead of B in phase when the point P is near s.

126. The starting of the synchronous motor.—Let A be the machine which is to be the generator and B the machine which is to be the motor. The machine A, which is driven continuously by an engine or water-wheel, is started. Machine B is then started by driving it with an engine or other independent mover, and its speed is carefully regulated until (*a*) it is *in synchronism with A, and* (*b*) *its electromotive force opposite to the electromotive*

force of A in phase. The circuit is then closed and the engine or other agent which has been used to bring machine B up to speed may be disconnected.

In carrying out this process it is necessary to use some sort of indicator for showing when A and B are in synchronism and when they are in the proper phase relation. For example, a lamp may be connected in circuit, as shown in Fig. 155. This lamp pul-

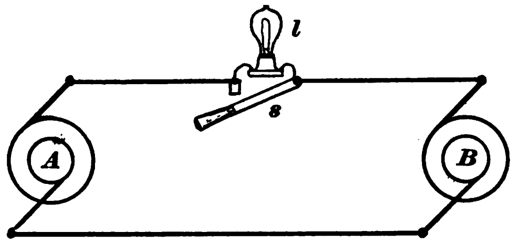


Fig. 155.

sates in brightness as machine B is being speeded up and the pulsations become slower and slower as the frequency of B approaches the frequency of A. When the lamp is at its maximum brightness the angle ϕ, Fig. 149, is zero, and when the lamp is at its minimum of brightness the angle ϕ is 180°. When the pulsations become very slow the machines are practically in synchronism and the switch s is closed when the lamp is at minimum brightness.

In practice the lamp l is connected in series with the secondaries of two transformers the primaries of which are connected to A and to B respectively. With this arrangement the lamp may be either at its maximum or at its minimum of brightness when the machines are in the proper phase for the closing of the switch, according to the connections of the transformers. In fact it is better to arrange the connections so that the lamp shall be at its maximum of brightness when the switch is to be closed, for the instant of maximum brightness is more sharply marked than the instant of minimum brightness.

127. Stability of running of synchronous motor.—Suppose the

machine A to be driven by means of a governed engine at constant speed, irrespective of its output; and suppose the motor B to be running steadily. If the load on B is suddenly increased this machine will run momentarily slower than A and fall behind A in phase. If this falling behind in phase *increases* the power which B takes from A, then B will fall only so far behind as to enable it to take in power sufficient to carry its increased load. If, on the other hand, this falling behind in phase *decreases* the power which B takes from A, then B will fall further and further behind A, fall out of synchronism and stop. In the first case the running of B is *stable*, in the second case it is *unstable*.

As the point P, Fig. 149, moves along the circle in the direction of the arrow ω the electromotive force A is getting farther ahead of B in phase or B is falling behind A. *If the motor intake of B increases as it falls behind B, then the running of B is stable, and vice versa*, as pointed out above. Now the projection of OP, namely OQ positive towards e, represents the intake of B; and this intake increases from s to M, Fig. 153, as B falls behind A, and decreases from M to s' as B falls behind A; therefore s to M is the region of stable motor running of B, and M to s' is the region of unstable motor running of B.

If B is running with given load as a motor the point P will take up a position between s and M such that the intake of B is sufficient to carry its load. If B is further loaded P moves further towards M; if B is unloaded P moves towards s. If B is loaded until P reaches M then further loading decreases the intake of B and the machine B therefore falls out of synchronism and stops, as has been explained.

Remark.—With given motor intake the point P may be in the region sM of stable running or in the region Ms' of unstable running. For the first case the current is smaller than for the second inasmuch as the resultant electromotive force is smaller in the first case.

128. Running of two alternators in parallel as generators.—

Two alternators, both engine-driven, run satisfactorily as generators when they are adjusted to synchronism (and to proper phase relation) and connected in parallel to receiving mains, as shown in Fig. 156. This arrangement is frequently used in practice.

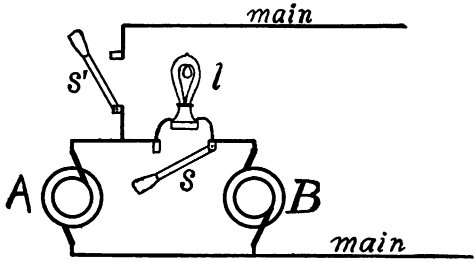


Fig. 156.

Machines A and B are started and connected together through the indicating lamp l. The machines are adjusted to synchronism, and when the lamp is at minimum brightness the switch s is closed. The switch s' is then closed, and the machines deliver current to the mains. If either machine should fall behind the other in phase its share of the load is reduced so that the running of the two machines is stable. The lamp l is in practice connected in series with the secondaries of two transformers, the primaries of which are connected to A and to B respectively, and the connections are so made that the proper conditions for closing the switch s are indicated by maximum brightness.

Machine A may be run alone when the output of the station is small, during the day, for example; and when the load increases to the full capacity of machine A, machine B may be started up, adjusted to synchronism with A and connected to the mains by closing the switch s, the switch s' being left closed during the whole operation.

Sharing of load between two engine-driven alternator generators in parallel.—(*a*) When one engine only is governed, the other engine being set at a fixed cut-off. In this case the output of the alternator driven by the fixed cut-off engine is constant and the variations of station output are met by the governed engine. If the station output falls below the constant output of the alternator, which is driven by the fixed cut-

off engine, the other alternator takes in power as a synchronous motor, it may even take in enough power to drive its engine and if the station output falls too low the fixed cut-off engine may cause the entire system to race. This arrangement is seldom used in practice.

(*b*) When both engines are governed the distribution of load between them is approximately as follows: Let a be the zero-load-speed of engine A, and let b be the zero-load-speed of engine B. Let s be the common speed of both engines when they are driving the two alternators. Let P' be the power delivered by engine A to its alternator, and P'' the power delivered by engine B to its alternator. Then, approximately:

$$P' = m(a - s) \tag{i}$$

and

$$P'' = n(b - s) \tag{ii}$$

These equations are based on the assumption that the speed of a governed engine falls off in proportion to its output. The quantity m is obtained by dividing the full load output of engine A by its full load drop in speed. The quantity n is found by dividing the full load output of engine B by its full load drop in speed.

The total station output determines the combined output $P' + P''$ of the two engines, and equations (i) and (ii) determine P', P'' and s.

The engines share the load equally only when their zero load speeds are equal and when their full load drop in speed is the same.

Remark.—Well designed engines fall off but very little in speed with increase of load.

129. Hunting action of the synchronous motor.—When the load on a synchronous motor is increased the motor slows up momentarily and falls behind the generator in phase. When the motor has fallen behind sufficiently to take in power enough to enable it to carry its load it is still running slightly below synchronism, it therefore falls still further behind and takes an excess of power from the generator which quickly speeds it above synchronism. It then gains on the generator in phase until it takes in less power than is required for its load when it again slows up and so on. This oscillation of speed above and below synchronism, called *hunting*, is similar to the hunting of a governed steam engine. It is frequently a source of great annoyance, especially where several synchronous motors (or rotary converters) are run in parallel from the same mains.

Remark.—The hunting of the synchronous motor is usually more troublesome in case of the rotary converter than it is in case of the synchronous motor with a belt load. In Chapter

XIII. the method employed for damping the hunting oscillations of a synchronous motor is described.

Theory of the hunting of the synchronous motor.—When a synchronous motor is running steadily it takes in power steadily from the mains and gives out power steadily on its belt (or from its direct current commutator in case of the rotary converter). The pulsations of power intake due to the alternations of electromotive force and current are extremely rapid in comparison with hunting oscillations and need not be considered, indeed these pulsations do not exist in case of polyphase machines.

The *mean position*, at a given instant, of the armature of a synchronous motor which is hunting is the position it would have at that instant if it were turning at a constant angular velocity. When the motor hunts its armature oscillates forwards and backwards through its mean position.

When the armature is in its mean position the power intake of the motor and its belt load (including friction losses and the like) are equal, and no unbalanced torque acts on the armature.

When the armature gets ahead of its mean position its intake is lessened, the belt load of the machine, which is assumed to be constant, exceeds the intake and an unbalanced retarding torque acts on the armature.

When the armature falls behind its mean position its intake exceeds its belt load and an unbalanced accelerating torque acts on the armature.

Let ψ be the angle between the mean position of the armature and its actual position at a given instant, and let T be the unbalanced torque acting on the armature at this instant. Our problem is to find the relation between ψ and T. This relation, when ψ is small is

$$T = -b\psi \tag{i}$$

where b is a constant. Therefore, from the laws of harmonic motion, we have

$$\frac{4\pi^2 K}{t^2} = b \tag{ii}$$

in which K is the moment of inertia of the rotating part of the machine and t is the period of the hunting oscillations.

Derivation of equation (i).—We shall derive equation (i) for a special case, namely, for the case in which the moments of inertia of the armatures (or rotating parts) of generator and synchronous motor are equal and for the particular phase angle $\phi = 180°$, see Figs. 149 and 152. In this case, namely, when $\phi = 180°$, a small change of P' is accompanied by an equal and opposite change of P'', so that equal unbalanced torques act at each instant on the armatures of machines A and B, and their moments of inertia being equal the ranges of the oscillations of the armatures of both machines are equal. That is, the armature of machine A is as much ahead of its mean position at each instant as the armature of machine B is behind its mean position at the same instant and *vice versa*. Therefore the change of the phase angle ϕ is equal to $2p\psi$, 2ψ being the angular displacement of one armature referred to the other and p being the number of pairs of field magnet poles in each machine.

Differentiating equation (89) with respect to ϕ, writing $2p\psi$ for $d\phi$, and after the differentiation is performed, putting $\phi = 180°$, we have

$$dP'' = -\frac{2p\psi AB}{\sqrt{R^2 + \omega^2 L^2}} \cdot \sin\theta$$

Now dP'' equals $2\pi n T$ where n is the speed of the machine and T is the unbalanced torque. Therefore

$$T = \frac{p\psi AB}{\pi n \sqrt{R^2 + \omega^2 L^2}} \cdot \sin\theta$$

The value of b, equation (i), is therefore

$$b = \frac{pAB}{\pi n \sqrt{R^2 + \omega^2 L^2}} \cdot \sin\theta$$

Substituting this value of b in equation (ii) and solving for t^2, we have

$$t^2 = \frac{4\pi^3 n K \sqrt{R^2 + \omega^2 L^2}}{pAB \sin\theta}$$

or, since

$$\sin\theta = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \quad \text{and} \quad \omega = 2\pi n$$

we have

$$t^2 = \frac{2\pi^2 K(R^2 + \omega^2 L^2)}{pABL} \tag{iii}$$

130. The efficiency of transmission of power by means of an alternating generator and a synchronous motor.—The ratio, P''/P', of the motor intake to the generator output is called the *efficiency of transmission.* This is not the net efficiency of the system inasmuch as power consumed in field excitation of the two machines and power lost by friction and by eddy currents and hysteresis are not considered. The conditions necessary for maximum efficiency of transmission depend upon which of the quantities A, B, ωL, R and P'' are open to choice or capable of adjustment. The quantities ωL and R are ordinarily fixed in value, while A and B may be changed more or less by varying the field excitation of the respective machines, and P'' may be varied by changing the load on the motor.

1. *When the electromotive force A of the generating alternator is adjustable,* maximum efficiency is obtained when the current (and also RI^2) is a minimum; values of B, ωL, R and P'' being given. This minimum current is obtained when A is adjusted until B and I are opposite to each other in phase.

Proof.—The motor intake is $P'' = BI \cos (BI)$, according to equation (86). Therefore, since P'' and B are given, the minimum value of I corresponds to maximum value of $\cos (BI)$. But the maximum (negative) value of $\cos (BI)$ is -1, and the corresponding value of the angle (BI) is 180°.

To calculate the value of A which will bring B and I opposite to each other in phase for the given values of B, ωL, R and P'',

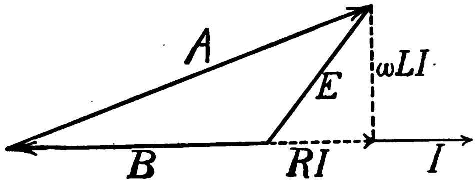


Fig. 157.

consider Fig. 157, in which I represents the current, and E the resultant electromotive force of machines A and B. From this figure we have

$$A^2 = (B + RI)^2 + \omega^2 L^2 I^2$$

from which A may be calculated when I is known. The value of I may be determined from the equation $P'' = BI \cos (BI)$.

2. *When the electromotive force B of the synchronous motor is adjustable*, maximum efficiency is obtained when the current (and also RI^2) is a minimum; values of A, ωL, R and P'' being given. This minimum current is obtained when B is adjusted until A and I are in phase with each other. The proof of this proposition is given below.

To calculate the value of B which will bring A and I into phase with each other for the given values of A, ωL, R and P'', consider Fig. 158, in which A represents the value of the elec-

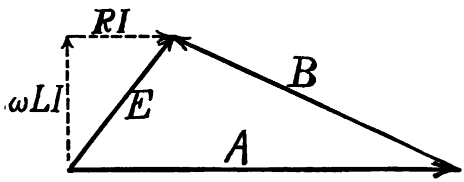


Fig. 158.

tromotive force of machine A, and E represents the resultant electromotive force of machines A and B. From this figure we have

$$(A - RI)^2 + \omega^2 L^2 I^2 = B^2$$

from which B may be calculated when I is known. The value of I may be calculated from the relation

$$AI \cos (AI) = P'' + RI^2$$

Proof that I is a minimum when B is adjusted until A and I are in phase with each other; A, ωL, R and P'' being given.—Plot the curve of which the ordinates represent values of ωLI and the abscissas represent values of $B \cos (BI)$. This curve is a hyper-

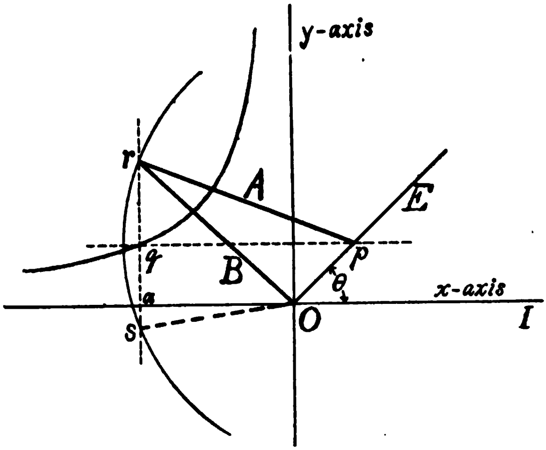


Fig. 159.

bola inasmuch as $B \cos (BI) \times I = P''$ or $B \cos (BI) \times \omega LI = \omega LP'' =$ *a constant*. This curve is plotted in Fig. 159 with the values of $B \cos (BI)$ laid off to the left inasmuch as P'', being an intake, is negative. The x-axis of reference OI, represents the current in the circuit; and the line OE, making with OI the known angle θ, but of unknown length, represents the resultant electromotive force of machines A and B.

Draw a horizontal line cutting OE in p and cutting the hyperbola in q. Then for that particular value of the current which

corresponds to the chosen ordinate aq ($= \omega LI$) the line Op represents the actual resultant electromotive force inasmuch as the vertical component E is equal to ωLI. Describe about the point p a circle of which the radius represents A. Then the lines Or and Os represent the two possible values of B for the chosen value of ωLI. From this diagram it is evident that for the shortest possible value of Op (smallest possible current) the circle described about p just touches the ordinate aq, in which case the line A is horizontal and parallel to OI, as shown in Fig. 160.

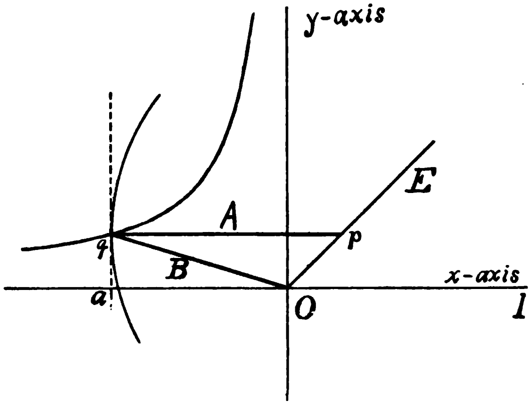


Fig. 160.

For any shorter value of Op (smaller value of E or I) the circle does not reach to the ordinate aq, which means that for so small a value of the current the given value of A is too small to supply the line losses RI^2 and the given motor intake P''.

3. *When P'' is adjustable*, maximum efficiency occurs when the differential coefficient of P''/P' with respect to ϕ is zero. From equations (89) and (90) we have

$$\frac{P''}{P'} = \frac{AB \cos(\phi - \theta) + B^2 \cos \theta}{AB \cos(\phi + \theta) + A^2 \cos \theta}$$

whence, applying the condition

$$\frac{d\left(\frac{P''}{P'}\right)}{d\phi} = 0$$

we have

$$A^2 \sin(\phi - \theta) - B^2 \sin(\phi + \theta) = 2AB \sin\theta \qquad (93)$$

which determines the value of ϕ for which the efficiency of transmission is a maximum.

Geometrical construction of equation (93).—Draw lines, Fig. 161, representing A^2 and B^2 to scale, the angle between them being 2θ. About the point p as a center describe a circle of which the radius represents $2AB \sin\theta$. From the point q draw a tangent to this circle. The angle between the line OS and this tangent is the required value of ϕ. Two tangents can be drawn from the point q. One of these tangents determines the value of ϕ (less than 180°) for which the efficiency of transmission is a maximum with machine A acting as a synchronous motor and the other tangent determines the value of ϕ (greater than 180°) for which the efficiency of transmission is a maximum with machine B acting as a synchronous motor. The machine A is distinguished as having the greater electromotive force. The angle ϕ is the lag of B behind A.

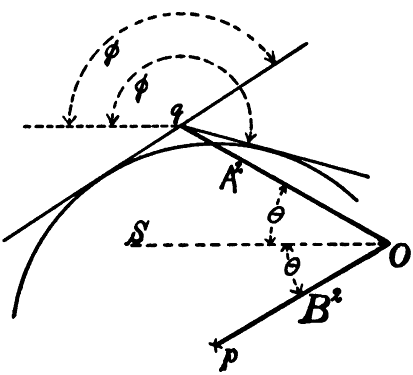


Fig. 161.

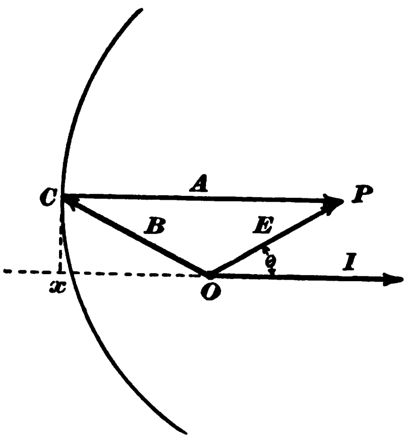


Fig. 162.

131. Value of B to give maximum intake of machine B with given current; A, ωL *and R, being given.*—Let I, Fig. 162, be the given current and

E $(= I\sqrt{R^2 + \omega^2 L^2})$ the resultant electromotive force. In order that the intake of B may be a maximum, $BI \cos (BI)$ or $B \cos (BI)$ must be a maximum. Now $B \cos (BI)$ is the projection of B on the current line OI. Describe a circle of radius A about the point P, Fig. 162; then Ox is the greatest possible value of $B \cos (BI)$ for the given current and OC is the required value of B. From the triangle OPC we have as the required value of B

$$B = \sqrt{A^2 + E^2 - 2AE \cos \theta}$$

Remark.—From Fig. 162 it is evident that A is in phase with I when B is adjusted to give maximum P'' with given I. It was shown in Article 130 that A is also in phase with I when B is adjusted to give minimum I with given P' (or with given P''). This correspondence is by no means self-evident.

132. Maximum intake of machine B; *A, B, ωL and R being given.*—P'' has its maximum negative value when $\cos (\phi - \theta) = -1$ and equation (89) becomes

$$P''_{max.} = \frac{B^2 \cos \theta - AB}{\sqrt{R^2 + \omega^2 L^2}} \qquad (94)$$

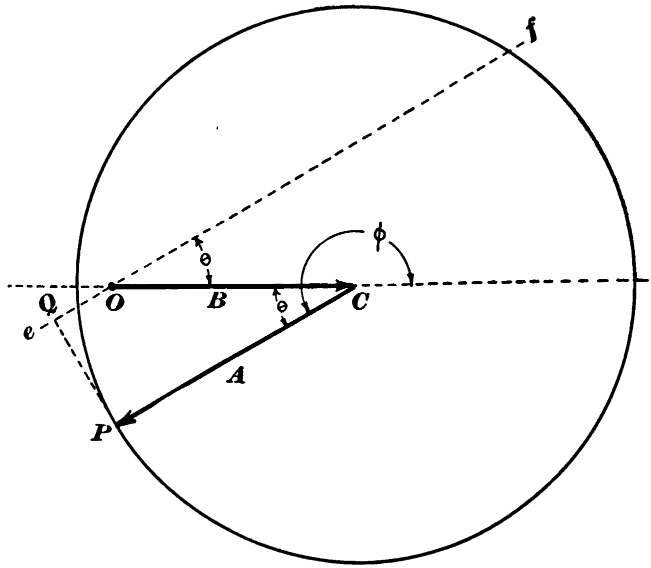


Fig. 163.

Fig. 163 shows the state of affairs when intake of B is at its greatest.

133. Greatest values of the electromotive force B for which machine B can act as a motor; *A, ωL and R being given.*—So long as

$$\frac{AB}{\sqrt{R^2+\omega^2L^2}}$$

is greater than

$$\frac{B^2}{\sqrt{R^2+\omega^2L^2}}\cdot\cos\theta$$

then P'' can have negative values according to equation (89). Therefore the limiting case is where

$$\frac{AB}{\sqrt{R^2+\omega^2L^2}}=\frac{B^2}{\sqrt{R^2+\omega^2L^2}}\cdot\cos\theta$$

or

$$B=\frac{A}{\cos\theta} \tag{95}$$

This limiting case is shown in Fig. 164.

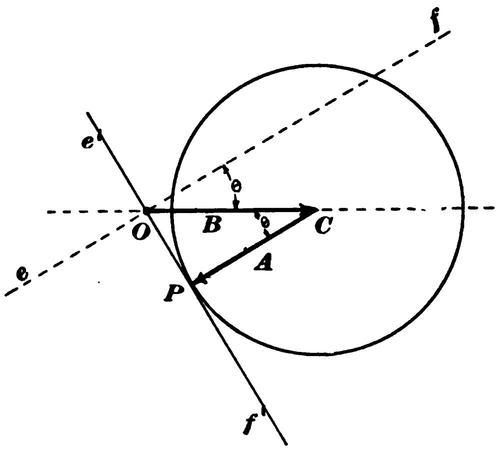


Fig. 164.

134. To find value of B for which the machine B may take in the greatest possible power from A; *A, ωL and R being given.*—

Equation (94) expresses the greatest intake of B for given values of A, B, ωL and R. It is required to find the value of B which will make this greatest intake a maximum. This value of B must render $B^2 \cos\theta - AB$ [the numerator of right-hand member of equation (94)] a maximum. Differentiating this expression with respect to B and placing the differential coefficient equal to zero we have

$$2B\cos\theta - A = 0$$

or

$$B = \frac{A}{2\cos\theta} \qquad (96)$$

Remark.—A comparison of equations (95) and (96) shows that the value of B for greatest possible intake of machine B is half the greatest value of B for which machine B can act as a motor at all. This is also the case with a direct-current motor. The greatest electromotive force such a motor can have is the electromotive force of the dynamo which drives it, and the value of its electromotive force to permit the greatest possible intake is one-half the electromotive force of the dynamo which drives it.

135. Excitation characteristics.—With given load on a synchronous motor (given value of P'') its electromotive force B may be changed by varying its field excitation, and for each value of B there is a definite value of the current I. Thus the abscissas of the curves, Fig. 165, represent values of I, and ordinates represent values of B for loads of zero, 100 kilowatts and 200 kilowatts respectively. These curves are called the *excitation characteristics* of the motor. Fig. 165 is based on the values $A = 1{,}100$ volts, $R = 1$ ohm and $\omega L = 0.58$ ohm. For the greatest possible intake, 302.7 kilowatts, the characteristic reduces to the point enclosed in the small circle. It was pointed out in Article 127 that with given load there are two values of I for each value of B, and that the larger value of I corresponds to unstable and the smaller value to stable running. The dotted portions of the curves, Fig. 165, correspond to the larger values of I. These

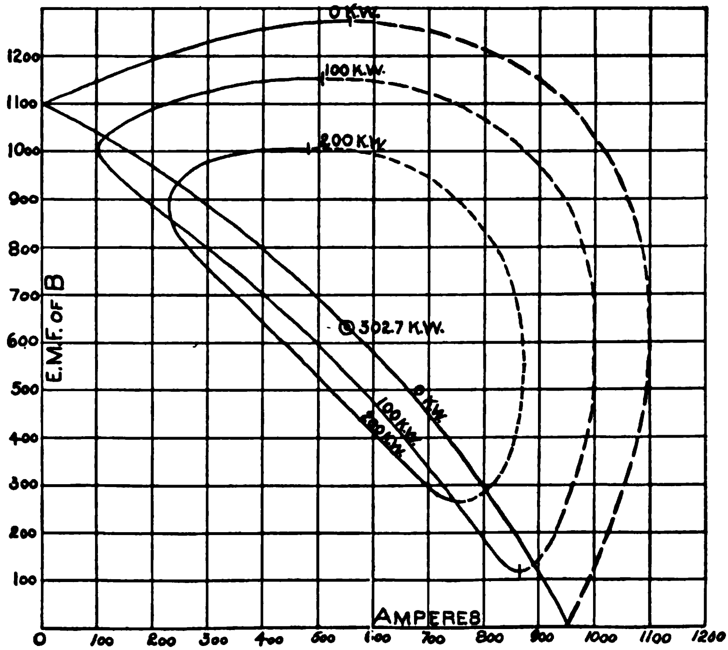


Fig. 165.

dotted portions cannot, of course, be determined by experiment, on account of the instability of running.

The equation to the excitation characteristics may be derived as follows: Let I, Fig. 166, represent the current and E the resultant electromotive force; the compo-

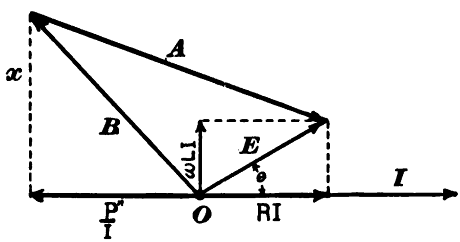


Fig. 166.

nents of E are RI and ωLI. The electromotive force E is the vector sum of A and B. as shown, and the component of B parallel to I is $\frac{P''}{I}$. From the right-angled triangles of the figures we have

$$B^2 = x^2 + \left(\frac{P''}{I}\right)^2 \qquad (a)$$

$$A^2 = \left(RI + \frac{P''}{I}\right)^2 + (x - \omega LI)^2 \qquad (b)$$

By eliminating x from these equations we have the required relation between B and I; P'', A, R and ωL being given. The curves, Fig. 165, were calculated graphically by means of the diagram, Fig. 149.

136. The negative reactance of the over-excited synchronous motor.—When the electromotive force of a synchronous motor is greater than the electromotive force of the generator which drives it, the motor is called an *over-excited* synchronous motor. Such a synchronous motor takes current which is ahead of the generator electromotive force in phase, so that an over-excited synchronous motor constitutes a receiving circuit of negative reactance like a condenser. This is especially the case when the motor load is light. This state of affairs is shown in Fig. 167, in which the A machine, which has the greater electromotive force, is the motor and is at zero load.

To calculate the effectiveness of an over-excited, light-running synchronous motor as a compensator for lagging currents when connected in parallel with an inductive receiving circuit, represent by the line A, Fig. 167, the electromotive force of the synchronous motor, by B the electromotive force between the mains at the receiving station, the angle θ being determined by the resistance and reactance of the armature of the synchronous motor. The component of I which is 90° ahead of B in phase is $I \sin \phi$, where ϕ is the angle at C of the triangle OCP and I is equal to the resultant electromotive force OP divided by the impedance of the motor.

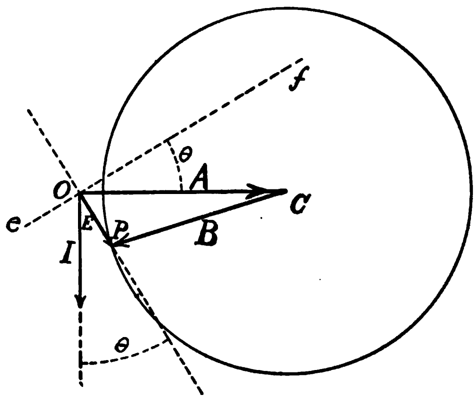


Fig. 167.

137. The polyphase synchronous motor.—The preceding articles refer explicitly to the single-phase synchronous motor, that is, to a single-phase alternator taking power as a motor from a single-phase generator. The entire discussion applies equally well, however, to the polyphase synchronous motor, that is, to a polyphase alternator taking power as a motor from a polyphase generator. In this case each armature winding of the motor takes current from one phase (or one armature winding) of the generator, and the total power intake is nP'', where n is the number of phases and P'' is the intake of each phase. When the preceding discussion is applied to the polyphase synchronous motor, the letters A, B, ωL, P', P'' and I refer to one phase only of the system.

138. The rotary converter.—The next chapter is devoted to the rotary converter, which, as ordinarily used, is at once a synchronous alternating-current motor and a direct-current dynamo. This machine usually runs as a polyphase synchronous motor, taking power from a polyphase alternating-current generator and giving out power in the form of direct current. When run in this way the rotary converter exhibits all the properties of the synchronous motor as outlined in the foregoing articles. The next chapter is devoted mainly to the discussion of the relations between the direct-current output and the alternating-current intake, to the relations between the direct electromotive force and the alternating electromotive force of the rotary converter and to armature heating.

PROBLEMS.

95. Plot curves showing the values of P', P'' and RI^2 for two synchronous alternators in series, the one alternator A having an electromotive force of 1,200 volts, the other machine B an electromotive force of 800 volts, the resistance of the circuit being 1 ohm and the reactance 2 ohms. Result similar to Fig. 151.

96. Two governed engines drive two alternators which are connected in multiple and feed one pair of mains. Engine A drops from 150 revolutions per minute at zero load to 145 revolutions per minute at 100 horse power. Engine B drops from 150 revolutions per minute at zero load to 147 revolutions per minute at full load of 75 horse power. The station output is such as to require a total of 125 horse power to be delivered by the engines. What power is delivered by each engine, and what is their common speed? What power is delivered by engine B when engine A is idle? Ans. (*a*) 70.2 H. P., (*b*) 54.8 H. P., (*c*) 18.7 H. P.

97. An alternator B, of which the electromotive force is 1,000 volts, takes 75 kilowatts from alternator A. To what value must the electromotive force of A be adjusted, so that the efficiency of transmission may be maximum, and what is the corresponding value of the current, resistance of circuit being 1 ohm and reactance of circuit being 0.58 ohm? Ans. 1,076 volts, 75 amperes.

98. The electromotive force of alternator A in problem 97 is 1,076 volts. To what value must the electromotive force of alternator B be adjusted to give maximum efficiency of transmission, and what is the corresponding value of the current, intake of B being 75 kilowatts? Ans. 1,009.8 volts, 74.9 amperes.

99. The electromotive force of alternator A is 1,100 volts, and that of alternator B is 1,000 volts, resistance of circuit is 1 ohm and reactance of circuit 0.58 ohm. What is the angular lag of B behind A, for which power is most efficiently transmitted from A to B, what is the intake of B under these conditions, what is the output of A, and what is the efficiency? Ans. 181.8°, $P'' = -92$ kilowatts, $P' = +101.8$ kilowatts, efficiency 91.3 per cent.

100. What is the angular lag ϕ of B behind A, in problem 95, for which power is most efficiently transmitted from B to A; what is the intake of A under these conditions, what is the output of

B, and what is the efficiency? Ans. 159.4°, $P' = -36.4$ kilowatts, $P'' = +156.7$ kilowatts, efficiency 23.2 per cent.

101. The electromotive force of A is 900 volts, that of B is 800 volts, resistance of circuit is 1 ohm, and reactance of circuit is 1 ohm. What is the maximum intake of machine B as a synchronous motor? Ans. 189.1 kilowatts.

102. What is the greatest value of B (A, ωL, and R being as in problem 97) which will permit machine B to act as a synchronous motor? Ans. 1,272 volts.

103. What value must B have in order that the maximum intake of machine B may be the greatest possible, and what is this intake? Ans. 636 volts; 220.3 kilowatts.

104. Given $A = 800$ volts, $R = 1$ ohm, $\omega L = 1$ ohm, $P'' = 30$ kilowatts; plot curve showing different values of I corresponding to different values of B. Result similar to Fig. 165.

105. An alternator of which the electromotive force is 150 volts is to be run as a motor from 110-volt mains. The resistance of the alternator armature is 1 ohm. What is the minimum amount of inductance required in the circuit, the frequency being 60 cycles per second? Ans. 0.00246 henry.

106. A condenser is connected in series with alternators A and B so that the total reactance of the circuit is -0.58 ohm. The resistance of the circuit is 1 ohm, the electromotive force of machine A is 1,100 volts, and the electromotive force of machine B is 1,000 volts. Plot curves showing the values of P', P'' and RI^2 for various values of ϕ.

107. An alternator A has an electromotive force of 1,100 volts, a resistance of 1 ohm, and a reactance of 0.58 ohm. The machine is driven as a synchronous motor with zero load from 1,000-volt mains. What is the value of the current? What is the component of this current which is 90° ahead of the supply electromotive force in phase? What capacity of condensers would take the same amount of leading current from the 1,000-volt mains at a frequency of 60 cycles per second? Ans. 268.7 amperes, 253.1 amperes, 4,574 microfarads.

CHAPTER XIII.

THE ROTARY CONVERTER.

139. The rotary converter.—An ordinary direct-current dynamo may be made into an alternator by providing it with collecting rings, as described below, in addition to its commutator. Such a machine is called a *rotary converter.*

The single-phase converter is provided with two collecting rings which, in case of a two-pole machine, are connected to diametrically opposite armature conductors.

The two-phase converter is provided with four collecting rings which, in case of a two-pole machine, are connected to armature conductors 90° apart.

The three-phase converter is provided with three collecting rings which, in case of a two-pole machine, are connected to armature conductors 120° apart.

Remark 1.—It is often convenient to refer to a rotary converter as a two-ring, three-ring, four-ring, or n-ring converter, as the case may be.

Remark 2.—In case of a multipolar machine the n collecting rings are connected to the armature as follows: Ring No. 1 is connected to all armature conductors which, for any given position of the armature, lie midway under the north poles of the field magnet. Let l be the distance between adjacent conductors of this first set, that is, the distance from north pole to the next north pole. Then ring No. 2 is connected to the armature conductors which are $\frac{1}{n}$th of l ahead of the first set; ring No. 3 is connected to the armature conductors which are $\frac{2}{n}$ths of l ahead

of the first set; ring No. 4 is connected to the conductors which are $\frac{3}{n}$ths of l ahead of the first set, and so on. This statement applies to multicircuit winding. In case of the two-circuit winding each collecting ring is connected to one armature conductor only. Fig. 168 shows a four-pole dynamo with two collecting rings each connected to two armature conductors. The machine when provided with these collecting rings is a four-pole single-phase rotary converter.

Use of the rotary converter.—The rotary converter may be used as an ordinary direct-current dynamo or motor; as an alternator or synchronous motor; it may be driven as a direct-current motor, the load being provided by taking alternating current from its collecting rings; or it may be driven as a synchronous alternating-current motor, the load being provided by taking direct current off the commutator. This last is the principal use of the machine. In most cases where power, transmitted to a distance by alternating current, is to be used in the form of direct current, the rotary converter is used for bringing about the conversion from alternating current to direct current. Thus, in many extended electric railway plants, it is found expedient to transmit the power as high pressure polyphase current from a central station to rotary converters stationed along the line of the railway; these rotary converters take the alternating current through step-down transformers and, in their turn, supply direct current at medium pressure to the trolley wires.

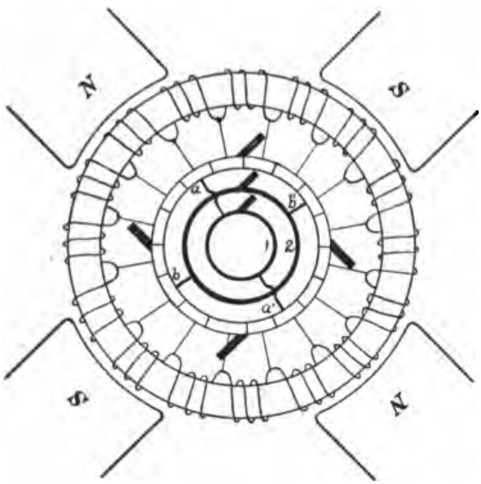


Fig. 168.

140. The starting of the rotary converter and its operation when used to convert alternating current into direct current.—When used in this way the rotary converter is a synchronous motor and it differs but little in its operation from the synchronous motor with a belt load.

Starting.—The machine may be started as a direct-current motor using storage batteries or other local source of direct current; or it may be started in precisely the same manner as a synchronous motor with a belt load as described in Article 126. The field magnet of a rotary converter is always excited by direct current taken from the machine itself.

Operation.—Let *B*, Fig. 169, be the effective alternating electromotive force of a rotary converter and *A* the electromotive force of the alternating generator. Fig. 169 is identical to Fig.

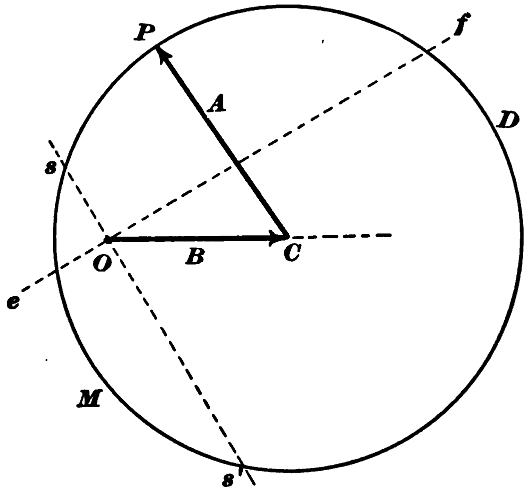


Fig. 169.

153, Chapter XII. When no direct current is taken from the converter its load is zero and the point *P*, Fig. 169, is at *s*. When direct current is taken from the converter the point *P* moves towards *M*. The alternating current taken by the converter, being proportional to the resultant electromotive force

OP, at first decreases and then increases * as the direct current load increases, and so on, exactly as in the case of a synchronous motor with a belt load. (See Article 122.)

It was pointed out in Chapter XII. that a synchronous motor may operate at comparatively high efficiency for a wide range of values of B (value of A given) if the alternating-current circuit has large reactance; in fact, B may even be larger than A, as pointed out in Article 133. If a considerable portion of the reactance is external to the armature of the converter then the electromotive force between the collecting rings of the converter changes with B and so also does the electromotive force of the machine. Therefore the direct electromotive force of a rotary converter may be varied at will † by changing the field excitation of the machine, although the electromotive force A of the alternating generator may be constant.

Hunting.—The hunting action of the sychronous motor is described in Article 129. The synchronous motor with a belt load does not often hunt, for, the inelasticity of the belt, the slack in the belt, and the friction of the driven machinery tend to steady the machine. The rotary converter, on the other hand, is peculiarly subject to hunting, and if the pulsations of the engine, which drives the alternator, are in unison with the hunting oscillations of the converter and alternator, hunting is sure to be produced. A method for damping the hunting oscillations of a rotary converter is described in Article 142.

141. Preliminary statement concerning armature current of a rotary converter.—Consider a given armature conductor of a rotary converter. A part of the current in this conductor is due to the alternating currents which flow into the armature at the collecting rings and a part is due to the direct current flowing out of the armature at the direct current brushes. The actual current in the conductor is the algebraic sum of these two parts,

* When B is less than A.

† The possible range of variation depends upon the reactance in the circuit external to the rotary converter.

and since these parts are generally opposite in sign, therefore the actual current in the conductor is rather small and so also is its magnetic effect and its heating effect.

142. Magnetic reaction of the armature of the rotary converter. *Distortion of field.*—The distortion of the magnetic field of a dynamo by the armature currents accompanies, and is in fact the cause of, the torque with which the field acts upon the armature. When the torque is in the direction of the rotation of the armature (motor action) the field is concentrated under the leading horns of the

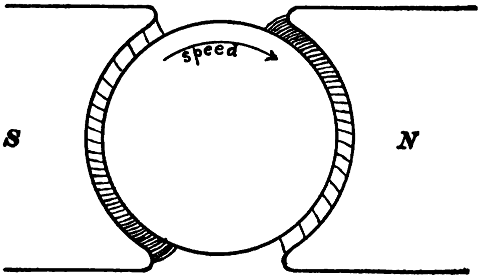


Fig. 170.

pole pieces as shown in Fig. 170. When the torque is opposite to speed the field is concentrated under the trailing horns of the pole pieces as shown in Fig. 171.

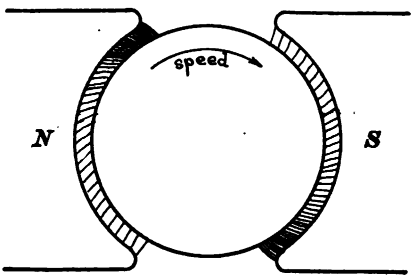


Fig. 171.

When a rotary converter is running steadily the speed of its armature is constant, and the only torque acting on the armature is the slight torque needed to overcome friction, therefore the field is scarcely at all distorted.

When a rotary converter hunts, its speed oscillates above and below synchronism so that a torque acts upon the armature, first in one direction and then in another, and the field is concentrated, first under the trailing horns and then under the leading horns of the pole pieces.

The damping of the hunting oscillations of a rotary converter is accomplished by a heavy copper frame *ccc*, Fig. 172, which is embedded in the face of each field pole as shown. The shifting

of the flux from one side to the other of the pole face, as described above, induces large currents in the circuits A and B, Fig. 172. These currents oppose the shifting of the flux and thereby damp the oscillations.

Demagnetizing action.—The demagnetizing action of the armature currents of a rotary converter may be considered as made up of the demagnetizing action of the direct current alone and of the alternating currents alone. The first is the same as in the direct-current dynamo and the second is considered in Article 79.

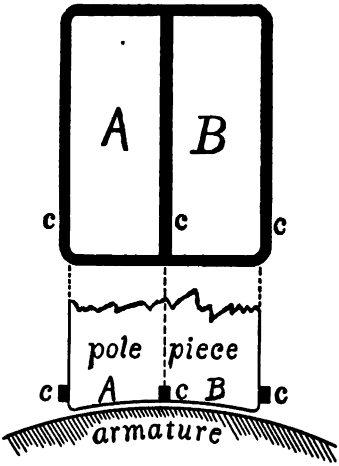


Fig. 172.

A very important effect due to the demagnetizing action of the armature currents in a rotary converter is the following. A converter takes direct current from constant electromotive force mains and delivers alternating current. When a short circuit occurs on the alternating current mains the converter speeds up indefinitely and may be destroyed. The enormous *lagging* alternating current weakens the field and, since but little mechanical effort is required, the speed increases just as the speed of an *unloaded* direct-current motor would increase by weakening its field.

143. Power rating of rotary converters.—The magnetic action (demagnetizing action and distorting action) of the armature currents of a rotary converter is never troublesome, so that the allowable output is limited by the permissible heating of the armature. The armature heating is rather small, as pointed out in Article 141, so that a given machine has a higher power rating as a rotary converter than as a direct-current dynamo, except in the case of the single-phase converter. The accompanying table gives the power ratings (based upon equal average armature

heating) of a given machine when used (*a*) as a direct-current dynamo, (*b*) as a single-phase converter, (*c*) as a three-phase converter, (*d*) as a two-phase (four-ring) converter, and (*e*) as a six-phase converter.

POWER RATINGS OF ROTARY CONVERTERS.*

a. Continuous-current dynamo.	*b.* Single-phase converter.	*c.* Three-phase converter.	*d.* Four-ring converter.	*e.* Six-phase converter.
1.00	.85	1.32	1.62	1.92

144. Electromotive force relations of the rotary converter.—Let E_0 be the electromotive force between the direct-current brushes and E_n the effective alternating electromotive force between adjacent collecting rings of an n-ring converter. The ratio $\frac{E_n}{E_0}$ has a characteristic value for each value of n.

Fundamental assumption.—Consider an armature conductor c, Fig. 173, at angular distance β from the axis of the field, as

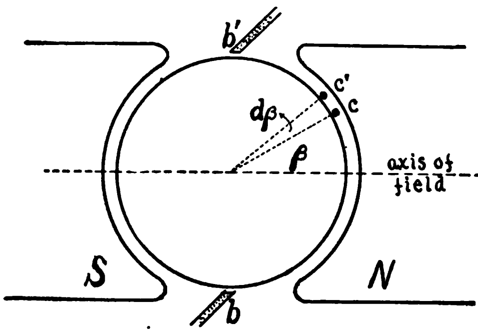


FIG. 173.

shown.† We assume that the electromotive force induced in the conductor c is proportional to $\cos \beta$ or equal to $C \cos \beta$ where

* These ratings are calculated as explained in Article 147, and in their calculation the losses in the machine and the wattless component of the alternating currents have been ignored. These ratings are therefore somewhat too large.

† The discussion in Articles 144 to 147 is given for the case of a two-pole machine. The results, however, apply to multipolar machines as well.

C is a constant. The results of this assumption are practically in accord with experiment.

The number of armature conductors between c and c' is proportional to, or say equal to, $d\beta$.

The electromotive force in each conductor is $C \cdot \cos\beta$, and the electromotive force in all the conductors between c and c' is:

$$de = C \cdot \cos\beta \cdot d\beta. \qquad (a)$$

Electromotive force E_0 between direct current brushes.—All the conductors between b and b' are in series between the direct-current brushes so that

$$E_0 = \int_{-90°}^{+90°} C \cdot \cos\beta \cdot d\beta$$

or

$$E_0 = 2C \qquad (b)$$

Effective electromotive force E_n between adjacent collecting rings of an n-ring converter.—The electromotive force between adjacent rings r and r', Fig. 174, is at its maximum value when the arc

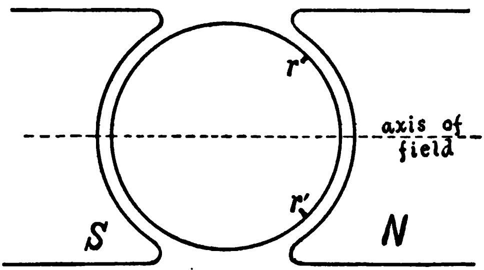


Fig. 174.

rr' is bisected by the axis of the field as shown. The angle between r and r' is $\frac{2\pi}{n}$ or half this angle is $\frac{\pi}{n}$. The maximum electromotive force $\sqrt{2}E_n$ between rings r and r' is therefore:

$$\sqrt{2}E_n = \int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} C \cdot \cos\beta \cdot d\beta = 2C \sin\frac{\pi}{n}$$

or since $2C = E_0$, we have

$$E_n = \frac{1}{\sqrt{2}} E_0 \sin \pi/n \qquad (97)$$

Examples.—The effective alternating electromotive force of a single-phase converter ($n = 2$) is:

$$E_2 = \frac{E_0}{\sqrt{2}} = .707 E_0 \tag{98}$$

The effective alternating electromotive force between adjacent rings of a three-phase converter ($n = 3$) is:

$$E_3 = \frac{\sqrt{3} E_0}{2\sqrt{2}} = .612 E_0 \tag{99}$$

The effective alternating electromotive force between adjacent rings of a two-phase converter ($n = 4$) is:

$$E_4 = \frac{E_0}{2} \tag{100}$$

The effective electromotive force between opposite rings of a two-phase converter is E_2.

145. Current relations of the rotary converter. *Fundamental assumptions.*—In the discussions of current relations we shall assume that the alternating current flowing through each section (between adjacent collecting rings) of the armature is exactly opposite in phase to the alternating electromotive force in that section, and that the intake and output of power are equal.

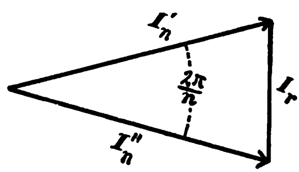


Fig. 175.

Let I_0 be the output of direct current and let I_n be the effective alternating current flowing in the armature between two adjacent collecting rings. The intake of power per phase is $E_n I_n$ or the total intake is $nE_n I_n$ and the power output is $E_0 I_0$. Therefore

$$E_0 I_0 = nE_n I_n$$

or substituting the value of E_n from equation (97) we have:

$$I_n = \frac{\sqrt{2} I_0}{n \sin \pi/n} \tag{101}$$

Current in each main.—The current I_r in each main or the current entering the armature at each collecting ring, is the vector difference between I_n' and I_n'', Fig. 175, so that

$$I_r = 2I_n \sin \pi/n \qquad (102)$$

Examples.—The effective alternating current in each half of the armature of a 2-pole single-phase converter ($n = 2$) is:

$$I_2 = \frac{I_0}{\sqrt{2}} \qquad (103)$$

and the effective alternating current entering at each collecting ring is

$$I_r = 2I_2 = 1.414 I_0 \qquad (104)$$

The effective alternating current flowing in the armature between adjacent collecting rings of a 2-pole three-phase converter ($n = 3$) is:

$$I_3 = \frac{2\sqrt{2}}{3\sqrt{3}} I_0 \qquad (105)$$

and the effective current entering at each collecting ring is

$$I_r = \sqrt{3} I_3 \qquad (106)$$

The effective alternating current flowing in the armature between adjacent collecting rings of a 2-pole two-phase converter ($n = 4$) is:

$$I_4 = \tfrac{1}{2} I_0 \qquad (107)$$

and the effective current entering at each collecting ring is

$$I_r = \sqrt{2} I_4 \qquad (108)$$

146. Instantaneous current in a given armature conductor of a rotary converter.—Let r and r', Fig. 176, be the points of attachment of adjacent collecting rings of an n-ring converter and let the line OM bisect the arc rr'. Consider an armature conductor c between r and r' and let the angle cOM be represented by α. The largest possible value of α is π/n or one-half of the angle between r and r'.

Let ωt be the angle between OM and the axis of the field. Then the alternating current between the collecting rings r and

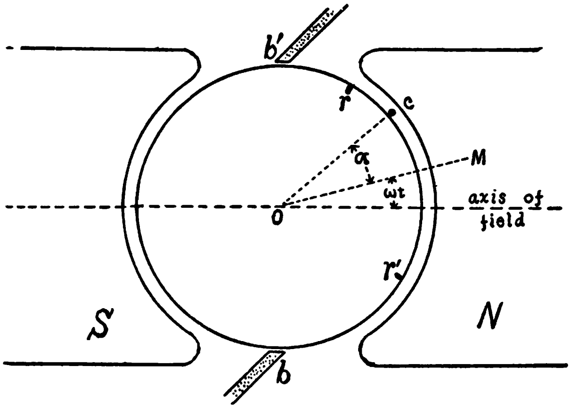


Fig. 176.

r', that is, the alternating current in the conductor c, is at its maximum value $\sqrt{2}I_n$ when $\omega t = 0$. Therefore the expression for the instantaneous value of this current is $\sqrt{2}I_n \cos \omega t$.

When the conductor c is at brush b', $\omega t = 90° - a$, when the conductor c reaches brush b, $\omega t = 270° - a$, and when conductor c reaches brush b' again $\omega t = 450° - a$ and so on. Each time the conductor c passes a brush the direct current $\frac{I_0}{2}$ (discussion applied to a 2-pole machine) in conductor c is reversed. Therefore the total current in conductor c is

$$i = \sqrt{2}I_n \cos \omega t \pm \frac{I_0}{2} \tag{109}$$

The $+$ sign is to be taken between $\omega t = 90° - a$ and $\omega t = 270° - a$, the $-$ sign is to be taken between $\omega t = 270° - a$ and $\omega t = 450° - a$, the $+$ sign again between $450° - a$ and $630° - a$, etc.

Remark.—The angle a determines simply the phase of the alternating current in the conductor at the instant that the direct current is reversed.

The meaning of equation (109) is made more clear perhaps by the curves shown in Figs. 177 (a) and (b). The upper curves in each figure are the component current curves; that is, the ordinates of the square-wave curve represent that part of the current in the given conductor which depends upon the direct cur-

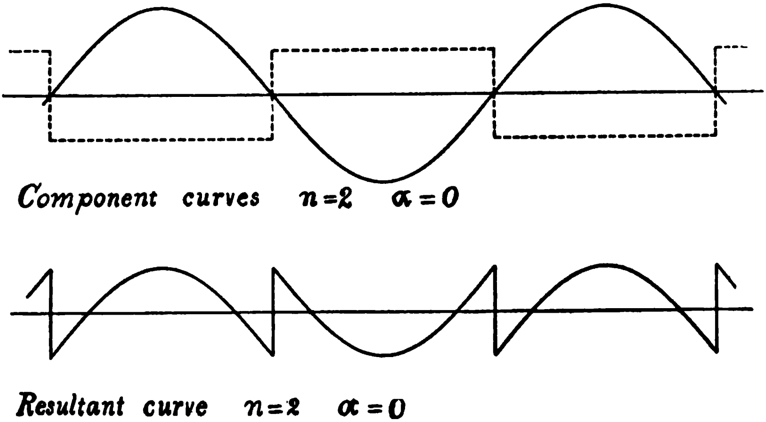


Fig. 177 (a).

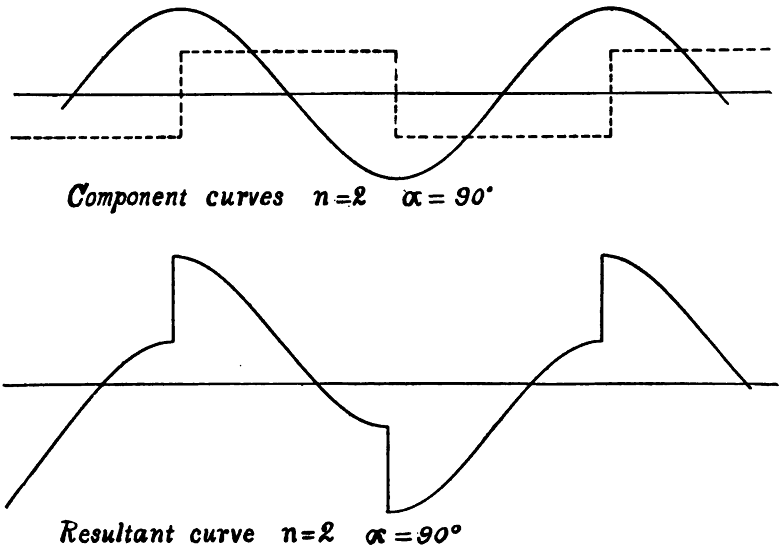


Fig. 177 (b).

rent output of the machine, and the ordinates of the sine curve represent that part of the current in the given conductor which depends upon the alternating current intake of the machine. The alternating currents are assumed to be exactly opposite in phase to the alternating electromotive forces of the machine.

147. The heating of the armature conductors of a rotary converter.—Equation (109) expresses the instantaneous value of the actual current in a given armature conductor of a rotary converter during the time that this conductor is passing from one direct-current brush to the other; that is, from $\omega t = -(90° + a)$ to $\omega t = 90° - a$. The current passes through a similar set of values during the next half revolution of the armature. The average rate at which heat is generated in the given conductor is proportional to the average value of i^2, equation (109), during the time $\omega t = -(90° + a)$ to $\omega t = 90° - a$. Therefore

the average rate of generating heat in the conductor c is proportional to:

$$\frac{I_0^2}{4}\left(1 - \frac{16 \cos a}{\pi n \sin \pi/n} + \frac{8}{n^2 \sin^2 \pi/n}\right) \qquad (110)$$

The rate at which heat would be generated in a given conductor by the direct current alone would be proportional to $I_0^2/4$, and equation (110) shows that the conductor c has

$$\left(1 - \frac{16 \cos a}{\pi n \sin \pi/n} + \frac{8}{n^2 \sin^2 \pi/n}\right)$$

times as much heat generated in it as an n-ring converter as would be generated in it by the direct current alone.

The conductors midway between the points of attachment of the collector rings ($a = 0$) are heated least, and the conductors near the points of attachment of the collector rings ($a = \pm \pi/n$) are heated most. For example, in a two-ring converter ($n = 2$) the conductors midway between the points of attachment of the collector rings ($a = 0$) have only 0.453 as much heat generated in them as would be generated in them by the direct current alone, and the conductors near the points of attachment of the

collector rings ($a = \pm 90°$) have three times as much heat generated in them as would be generated in them by the direct current alone.

The average heating of the entire armature of a rotary converter.—The average heating over the entire armature is found by integrating the equation (110) with respect to a from $a = -\frac{\pi}{n}$ to $a = +\frac{\pi}{n}$ and dividing the result by $\frac{2\pi}{n}$. This gives:

Average heating of armature of n-ring converter is proportional to: $$\frac{I_0^2}{4}\left(1 - \frac{16}{\pi^2} + \frac{8}{n^2 \sin^2 \pi/n}\right) \qquad (a)$$

The average heating is therefore $\left(1 - \frac{16}{\pi^2} + \frac{8}{n^2 \sin^2 \pi/n}\right)$* times as great as the heating of the armature by the direct current alone. Therefore an n-ring converter can put out

$$\frac{1}{\sqrt{1 - \frac{16}{\pi^2} + \frac{8}{n^2 \sin^2 \pi/n}}}$$

times as much direct current as the same machine can when used as a simple dynamo, for the same total armature heating. The table given in Article 143 is calculated in this way.

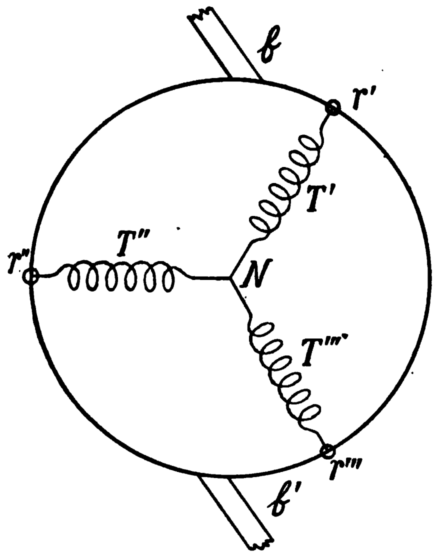


Fig. 178.

148. Use of rotary converter for supplying current to the Edison three-wire system.—T', T'', and T''', Fig. 178, represent the three secondaries of the step-down transformer which are Y-connected to three collecting rings, r', r'', and r''', of a three-ring converter. *The electromo-*

* Generally less than unity.

tive force between the neutral point N and either direct-current brush is constant and equal to half the electromotive force from brush to brush. Therefore the middle wire of Edison's three-wire system may be connected to N, the outside wires being connected to the direct-current brushes b and b'.

Discussion.—The electromotive force between brush b and the neutral point N is the sum of the electromotive forces b to r' and r' to N. Let E_0 be the steady electromotive force b to b', and let time be reckoned from the instant that r' is at b. Then the electromotive force br' fluctuates between zero (when $t = 0$) and E_0, and its instantaneous value is

$$\overline{br'} = \frac{E_0}{2} - \frac{E_0}{2} \cos \omega t$$

The effective electromotive force E_3 between rings is

$$E_3 = \frac{\sqrt{3}\,E_0}{2\sqrt{2}} \tag{99 bis}$$

and the effective electromotive force in one of the Y-connected units T', T'', and T''' is $E_3 \div \sqrt{3}$ or $E_0 \div 2\sqrt{2}$, so that the maximum electromotive force in T' is $E_0 \div 2$. Now the electromotive force in T' is at its maximum value when the ring r' is at b, that is, when $t = 0$ or when $\omega t = 0$. Therefore the instantaneous electromotive force in $r'N$ is

$$\overline{r'N} = \frac{E_0}{2} \cos \omega t$$

Therefore

$$\overline{br'} + \overline{r'N} = \frac{E_0}{2}$$

PROBLEMS.

108. The plain multicircuit ring-wound armature of a six-pole direct-current dynamo has 360 conductors on its face. These conductors are numbered from 1 to 360.

(*a*) One collecting ring of a rotary converter is connected to conductor No. 1. To what other conductors must this ring be connected? Ans. A to 121 and 241.

(*b*) To what conductors must a second ring (B) be connected to give a two-ring converter? Ans. B to 61, 181 and 301.

(*c*) To what conductors must two additional rings B and C be connected to give a three-ring converter? Ans. B to 41, 161 and 281, C to 81, 201 and 321.

(*d*) To what conductors must three additional rings B, C and D be connected to give a four-ringed? Ans. B to 31, 151 and 271, C to 61, 181 and 301, D to 91, 211 and 331.

(*e*) To what conductors must four additional rings B, C, D and E be connected to give a five-ring converter? Ans. B to 25, 145 and 265, C to 49, 169 and 289, D to 73, 193 and 313, E to 97, 217 and 337.

109. A four-pole, two-circuit, single drum winding has 102 conductors numbered consecutively from 1 to 102. The conductors are connected as follows: 1–26–51–76–101 . . . 28–53–78 and back to 1. To what conductors must the three rings A, B and C of a three-ring converter be connected? Ans. A to No. 1, B to No. 35, and C to No. 69.

110. Make a diagram of the following winding and show three collecting rings connected to conductors 1, 19 and 37. The winding is a four-pole, two-circuit, single drum winding with 54 conductors connected up as follows: 1–14–27–40–53 . . . 3–16–29–42 and back to 1.

111. A 50-kilowatt direct-current dynamo is to be used as an n-ring converter. What is its capacity rating when $n = 2$, when $n = 3$, when $n = 4$, and when $n = 6$? Ans. 42.5, 66, 81, and 96 kilowatts.

112. A three-ring converter is to supply direct current at 500 volts to a street-car line. At what voltage must the alternating currents be delivered to the machine? The step-down transformation is accomplished by three similar transformers with their

primaries Δ-connected to the high voltage mains and their secondaries Y-connected to the three rings of the converter. What is the ratio of transformation of each transformer, the line voltage being 10,000 volts? Ans. 306 volts. $\frac{N''}{N'} = 0.0530$.

113. A two-ring converter with negligible armature resistance and reactance takes current over a 0.2-ohm line through a reaction coil having 0.3 ohm reactance. The effective alternating electromotive force at the generator terminals is kept constant at 120 volts. The excitation of the converter can be varied at will. Find greatest value of alternating electromotive force of converter for which it can run as a synchronous motor, and find the corresponding value of the direct electromotive force of the converter.

Find the greatest direct-current load which the converter can carry without dropping out of synchronism (*a*) when the direct voltage of the converter is at its maximum and (*b*) when the direct voltage of the converter is 5 volts, 10 volts, 15 volts, 20 volts, and 25 volts respectively less than the maximum. In this problem ignore friction and other losses in the machine. Ans. 216.25 volts, 306 volts, (*a*) zero, (*b*) 4.24 amperes, 7.59 amperes, 11.53 amperes, 15.32 amperes, 19.15 amperes.

114. Plot the curve of which the ordinates represent the instantaneous values of the current in an armature conductor of a four-pole, three-ring converter with multicircuit winding, I being 250 amperes: (*a*) when the conductor is adjacent to the connection of a ring, and (*b*) when the conductor is midway between the connections of two rings.

115. A 100-kilowatt direct-current generator is provided with 5 collector rings. Find its power rating as a rotary converter, based upon average armature heating. Ans. 181 kilowatts.

CHAPTER XIV.

THE INDUCTION MOTOR.

149. The induction motor.—It has already been pointed out that the successful employment of alternating current for motive purposes depends upon the use of the *induction motor* driven by polyphase currents. The induction motor consists of a primary member and a secondary member, each with a winding of wire. The primary member is usually stationary, and is often called the *stator*. The secondary member is usually the rotating member,

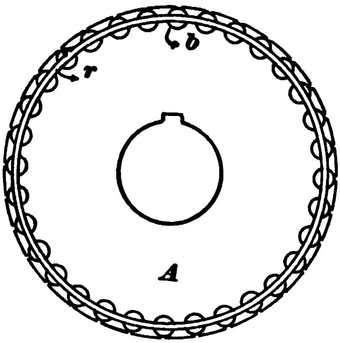


Fig. 179.

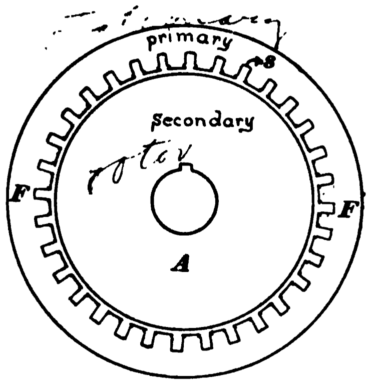


Fig. 180.

and is often called the *rotor*. Fig. 179 shows a rotor of the *squirrel-cage* type. It consists of a drum A built up of circular sheet-iron disks; near the periphery of this drum are a number of holes parallel to the axis of the drum; in these holes heavy copper rods b are placed, and the projecting ends of these rods are soldered to massive copper rings, r, one at each end of the drum. Another type of rotor is described later.

The stator is a laminated iron ring, *FF*, Fig. 180, closely surrounding the rotor. This ring is slotted on its inner face, as shown, windings are arranged in these slots, and these windings receive currents from polyphase supply mains. These polyphase currents produce in the stator a rotating state of magnetism, the action of which on the rotor is the same as the action of an ordinary field magnet in rotation. Thus Fig. 181 shows a squirrel-cage rotor *A* surrounded by an ordinary field magnet rotating in the direction of the curved arrows.

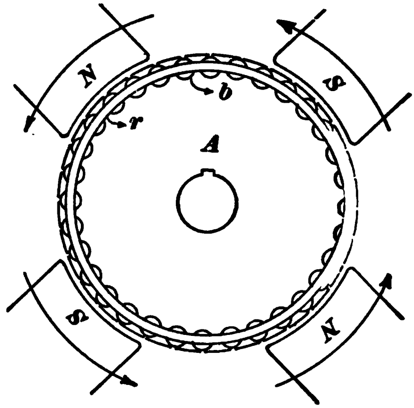


Fig. 181.

This motion of the field magnet induces currents in the short-circuited copper rods of the rotor; the field magnet exerts a dragging force on these currents and causes the rotor to rotate. No electrical connections of any kind are made to the rotor. The next article describes the stator windings and explains the manner in which these windings produce the rotating state of magnetism in the stator.

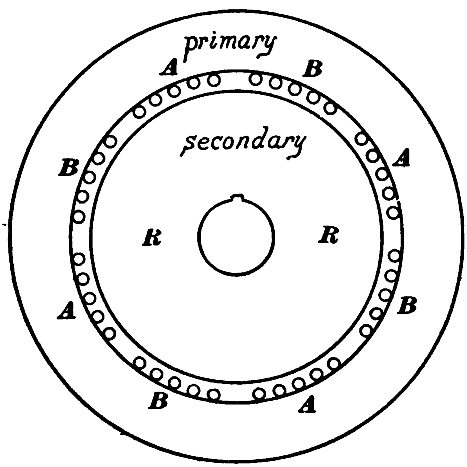


Fig. 182.

150. Stator windings and their action.—The stator windings are arranged in the slots *s*, Fig. 180, in a manner exactly similar to the arrangement of the wind-

ings of the two-phase or three-phase alternator armature, according as the motor is to be supplied with two- or three-phase currents.

Fig. 182 shows an end view of a four-pole two-phase induction motor. In this figure the outline, only, of the rotor is shown; the stator conductors are represented in section by the small circles; the slots are omitted for the sake of clearness; and the end connections of half the stator conductors are shown

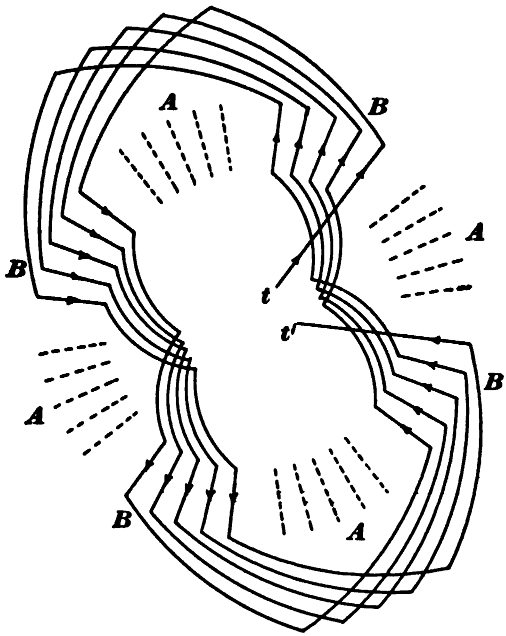


Fig. 183.

in Fig. 183. The stator conductors are arranged in two distinct circuits. One of these circuits includes all of the conductors marked A and it receives current from one phase of a two-phase system; the other circuit includes all of the conductors marked B and it receives current from the other phase of the two-phase system. The terminals of the B circuit are shown at tt', Fig. 183. The conductors which constitute one circuit are so connected that the current flows in opposite directions in adjacent groups

of conductors as indicated by the arrows in Fig. 183. The radial lines in Fig. 183 represent the stator conductors and the curved lines represent the end connections, as in the winding diagrams, Figs. 103 to 110.

The action of a band of conductors between two masses of iron is shown in Figs. 184 and 185. The small circles in these

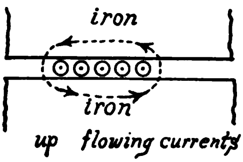


Fig. 184.

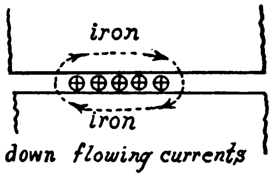


Fig. 185.

figures represent the conductors in section; conductors carrying down-flowing currents are marked with crosses, those carrying up-flowing currents are marked with dots, and those carrying no current are left blank. The action of the currents in these bands of conductors is to produce magnetic flux along the dotted lines in the directions of the arrows.

The lines A' and B' in Figs. 186, 187 and 188 are supposed to rotate and their projections on the fixed line ef represent the

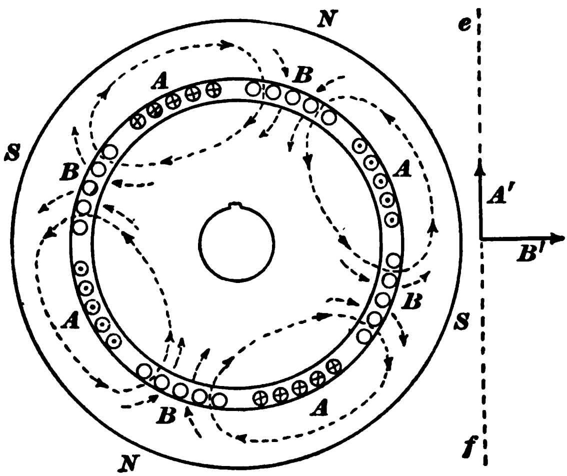


Fig. 186.

instantaneous values of the alternating currents in the *A* and *B* conductors respectively.

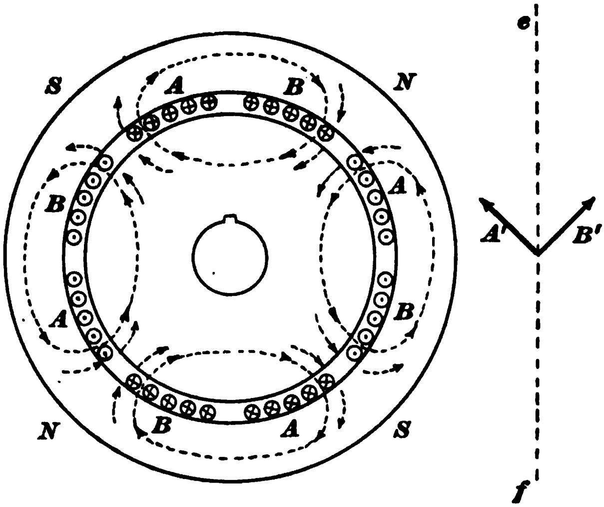


Fig. 187.

Fig. 186 shows the state of affairs when the current in conductors *A* is a maximum and the current in conductors *B* is zero. The dotted lines indicate the trend of the magnetic flux. This

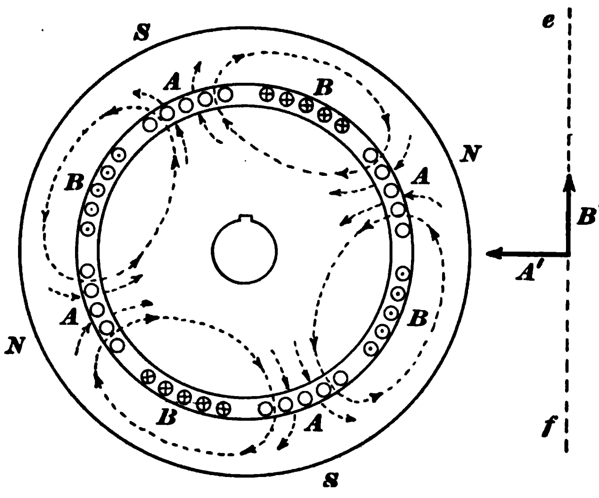


Fig. 188.

flux enters the rotor from the stator at the points marked N and leaves the rotor at the points marked S.

Fig. 187 shows the state of affairs, $\frac{1}{8}$ of a cycle later, when the current in the B conductors has increased and the current in the A conductors has decreased to the same value. The points N and S have moved over $\frac{1}{16}$ of the circumference of the stator ring.

Fig. 188 shows the state of affairs, after another eighth of a cycle, when the current in the B conductors has reached its maximum value and the current in the A conductors has dropped to zero. The points N and S have moved again over $\frac{1}{16}$ of the circumference of the stator ring.

This motion of the points N and S is continuous, and these points make one complete revolution (in a four-pole motor) during two complete revolutions of the vectors A' and B' or while the alternating currents supplied to the stator windings are passing through two cycles. In general

$$n = \frac{f}{p}$$

in which n is the revolutions per second of the stator-magnetism, p is the number of pairs of poles N and S, and f is the frequency of the alternating currents supplied.

Three-phase stator winding.—When an induction motor is driven by three-phase currents, the stator conductors are arranged in three distinct circuits A, B and C, which are either Δ- or Y-connected to the supply mains. Fig. 189 shows the complete connections, for four poles, of the A circuit with its terminals tt'. The B and C circuits are similarly connected.

In general, the n-phase stator winding for $2p$ poles has $2pn$ equidistant bands of conductors. The 1st, $(n + 1)$th, $(2n + 1)$th, etc., bands are connected in one circuit, so that currents flow oppositely in adjacent bands, and this circuit takes current from one phase of the n-phase system. The 2d, $(n + 2)$th, $(2n + 2)$th, etc., bands are similarly connected in another circuit and take

current from the second phase of the n-phase system. The 3d, $(n+3)$th, $(2n+3)$th, etc., bands are similarly connected in

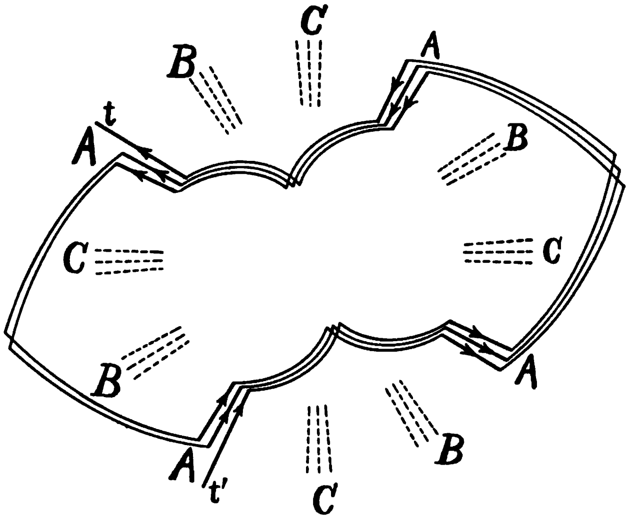


Fig. 189.

another circuit and take current from the third phase of the n-phase system; and so on.

151. The production of a rotating magnetic field by means of two-phase currents without an iron core.—A coil AA, Fig. 190, with its center at p and its plane at right angles to a, is placed inside of a larger coil BB, the center of this coil being likewise at p and its plane at right angles to b.

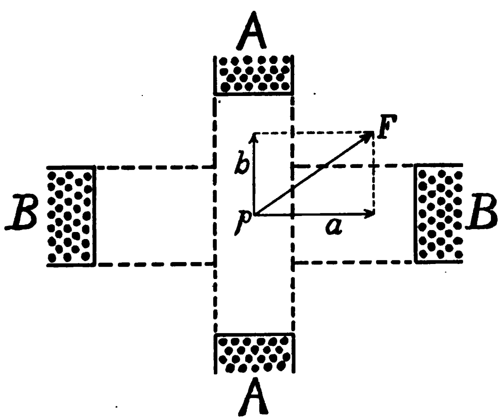


Fig. 190.

Let

$$i' = A \sin \omega t$$

be the alternating current flowing in coil AA, and

$$i'' = B \cos \omega t$$

be the current flowing in coil BB. The magnetic field, a, at p due to coil AA is parallel to the line a and proportional to i'. We may, therefore, write

$$a = F \sin \omega t$$

where F is a constant.

The magnetic field b at p due to coil BB is parallel to the line b and proportional to i'', so that if the number of turns of wire in coil BB is properly chosen we may write

$$b = F \cos \omega t$$

The resultant field at p (equal to $\sqrt{a^2 + b^2}$) is constant in magnitude and it rotates at an angular velocity ω; a is its x-component and b is its y-component.

152. Preliminary discussion of the action of the induction motor.—The complete theoretical discussion of the action of the induction motor is given later and is in many respects similar to the theory of the transformer. Many important details of the action of the induction motor, however, are most easily explained by looking upon the induction motor as *a rotor influenced by a rotating field magnet.*

Torque and speed.—Let n be the revolutions per second of the field and n' the revolutions per second of the rotor. When $n = n'$ the rotor and field turn at the same speed, so that their relative motion is zero; no electromotive force is then induced in the rotor conductors and no current, and therefore the rotating field exerts no torque upon the rotor. As the speed of the rotor decreases the relative speed of rotor and field increases, and therefore the electromotive force induced in the rotor conductors, the currents in the conductors, and the torque with which the field drags the rotor, all increase. If the whole of the field flux were to pass into the rotor and out again in spite of the demagnetizing action of the currents in the rotor conductors, then the torque would increase in strict proportion to $n - n'$, but in fact a larger and larger portion of the field flux passes through the space between stator and rotor conductors as the speed of the rotor decreases and this magnetic leakage causes the torque to increase

more and more slowly as $n - n'$ increases, in some cases * reaching a maximum value and then decreasing with further increase of $n - n'$.

Fig. 191 shows the typical relation between torque and speed of an induction motor. Ordinates of the curve represent torque

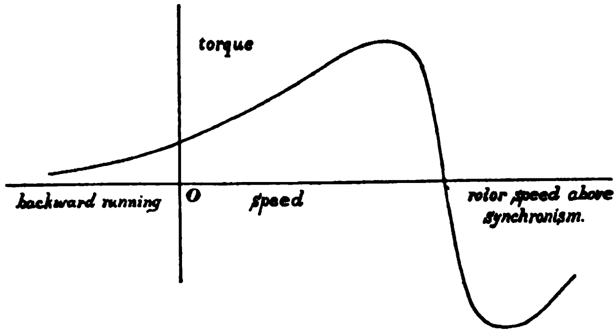


Fig. 191.

and abscissas measured from O represent rotor speeds. The rotor is said to run above synchronism when it is driven so that $n' > n$.

Discussion of torque-speed curve.—The torque acting on the rotor is proportional to the product of the rotor current i into the flux Φ', which passes into the rotor from the rotating field magnet. Therefore we may write

$$T = k\Phi' i \tag{i}$$

The field flux Φ of the induction motor is fixed in value or constant, and the difference $\Phi - \Phi'$ is proportional to i. Therefore we may write $(\Phi - \Phi') = k'i$, or

$$\Phi' = \Phi - k'i \tag{ii}$$

The rotor current i is proportional to $(n - n')$ and to Φ'. Therefore we may write

$$i = k''(n - n')\,\Phi' \tag{iii}$$

* In every case, if one makes $n - n'$ large enough by driving the rotor backwards so that n' becomes negative.

Eliminating Φ' and i from equation (i) with the help of equations (ii) and (iii) and reducing, we have

$$T = \frac{a(n - n')}{1 + b(n - n')^2} \tag{111}$$

in which a and b are constants depending upon Φ, k, k' and k''. This equation is plotted in Fig. 191, in which the abscissas represent speed of rotor n'. The curve crosses the x-axis, for $n' = n$.

Use of starting resistance in the rotor windings.—The speed of rotor for which the maximum torque occurs depends upon the resistance of the rotor windings, and it is advantageous to provide at starting such resistance in these windings as to produce the maximum torque at once, this resistance being cut out as the motor approaches full speed.

Efficiency and speed.—Let T be the torque with which the rotating field drags on the rotor; then $2\pi n'T$ is the power taken up by the rotor to be given off its belt pulley. Also T is the reacting torque which opposes the rotation of the field, so that, ignoring friction, $2\pi nT$ is the power required to maintain the rotation of the field. Therefore, ignoring friction losses, $2\pi nT$ is the power intake and $2\pi n'T$ is the power output of the motor, so that

$$\eta = \frac{2\pi n'T}{2\pi nT} = \frac{n'}{n} \tag{112}$$

is the efficiency of the machine. This equation shows that the efficiency of an induction motor is zero when the rotor stands still, that it increases as the rotor speeds up and approaches 100% (ignoring field losses and friction) as the rotor speed approaches the field speed. The ratio $\frac{n'}{n}$ ranges from .85 to .95 or more in commercial induction motors under full load.

Efficiency and rotor resistance.—For a given difference $n - n'$ between field speed and rotor speed, given electromotive force is induced in the rotor conductors, and the less the rotor resistance

the greater the current produced by this electromotive force and the greater the torque. Therefore a given induction motor will develop its full load torque for a small value of $n-n'$ or for a large value of $\frac{n'}{n}$ (efficiency) if its rotor resistance is small. High efficiency depends, therefore, upon low rotor resistance.

153. The induction generator.—When the rotor of an induction motor is driven above synchronism ($n' > n$), by an engine for example, the torque is reversed and opposes the motion of the rotor so that $2\pi n'T$ is input and $2\pi nT$ is output. That is, the machine takes power from the engine to drive its rotor and gives out power from its stator windings. This output of power is in the form of polyphase currents the frequency of which is fixed by the frequency of the alternator (or synchronous motor) which is connected to the stator windings.

154. The driving of induction motors by single-phase alternating current.—This is accomplished by connecting the two stator circuits A and B (case of two-phase motor) in parallel to the single-phase supply mains at the same time connecting in series with A a resistance R (see Fig. 192). The currents in the circuits A and B then differ in phase on account of the dissimilarity of the circuits, and the motor starts. When the motor is well under way one of the windings A or B may be *open-circuited*, the other, only, being left connected to the mains. A two-phase, or even a three-phase motor operates fairly well under these conditions, except that excessive current is required at starting to give a good starting torque. The resistance R may be replaced with advantage by a condenser, especially in case of a small motor.

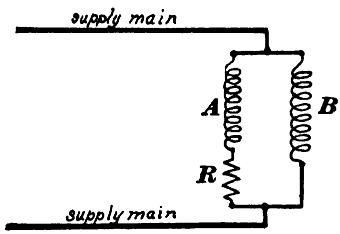


Fig. 192.

155. The action of the polyphase alternator as an induction motor when being started as a synchronous motor.—The winding

of the polyphase armature is identical to the stator or primary winding of an induction motor. When, at starting, the armature is connected to the polyphase supply mains a rotating state of magnetism is set up in the armature core. This rotating magnetism exerts a dragging torque on the field magnet, especially if the field coils are short-circuited, and the reacting torque of the field upon the armature sets the latter rotating in a direction opposite to the direction of its rotating magnetism.

156. The induction wattmeter is essentially a two-phase induction motor, of which the driving torque is proportional to the watts delivered to the receiving circuit. The armature disk is large and one edge of it moves between the poles of a permanent steel magnet (not shown in the figure) which causes the speed to be proportional to the driving torque as in a Thompson wattmeter.

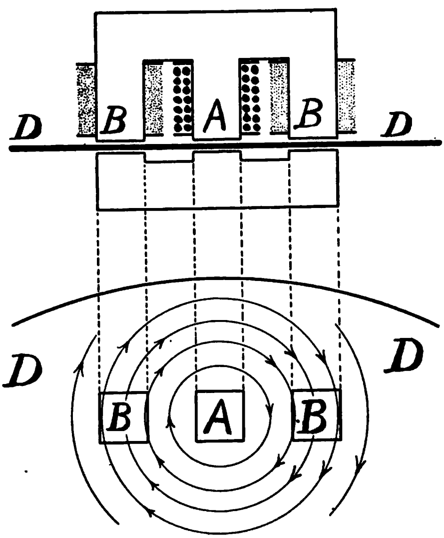


Fig. 193.

The total current delivered to the receiving circuit passes through the coil of coarse wire on the lug A of the laminated iron core, Fig. 193. The lugs BB are wound with fine wire which is connected across the mains.

The alternating current I in coil A produces a magnetic flux in lug A, which is proportional to and in phase with I. This flux induces electromotive force in the disk DD, which is 90° ahead of the flux in phase, and this electromotive force produces current in the disk, which is in phase with it. Therefore the eddy current in the disk is proportional to and 90° ahead of I. This

eddy current flows along the circular lines shown in the figure, passing under the ends of the lugs BB.

The electromotive force E between the mains produces in coil BB a current which is proportional to and nearly 90° behind E. This current in its turn produces through B and B a flux which is 90° behind and proportional to E. The phase difference between this flux and the eddy current in the disk is equal to the phase difference θ between E and I.

The eddy current in passing under the lugs BB is pushed sideways by the flux from BB with a force the average value of which is proportional to the product of *maximum eddy current* × *maximum flux* × *cosine of phase difference between the two*, which, according to the statements above, is proportional to $EI \cos \theta$, or to the power delivered to the receiving circuit. The speed of the disk is therefore proportional to power delivered and the total revolutions of the disk in a given time is proportional to the total work delivered.

General Theory of the Induction Motor.

157. The general alternating-current transformer.—The general theory of the induction motor is best developed by considering at once the most general type of machine, a multipolar multiphase motor, of which the rotor is wound in precisely the same way as the stator, the rotor windings being connected to collecting rings, so that the currents induced in the motor windings may be available for outside purposes. Such a machine we will call the general alternating-current transformer. Thus, a $2p$-pole, q-phase machine would have its stator conductors arranged in q distinct circuits, each taking current from one phase of a q-phase system; furthermore, each circuit would include $2p$ equidistant groups of conductors so connected that a current in that circuit would flow in opposite directions in adjacent groups. The rotor conductors would be similarly arranged in q* distinct cir-

* Stator and rotor are not necessarily wound for the same number of phases, but the discussion is simplified by such an arrangement.

cuits, each connected to a pair of collecting rings and supplying current to an outside receiving circuit.

Of course, no such induction motors * are ever actually built, but it is important to have clearly in mind the details of the machine to which the following discussion applies.

Fig. 194 shows a little more than one-sixth part of the circumference of a six-pole, three-phase machine. The three groups of stator conductors, A, B and C, belong one to each of the three circuits formed by the stator windings, and the three groups of rotor conductors, A', B' and C', belong one to each of the three circuits formed by the rotor windings.

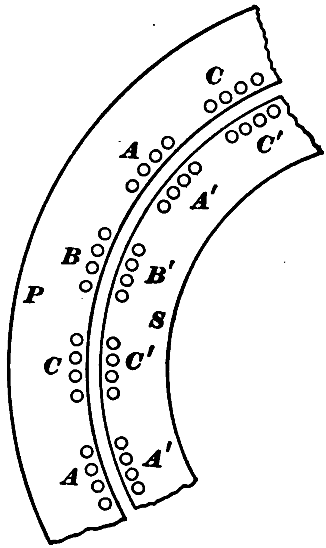


Fig. 194.

When the rotor of such a machine is stationary the machine acts simply as a transformer taking q-phase currents from the supply mains into its stator windings and giving out q-phase currents of the same frequency from its rotor windings.

Remark. — The immediately following discussion is based upon the ideal induction motor, of which the primary and secondary windings have no resistance; the magnetic flux through the stator windings all passes into the rotor, and the magnetizing current is negligible. Throughout the discussion the stator and rotor are supposed to

* Steinmetz has proposed the use of such induction motors for street railway work. Two similar motors are used on each car, one geared to each axle. At starting and for slow running, motor No. 1 takes currents into its stator windings from polyphase trolley wires, and supplies polyphase currents from its rotor windings to the stator of motor No. 2, the starting resistance being connected in the rotor circuits of motor No. 2. With such an arrangement the limit of speed is one-half of synchronous speed ($n' = \frac{1}{2}n$), and the efficiency at given speed is doubled. For fast running both motors take current directly from the trolley wires.

be wound with the same number of conductors and for the same number of phases.

158. Rotor electromotive forces referred to stator.—The electromotive forces induced in the rotor conductors, and also the electromotive forces induced in the stator conductors, may be ascribed to the *rotating stator magnetism.* Let n be the speed of the stator magnetism, and n' the speed of the rotor. Then the speed of the stator magnetism referred to stator conductors is n, and the speed of the stator magnetism referred to rotor conductors is $n - n'$. Consider (a) the varying electromotive force which is induced in a given stator conductor, and (b) the instantaneous values of electromotive force induced in the various rotor conductors as they pass the given stator conductor. These two electromotive forces are at each instant in the ratio of n to $n - n'$, because they are produced by the same lines of force sweeping past the stator conductor at speed n and sweeping past the successive rotor conductors at speed $n - n'$. Therefore the two electromotive forces (a) and (b) above are of the same frequency, their ratio is n to $n - n'$, and they are in phase with each other. Of course the electromotive force induced in one of the circuits of the stator winding is equal and opposite to the impressed electromotive force E' which acts on that circuit.

Rotor electromotive forces referred to rotor.—When the rotor is at a standstill the relative speed of rotor and stator-magnetism is n, and the electromotive force induced in a *given rotor conductor* is equal to the electromotive force E induced in a stator conductor and of the same frequency f. When the rotor runs at speed n' the relative speed of rotor and stator magnetism drops to $n - n'$, and the electromotive force induced in a *given rotor conductor* is decreased in the ratio n to $n - n'$ both in value and in frequency, its value becoming sE and its frequency sf where

$$s = \frac{n - n'}{n} \tag{113}$$

This quantity, s, is much used in the theory of the induction motor, and is called the *slip*.

159. Rotor currents referred to stator.—When the secondary or rotor circuits are open, the primary or stator current m in each primary circuit is called the magnetizing current. Let I'' be the current in each rotor circuit when the rotor circuits are closed, and let I' be the additional current, over and above m, which flows in each stator circuit. Then, as in the case of the transformer, *the magnetizing action of I'' is balanced at each instant by the magnetizing action of I'.* In the following discussion the currents m are not considered.

Consider (a) the current in a given stator conductor and (b) the instantaneous values of current in the various rotor conductors as they pass the given stator conductor. These two currents are at each instant equal and opposite, otherwise the magnetizing action of the one could not be at each instant balanced by the magnetizing action of the other. Therefore the two currents (a) and (b) above are of the same frequency, they are equal to each other, and they are opposite to each other in phase.

Rotor currents referred to rotor.—When the rotor is at standstill, the current in a *given rotor conductor* has the same value I and the same frequency f as the current in a stator conductor. When the rotor speed is n' the current in the given rotor conductor remains equal in value to the current in a stator conductor, but it drops in frequency in the ratio of $(n - n') \div n$, that is, its frequency becomes sf.

Remark.—When one is studying the mutual action of stator and rotor it is convenient to refer rotor electromotive forces and rotor currents to the stator.

160. The vector diagram of the induction motor.—Let E_1, Fig. 195, represent the electromotive force acting upon one circuit of the stator winding. The secondary electromotive force E_2* per

* The precise meaning of E_2 is most clearly stated when there are $2pq$ conductors on stator and rotor, one for each band of actual conductors, so that $2p$ conductors con-

circuit is equal to sE_1 and opposite to E_1 in phase. This electromotive force E_2 has a frequency sf referred to the outside receiving circuit to which it supplies current. Let r be the resistance of this circuit, and let L be its inductance, or $sx\,(= 2\pi sfL)$ its reactance. Then I_2 is equal to $E_2 \div (r + j \cdot sx)$ and $\tan\theta = sx/r$. The current I_1 in the stator circuit is equal and opposite to I_2.

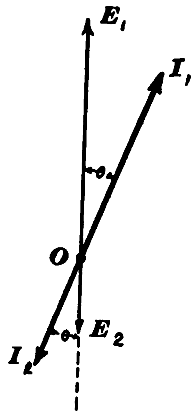


Fig. 195.

The input of power per phase, that is, the power delivered to each stator circuit, is $E_1I_1 \cos\theta$; and the electrical output of power per phase, that is, the output from each rotor circuit, is $E_2I_2 \cos\theta$ or $sE_1I_1 \cos\theta$, since $E_2 = sE_1$ and $I_1 = I_2$. The difference between electrical input and electrical output is mechanical output, or power delivered by belt. This is $(1 - s)E_1I_1 \cos\theta$, or $(1 - s)/s$ times the electrical output.

Example.—A four-pole, three-phase machine takes current at 220 volts and 60 cycles per second. When the rotor is stationary it gives out three-phase currents at full frequency, the amount of output depending, of course, upon the resistance and reactance of the receiving circuits. The speed of the stator magnetism is 30 revolutions per second. When rotor speed is 20 revolutions s $[= (n - n')/n]$ is equal to $\frac{1}{3}$, so that the rotor delivers three-phase currents at $73\frac{1}{3}$ volts and at 20 cycles per second, and of the total power delivered to the machine $\frac{1}{3}$ is electrical output and $\frac{2}{3}$ is mechanical output.

161. The actual induction motor.—The behavior of the actual induction motor deviates from the above described ideal action, because of the resistance of the stator windings, because of eddy current and hysteresis losses in the iron and because of magnetic

stitute one circuit. Then E_1 is the electromotive force in a given stator circuit and E_2 is the electromotive force in the successive rotor circuits as they pass by the given stator circuit. E_2 is opposite to E_1, equal to sE_1, and referred to the stator its frequency is the same as the frequency of E_1.

leakage. The effect of each of these things is rather small,* therefore their mutual influences may be neglected. The effect of each will consequently be considered by itself. The effects will first be discussed in a general way with the help of the vector diagram, after which the general complex equations of the induction motor will be established and Steinmetz's solution outlined. The discussion will be directed mainly to the mechanical behavior of the machine, that is, to the relation between torque and speed.

The effect of magnetic leakage is equivalent to an outside inductance connected in series with the stator circuit. The rotor windings may be considered as non-inductive.

162. Effect of magnetic reluctance, eddy currents and hysteresis upon the action of an induction motor.—The ideal induction motor takes no current from the mains into its stator windings when the motor is running at synchronous speed ($f' = f$). The actual induction motor running in synchronism takes sufficient current to overcome the magnetic reluctance of the iron (stator and rotor); and an amount of power equal to the hysteresis and eddy current loss in the stator iron only, inasmuch as the magnetic state of the rotor is constant, since it rotates with the stator magnetism. When the actual induction motor is running at any given speed it takes from the mains the above current and power in excess of what an ideal motor would take at same speed. Further, there is eddy current and hysteresis loss in the rotor iron when it runs below synchronism and the effect of this loss is to slightly increase the torque.

163. Effect of stator resistance upon the action of an induction motor.—When the rotor is running nearly in synchronism with the stator magnetism the currents in the stator windings are very small and no perceptible portion of the supply electromotive force is needed to overcome the stator resistance. As the rotor is slowed up the stator currents increase and a larger and larger portion of the supply electromotive force is needed to overcome

* Not, however, so small as in the simple transformer.

the stator resistance. The result is that the core flux falls off* slightly, and also the torque acting upon the rotor falls short of its ideal value, inasmuch as this torque depends upon both flux and rotor currents.

164. Effect of magnetic leakage upon the action of an induction motor.—As in case of the simple transformer the effect of magnetic leakage is the same as the effect of an outside inductance connected in series with the primary (stator) windings, a separate inductance for each stator circuit. Let P be the value of each inductance and let x ($= \omega P$) be its reactance value.

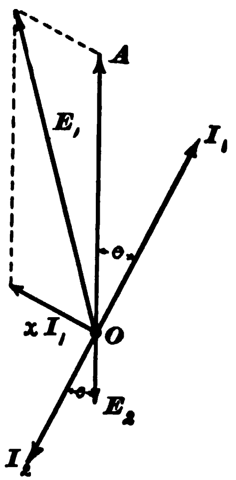


Fig. 196.

The diagram of Fig. 196 represents the action of an induction motor, in so far as it is affected by magnetic leakage; A and E_2 ($= sA$) are the electromotive forces induced in stator and rotor windings respectively by the magnetic flux which passes through both. The current I_2 is determined by the resistance and reactance† of the secondary circuit. The primary current I_1 is equal and opposite to I_2. The line xI_1, at right angles to I_1, represents that part of the total primary electromotive force which is used to overcome the leakage inductance P or to balance the electromotive force induced in each primary circuit by the leakage flux.

165. Calculation of leakage reactance.—The leakage reactance x per circuit is equal to $\omega P (= 2\pi f P)$ where P is the leakage inductance per circuit. This leakage inductance is calculated, as in case of the simple transformer, by equation (75), namely

* Inasmuch as the portion of the supply electromotive force which is balanced by the induced electromotive force in the stator windings is decreased, and, therefore the harmonically varying flux which induces this electromotive force must decrease exactly as in the simple transformer.

† Under practical conditions the rotor circuits are non-inductive and the angle θ, Fig. 196, is zero, as in case of the simple transformer feeding a non-inductive receiving circuit.

$$P = \frac{4\pi N'^2 \lambda}{l}\left(\frac{X}{3} + \frac{Y}{3} + g\right) \qquad (75) \text{ bis}$$

This equation gives P in centimeters, all dimensions being in centimeters. In this equation λ is the length, parallel to the shaft, of the rotor or stator; l is the sum of the widths of all the slots in which the windings of one stator circuit are wound; X is the depth of the stator slots; Y is the depth of the rotor slots, and g is the clearance space between stator and rotor.[1] This equation assumes that stator and rotor slots are of the same width, that they are wound full of wire, and that the permeability of the iron lugs between the slots is very great, so that reluctance of iron is negligible.

166. Formulation of the complex equations of the induction motor.—The following discussion is taken from Steinmetz * with the same changes as are mentioned in Article 120. Stator and rotor are supposed to be wound for the same number of phases.

Let N' = number of stator conductors per phase.

N'' = number of rotor conductors per phase.

$a = \frac{N'}{N''}$

r_1 = resistance of stator per circuit.

r_2 = resistance of rotor per circuit, including outside resistance (non-inductive).

x = reactance value ωP of primary leakage inductance per phase.

$Z = r_1 + jx$.

E' = primary impressed electromotive force per phase.

A = that part of E' which is used to balance the electromotive force induced in each stator circuit by the magnetic flux which passes through the rotor.

B = secondary induced electromotive force per phase.

I' = total primary current per circuit.

M = magnetizing current per circuit.

I'' = secondary current per circuit.

* "Alternating Current Phenomena," third edition, p. 221.

$Y_1 = g_1 - jb_1 = \frac{M}{A}$ = admittance of motor per phase at zero load, that is, at synchronous speed.

$s = \frac{n - n'}{n}$, where n is the speed of stator magnetism and n' is speed of rotor.

Remark.—In calculating g_1 and b_1 use equations (71) and (72), in which W_e and W_h are eddy current loss and hysteresis loss in stator iron only, and G is magnetic reluctance of stator iron, air gap, and rotor iron. For calculations near standstill it would be more accurate to use for W_e and W_h the losses in both stator and rotor.

The ratio of A to B is a when the rotor stands still, and a/s when the rotor speed is n'. Further, A and B are opposite in phase, so that

$$A = -\frac{aB}{s} \tag{i}$$

The secondary current is

$$I'' = \frac{B}{r_2} \tag{ii}$$

The part of I' which corresponds to I'' is equal to $-I''/a$ or $-\frac{B}{ar_2}$, which, added to $M(=Y_1A)$ gives the total primary current. That is,

$$I' = Y_1A - \frac{B}{ar_2} \tag{iii}$$

The electromotive force used to overcome the impedance of the stator windings per circuit is ZI', which, added to A, gives E'. That is,

$$E' = A + ZI' \tag{iv}$$

With the help of equation (i), I' and E' may be expressed in terms of B, giving

$$I'' = \frac{B}{r_2} \tag{114}$$

$$I' = -B\left(\frac{1}{ar_2} + \frac{aY_1}{s}\right) \tag{115}$$

$$E' = -B\left(\frac{a}{s} + \frac{Z}{ar_2} + \frac{aY_1Z}{s}\right) \tag{116}$$

It is desirable to express I' and I'' in terms of E' as follows:

$$I'' = -\frac{E'}{r_2\left(\frac{a}{s} + \frac{Z}{ar_2} + \frac{aY_1Z}{s}\right)} \tag{117}$$

$$I' = +\frac{\left(\frac{1}{ar_2} + \frac{aY}{s}\right)E'}{\frac{a}{s} + \frac{Z}{ar_2} + \frac{aY_1Z}{s}} \tag{118}$$

These complex equations are most easily handled by taking E' as the reference axis. Then E' becomes a simple quantity, and the components of I' and I'' are easily found by separating the real and imaginary parts of (117) and (118) respectively.

When the components of I'' are thus found, the product of r_2 into the sum of the squares of the components of I'' gives the electrical output of the rotor per phase, and the product of this electrical output per phase by $\frac{1-s}{s}$, as stated in Article 159, gives the mechanical output per phase.

It is convenient to express torque in terms of the power which the torque would develop at synchronous speed n. This is sometimes called "synchronous watts" for brevity. It exceeds the actual mechanical output in the ratio of n to n'. But n/n' is equal to $\frac{1}{1-s}$, therefore mechanical output multiplied by $\frac{1}{1-s}$, gives torque expressed in synchronous watts.

Proceeding as above we find

$$\text{Component of } I'' \text{ parallel to } E' = -\frac{asuE'}{u^2+v^2}$$

$$\text{Component of } I'' \text{ perpendicular to } E' = +\frac{asvE'}{u^2+v^2}$$

$$\text{Numerical value of } I'' = \sqrt{\frac{a^2s^2E'^2}{u^2+v^2}}$$

in which

$$u = a^2 r_2 + s r_2 + a^2 r_2 (g_1 r_1 + b_1 x)$$
$$v = s x + a^2 r_2 (g_1 x - b_1 r_1)$$

Therefore the electrical output per phase is $\dfrac{a^2 s^2 r_2 E'^2}{u^2 + v^2}$, which multiplied by $\dfrac{1-s}{s}$ gives the mechanical output per phase, and this multiplied by the number of phases q gives the total mechanical output P, for which the expression is

$$P = \frac{s(1-s)cE'^2}{d + hs + ks^2} \tag{119}$$

in which c is written for $a^2 r_2 q$

d is written for

$$a^4 r_2^2 (1 + 2g_1 r_1 + 2b_1 x + g_1^2 r_1^2 + b_1^2 x^2 + g_1^2 x^2 + b_1^2 r_1^2)$$

h is written for $2a^2 r_2 (r_1 + g_1 r_1^2 + g_1 x^2)$

and

k is written for $r_1^2 + x^2$

Multiplying P by $\dfrac{1}{1-s}$ gives the torque T in terms of synchronous watts. Therefore

$$T = \frac{scE'^2}{d + hs + ks^2} \tag{120}$$

The ordinates of the curves in Fig. 197* represent the values of P and of T (in kilogram-meters) and the abscissas represent both slip $s \left(= \dfrac{n - n'}{n} \right)$ and speed in per cent. of synchronism $\left(\dfrac{n'}{n} \right)$. The curves are calculated for a three-phase, eight-pole, 20-horse-power (rated) induction motor Y-connected to 110-volt 60-cycle mains, so that $E' = 63.6$ volts. The rotor winding has been reduced to an equivalent winding for which $a = 1$:

$$g_1 = 0.1 \, \frac{\text{ampere}}{\text{volts}}$$

* Taken from Steinmetz, "Alternating Current Phenomena." Calculations verified by P. L. Anderson and L. A. Freudenberger.

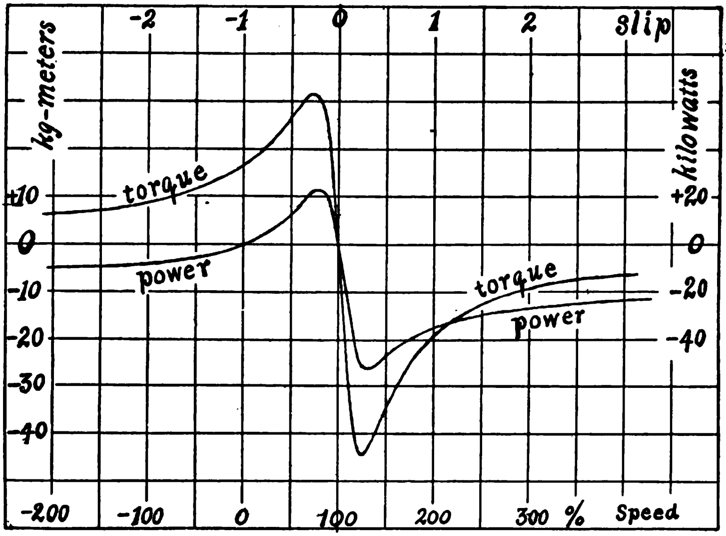


Fig. 197.

$$b_1 = 0.4 \frac{\text{ampere}}{\text{volts}}$$

$$r_1 = 0.03 \text{ ohm}$$

$$x = 0.175 \text{ ohm}$$

$$r_2 = 0.045 \text{ ohm}$$

The ordinates of the curves in Fig. 198* represent speed, efficiency, power factor, torque and primary current for various values of power output. The curves are all tangent to the vertical line of which the abscissa represents the maximum power output. These curves are calculated for a three-phase, eight-pole induction motor of about 6-horse-power rating, Δ-connected to 110-volt mains so that E' is equal to 110 volts.

$$g_1 = 0.01 \frac{\text{ampere}}{\text{volts}}$$

* Taken from Steinmetz, "Alternating Current Phenomena." Calculatious verified by P. L Anderson and L. A. Freudenberger.

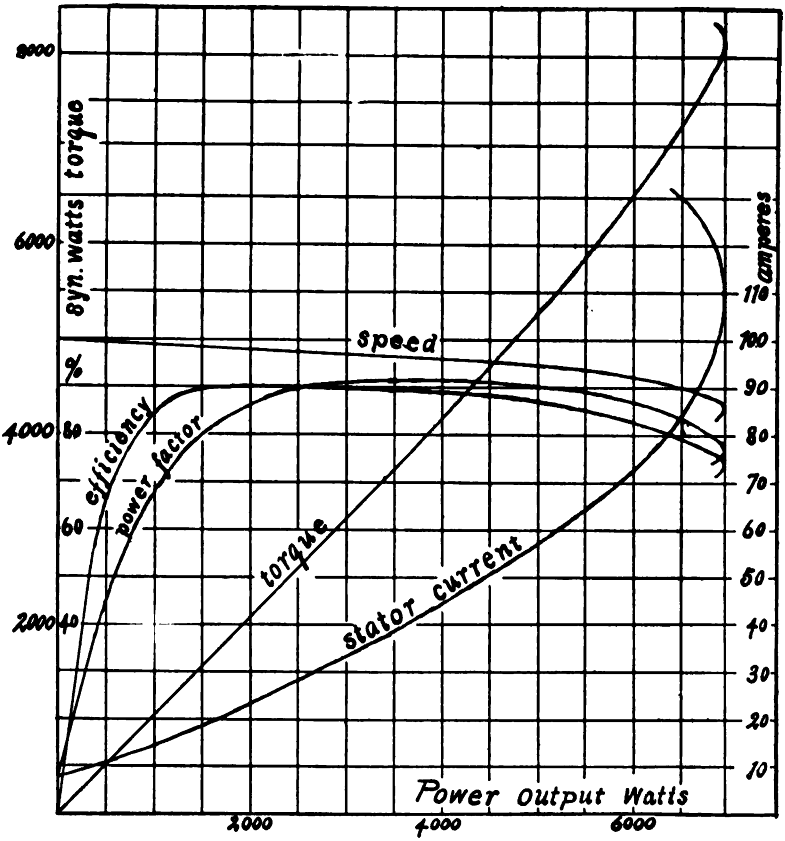


Fig. 198.

$$b_1 = 0.1 \frac{\text{ampere}}{\text{volts}}$$

$$r_1 = 0.1 \text{ ohm}$$

$$x = 0.6 \text{ ohm}$$

$$r_2 = 0.1 \text{ ohm}$$

A mere indication of the steps by which the curves of Fig. 198 were calculated must suffice. Choose a series of values of s. For each of these calculate mechanical power, speed and torque,

from equations (119) and (120). Separate the components of I' referred to E', equation (118), and calculate the values of these components for each of the chosen values of s. From these components the numerical value of I', the power factor, the power intake and the efficiency of the motor, may be readily calculated. The power factor is the cosine of the angle between E' and I' and the tangent of this angle is equal to the ratio of the two components of I'.

167. Maximum torque.—In the following discussion the magnetizing current is ignored for the sake of simplicity, so that $g_1 = b_1 = 0$. Also the stator and rotor conductors are supposed to be equal in number, so that $a = 1$. In this case the expression for torque, equation (120), becomes

$$T = \frac{q r_2 s E'^2}{r_2^2 + 2 r_1 r_2 s + (r_1^2 + x^2) s^2} \tag{121}$$

The value of slip s which gives maximum T is found by the condition $\frac{dT}{ds} = 0$, which gives as the value of s for maximum torque

$$s = \pm \frac{r_2}{\sqrt{r_1^2 + x^2}} \tag{122}$$

Substituting this value of s in the expression for T, we have

$$T_{\max} = \pm \frac{q E'^2}{2(r_1 \pm \sqrt{r_1^2 + x^2})} \tag{123}$$

The larger value of T is the negative value and it occurs when rotor speed is above synchronism, as shown in Fig. 191.

168. Starting torque.—At starting $s = 1$, which, substituted in equation (121), gives the value of the starting torque T_0

$$T_0 = \frac{q r_2 E'^2}{r_2^2 + 2 r_1 r_2 + r_1^2 + x^2} \tag{124}$$

Now the value of maximum torque is, according to equation (123), independent of the value of r_2, but variations of the value

of r_2 make this maximum torque occur for different values of s, according to equation (122). Therefore maximum torque occurs at starting when the value of r_2 is such as to give $s = 1$ in equation (122).

Therefore to give maximum torque at starting we must have

$$r_2 = \sqrt{r_1^2 + x^2}$$

or rotor resistance per circuit must be equal to primary impedance per circuit. The value of maximum torque at starting is

$$T_{\max} = + \frac{qE'^2}{2(r_1 + \sqrt{r_1^2 + x^2})}$$

169. The graphical solution of the transformer and of the induction motor.—Consider a transformer of which the secondary delivers current to a non-inductive receiving circuit of variable resistance R''. The primary current is that which would be delivered to a circuit of inductance P and of which the resistance is $R' + (N'/N'')^2R''$, according to Article 98, where P is the

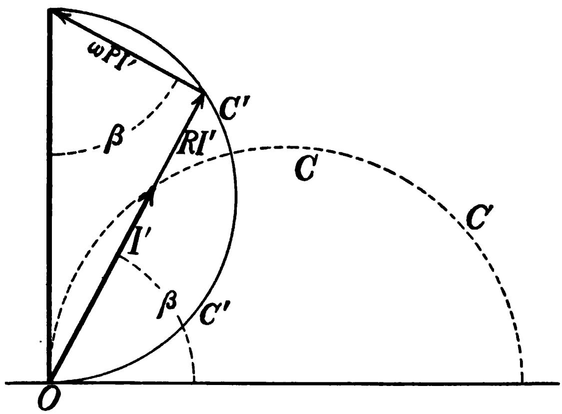


Fig. 199.

primary inductance equivalent of magnetic leakage. The two electromotive force components RI' and $\omega PI'$ are at right angles to each other, and therefore the termini of RI' and of $\omega PI'$ trace the circle $C'C'$. But ωP is constant. so that the vector I'

is proportional to the vector $\omega PI'$, and the angles β, β are equal. Therefore the terminus of I' traces the circle CC as the secondary resistance varies. The current here considered is the primary current over and above the magnetizing current M. In case the magnetizing current is considered the locus of I' is as shown in Fig. 200, in which A is the part of the primary current above considered, M is the magnetizing current, I' is the total primary current, and E' is the primary impressed electromotive force. The part A of the primary current is, for a 1 : 1 transformer, equal and opposite to the secondary current I'', as indicated.

The detailed electromagnetic action of the induction motor running at a slip s with a fixed value r_2 of secondary resistance is fully represented, when the motor is stopped, by increasing its secondary resistance to r_2/s. For, when the motor is stopped, the electromotive force in the secondary becomes $1/s$ times as great, so that secondary current, primary current, and, in fact, every detail of electromagnetic behavior remains unchanged, except that there is a negligible increase of eddy current and hysteresis loss in the

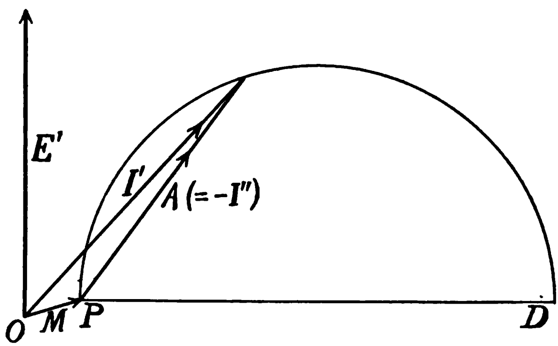


Fig. 200.

rotor. In consequence of this fact, the behavior of the induction motor may be studied by means of the diagram, Fig. 200.

Let us consider the application of this diagram to an induction motor, of which the primary and secondary turns are equal. We will use the same notation as in Article 166. Draw a line representing E'. Lay off the line representing $M\,[= E'(g_1 - jb_1)]$.

The terminus of M is one point on our circle. From the terminus of M draw the line PD at right angles to E'. At standstill the primary equivalent resistance of the motor is $r_1 + r_2$, and the corresponding value of primary current is

$$I' = \frac{E}{r_1 + r_2 + jx}$$

Knowing r_1, r_2 and x, this current, together with M, may be laid off, giving another point on the circle. The circle is thus determined, since its center must lie on the line PD.

To calculate a set of corresponding values of I', I'', T, s, power intake, power output and power factor, proceed as follows. Choose a point on the circle and scale off the values of I' and I''. Also scale off the vertical component of I'. This component of I' multiplied by E' gives total power delivered to the motor. From this subtract core losses (represented by the current M) and resistance loss I'^2r_1. This gives power delivered to the rotor. This power delivered to the rotor is all lost in the resistance r_2/s. It is therefore equal to I''^2r_2/s, from which s may be calculated. Of the total power delivered to the rotor, viz., I''^2r_2/s, the fractional part s or I''^2r_2 is lost in rotor resistance and the fractional part $1 - s$ or $I''^2r_2(1 - s)/s$ is mechanical output.

Remark.—In this, as in all previous discussions of the transformer and the induction motor, the mutually disturbing influences of coil resistances, magnetizing current and magnetic leakage are ignored, that is, the effect of each of these things is considered to be that which would exist if the transformer were ideal except for this one thing. The mutually disturbing influence of magnetizing current and magnetic leakage is partly allowed for by making the line I'', Fig. 200, represent the secondary current to a slightly greater scale (more amperes per inch) than that to which the line I' represents the primary current. In short, if I' represents primary current one ampere per inch, then I'' should represent secondary current v amperes per inch, where v is the number of amperes in a rotor circuit required

to balance in the stator the magnetizing action of one ampere in a stator circuit.

PROBLEMS.

116. The winding of an ordinary ring-wound direct-current armature has 24 sections. These sections are disconnected from the commutator and numbered in order from 1 to 24. Specify the manner in which these 24 coils are to be connected to two-phase mains to produce in the ring a rotating state of magnetism with four poles, with six poles, and with twelve poles. Specify the connections of the 24 coils to three-phase mains to produce in the ring a rotating state of magnetism with two poles, with four poles, and with eight poles. State the speed of the magnetism in each case in revolutions per second, the frequency of the polyphase currents being 60 cycles per second.

Sample answer for 4-pole 3-phase connections.—Phase A is connected as follows: Positive main to + (section 1) − to + (section 2) − to − (section 7) + to − (section 8) + to + (section 13) − to + (section 14) − to − (section 19) + to − (section 20) + to negative main.

Phase B is connected as follows: Positive main to + (section 3) − to + (section 4) − to − (section 9) + to − (section 10) + to + (section 15) − to + (section 16) − to − (section 21) + to − (section 22) + to negative main.

Phase C is connected as follows: Positive main to + (section 5) − to + (section 6) − to − (section 11) + to − (section 12) + to + (section 17) − to + (section 18) − to − (section 23) + to − (section 24) + to negative main.

Speed: Two-phase, four poles 30 revolutions per second.
six poles 20 revolutions per second.
twelve poles 10 revolutions per second.
Three-phase, two poles 60 revolutions per second.
four poles 30 revolutions per second.
eight poles 15 revolutions per second.

117. The armature of an ordinary direct current dynamo is

short circuited between the brushes when the resistance of the armature circuit is 0.036 ohm. The number of armature conductors is 260, and the flux through the armature is 1,500,000 lines, which is assumed to be invariable. The field magnet is set rotating about the shaft as an axis at a speed of 25 revolutions per second, carrying the direct-current brushes with it. Calculate the torque dragging upon the armature when its speed is 24 revolutions per second and when its speed is 23 revolutions per second. What portion of the power expended in driving the field is available at the armature belt, and what portion is lost in heating the armature, friction losses being ignored?

Suggestion.—This problem may be solved by use of the ordinary equations of the direct-current dynamo.

Ans. (*a*) Torque = 4.37 kg.-meters, 96 per cent. of the power is available at the armature belt and 4 per cent. is lost in heating the armature.

(*b*) Torque = 8.74 kg.-meters, 92 per cent. of the power is available at the armature belt and 8 per cent. is lost in heating the armature.

118. An ideal three-phase induction motor takes 5 amperes of current into each phase of its Δ-connected primary member at 200 volts and 60 cycles per second. The rotor, which has a three-phase winding, Δ-connected to collecting rings, supplies 25 amperes of current to each of three similar circuits, each having a power factor equal to 0.75. The rotor is running at ⅔ synchronous speed. What is the ratio of the stator to the rotor turns? What is the rotor terminal voltage? What is the total intake of power? What is the total electrical output of power? What is the mechanical output of power? Ans. (*a*) Ratio of turns is 5 : 1. (*b*) Rotor terminal voltage is $\frac{1}{3} \times \frac{1}{5} \times 200$ volts. (*c*) Total intake of power is 2,250 watts. (*d*) Total electrical output is 750 watts. (*e*) Total mechanical output is 1,500 watts.

119. An ideal induction motor has a three-phase stator winding, 56 conductors to each phase, and a two-phase rotor winding,

132 conductors to each phase. The machine, running at half synchronous speed, takes 25 amperes into each of its stator circuits at 220 volts. Required the electromotive force induced in each rotor circuit and the current flowing in each rotor circuit. Ans. 244.6 volts, 16.85 amperes.

Suggestion.—In the case of equi-phase stator and rotor windings the electromotive forces induced in each circuit of stator and rotor respectively are proportional to the number of stator and rotor conductors. When stator and rotor are wound for different numbers of phases this simple relation between primary and secondary electromotive forces does not hold. Thus a given intensity of rotating stator magnetism will induce in each circuit of a three-phase distributed stator winding only $\frac{955}{1000}$ as much electromotive force as would be induced in a concentrated winding having the same number of conductors; and in each circuit of a two-phase distributed rotor winding only $\frac{901}{1000}$ as much electromotive force as would be induced in a concentrated winding having the same number of conductors. (See Article 84.) Therefore the electromotive force acting on each circuit of a three-phase distributed stator winding is to the electromotive force induced in a two-phase distributed rotor winding as 955 is to 901, the number of conductors per circuit being the same in stator and rotor.

Furthermore, the effective magnetizing action of polyphase currents in a polyphase stator or rotor winding depends upon the number of phases as well as upon the number of conductors and strength of current. On account of this difference of effective magnetizing action, the stator and rotor currents must be different in order that their magnetizing actions may balance. This relationship between stator and rotor currents is, however, most easily arrived at by considering that the intake and output are equal, so that the current in each circuit of a two-phase distributed rotor winding is $\frac{3}{2} \times \frac{955}{901}$ as great as the current in each circuit of a three-phase stator winding, the number of conductors per circuit being the same in stator and rotor.

CHAPTER XV.

TRANSMISSION LINES.

170. Introductory.—Power may be transmitted by the pumping of water. If great pressure is used a given amount of power may be transmitted by a small flow of water through a small pipe. In every case, however, there is a loss of power on account of friction in the pipe. The smaller the pipe the greater this loss and the less the first cost; the best size of pipe is that for which neither the first cost nor the continuous loss of power by friction is excessive.

Similarly, a given amount of power may be transmitted by a small electric current through a small wire by using a large electrical pressure or electromotive force. In every case, however, there is a loss of power on account of the resistance of the wire. The smaller the wire the greater this loss and the less the first cost of the line; the best size of wire is that for which neither the first cost nor the continuous loss of power by resistance is excessive.

It is only by using very large electromotive forces that long distance transmission lines may be made at a reasonable cost, the loss due to resistance being at the same time reasonably small. The highest electromotive force that can be satisfactorily used upon a pole line exposed to the air is about 60,000 volts, inasmuch as the leakage from wire to wire (outgoing and returning wires) in the form of brush or spark discharge becomes excessive at higher electromotive forces, unless the wires are very large and very far apart. For transmission within a radius of two or three miles 1,000 and 2,000 volts are usually employed.

171. Power loss and electromotive force loss in line.—If, say, 10 per cent. of the power output of a direct-current dynamo is lost in the line, then 10 per cent. of the electromotive force of the dynamo is also lost in the line and 90 per cent. only is effective at the receiving circuit. With alternating current, however, the receiving circuit may receive, say, 90 per cent. of the power output of the dynamo, while the effective electromotive force at the receiving circuit may be more or less than 90 per cent. of the electromotive force of the dynamo. The difference (numerical) between dynamo electromotive force and the electromotive force at the receiver circuit is called the *line drop* and this line drop is of more practical importance than the power lost in the line, inasmuch as nearly all receiving apparatus needs to be supplied with current at approximately constant electromotive force. This is usually provided for by over-compounding the dynamo so as to keep the receiver electromotive force constant. Thus, if the line is designed to give 10 per cent. drop, the dynamo would be 10 per cent. over-compounded.

172. Line resistance.—The resistance of a wire for alternating currents may in all practical cases be taken to be the same as the resistance of the same wire for direct current. The fact is, however, that the alternating current near the axis of a wire lags in phase behind the current near the surface of the wire, and the resistance of the wire is therefore larger for an alternating current than for a direct current.*

173. Line reactance.—The reactance of a transmission line (outgoing and returning wires side by side) is greater the smaller the wires and the further they are apart, and is proportional to the length of the line and to the frequency. The accompanying table gives the resistance and reactance per half mile of transmission line.

* See Merritt, *Physical Review*, Vol. 5, p. 47.

RESISTANCE AND REACTANCE OF ONE MILE OF WIRE (½ MILE OF TRANSMISSION LINE) (EMMET).

Size of wire B. & S. gauge.	Resistance in ohms.	Reactance in Ohms.					
		At 60 cycles per sec.			At 125 cycles per sec.		
		Wires 12 inches apart.	Wires 18 inches apart.	Wires 24 inches apart.	Wires 12 inches apart.	Wires 18 inches apart.	Wires 24 inches apart.
0000	.259	.508	.557	.591	1.06	1.17	1.23
000	.324	.523	.573	.607	1.09	1.20	1.26
00	.412	.534	.588	.618	1.12	1.23	1.29
0	.519	.550	.603	.633	1.15	1.26	1.32
1	.655	.565	.614	.648	1.18	1.28	1.35
2	.826	.580	.629	.663	1.21	1.31	1.38
3	1.041	.591	.644	.674	1.24	1.34	1.41
4	1.313	.606	.656	.690	1.26	1.37	1.44
5	1.656	.620	.670	.704	1.30	1.40	1.47
6	2.088	.633	.685	.720	1.32	1.43	1.49
7	2.633	.647	.700	.730	1.35	1.46	1.52
8	3.320	.662	.712	.742	1.38	1.48	1.55
9	4.186	.677	.727	.761	1.41	1.51	1.58
10	5.280	.688	.742	.776	1.44	1.54	1.62

174. Line capacity.—The two wires of a transmission line constitute a condenser which is charged and discharged as the electromotive force between the wires alternates. The current which is delivered to the line by the generator when the receiving circuit is disconnected is called the *charging current* of the line. It is nearly 90° ahead of the generator electromotive force in phase.

The capacity in farads per mile of a two-wire line is approximately

$$C = \frac{4.52 \times 10^{-8}}{\log_\epsilon \left(\frac{d}{r}\right)} \qquad (125)$$

in which r is the radius of each wire and d is the distance of the wires apart from center to center.

The combined effect of line resistance, line reactance and line capacity, is quite complicated and its full discussion is beyond the scope of this text.

An instructive step-by-step graphical solution of the general problem involving also leakage from line to line is given by Steinmetz. (See "Alternating Current Phenomena," third edi-

tion, pages 47 to 51.) The complete algebraic solution of the problem is given by Steinmetz. (See "Alternating Current Phenomena," third edition, pages 163 to 192.)

When it is necessary for practical purposes to consider line capacity as well as line resistance and line reactance, the capacity effect of the line may be sufficiently well represented by condensers located at one or more points along the line. The problem is thus simplified, and can be solved with the help of the formulæ for series and parallel connections as outlined in Chapter VII. of this text. (See Steinmetz, "Alternating Current Phenomena," third edition, pages 158 to 163. Also see F. A. C. Perrine and F. G. Baum, *Transactions American Institute of Electrical Engineers*, Vol. XVII., June and July, 1900.)

The remainder of this chapter is devoted to the comparatively simple problem of the influence of line resistance and line reactance upon the electromotive force drop in alternating-current transmission lines. The method here outlined is sufficiently accurate for all practical calculations on short lines.

175. Interference of separate transmission lines.—When more than one transmission line (more than two wires) is strung on the same poles the alternating current in each line induces electromotive forces in the other lines and affects the line drop. This interference of one line upon another is obviated by crossing the lines at every second or third pole, as shown in Figs. 201, 202 and 203. Fig. 201 shows the arrangement of a single-phase alternating-current line to avoid inductive effects upon any

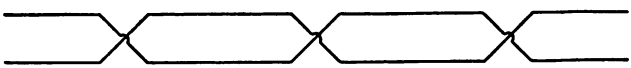


Fig. 201.

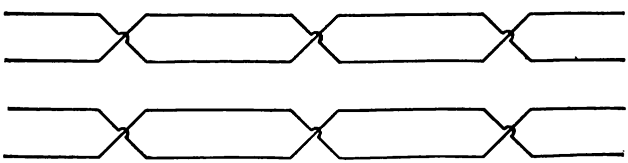


Fig. 202.

other lines that may be in the neighborhood; Fig. 202 shows the arrangement of four wires for transmitting two-phase currents,

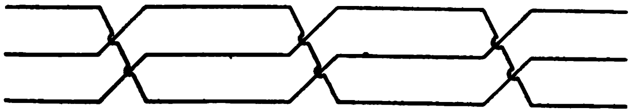


Fig. 203.

and Fig. 203 shows the arrangement of three wires for transmitting three-phase currents.

Remark.—Transmission lines also affect neighboring lines by charging and discharging them electrostatically with the pulsations of electromotive force; and by leakage currents due to incomplete insulation.

176. Calculations of a transmission line to give a specified line drop (*single-phase*).—A transmission line is usually designed to deliver a prescribed amount of power P at prescribed electromotive force E to a receiver circuit of which the power factor, $\cos\theta$ (see Article 52), is given. The line drop, frequency, length of line and distance apart of wires are also given.

The generator electromotive force E_0 is equal to the sum (numerical sum) of E and line drop.

The full load current I is found from $EI\cos\theta = P$.

The component of E parallel to I is $E\cos\theta$, and the component of E perpendicular to I is $E\sin\theta$.

By treating the problem at first as a direct-current problem the approximate resistance r' of the line is found, namely, $r'I =$ line drop. From this approximate resistance and length of line, the approximate size of wire and line reactance x are found from the table; and since the line reactance varies but little with size of wire the value of x need not be further appropriated.

The component of E_0 parallel to I is $E\cos\theta + rI$ where r is the true resistance of the line, and the component of E_0 perpendicular to I is $E\sin\theta + xI$. Therefore

$$E_0^2 = (E\cos\theta + rI)^2 + (E\sin\theta + xI)^2$$

or

$$r = \frac{\sqrt{E_0^2 - (E \sin \theta + xI)^2} - E \cos \theta}{I} \qquad (126)$$

From this equation the true line resistance r may be found and thence the correct size of wire.

Example:

$E = 20{,}000$ volts
$P = 1{,}000$ kilowatts
$\cos \theta = .85$ = power factor of receiving circuit
$E_0 = 23{,}000$ volts, or line drop $= 3{,}000$ volts
frequency = 60 cycles per second
distance = 30 miles
distance apart of wires = 18 inches

From these data we find:

$I = 58.8$ amperes
$r' = 51$ ohms

Therefore, from the table we find that, approximately, a No. 2 B. & S. wire is required so that $x = 37.7$ ohms.

Further

$$E \cos \theta = 17{,}000 \text{ volts}$$
$$E \sin \theta + xI = 12{,}700 \text{ volts}$$

and from equation (126) we find

$$r = 37.3 \text{ ohms}$$

from which the correct size of wire is found to be approximately a No. 1 B. & S.

177. Calculation of double line for two-phase transmission (*four wires*).—In this case each line is calculated to deliver half the prescribed power. Thus, if it is desired to deliver 1,000 kilowatts at 20,000 volts two-phase, at a frequency of 60, line drop of 3,000 volts, etc., then each line is calculated as a single-phase line to deliver 500 kilowatts at 3,000 volts line drop, the lines being, of course, arranged as shown in Fig. 202.

178. Calculation of a three-wire transmission line for three-phase currents.—The calculation will be carried out for the case of Y-connected generator and Y-connected receiver as shown in Fig. 204, for the reason that the relation between E_0, E, and line current is then the simplest.

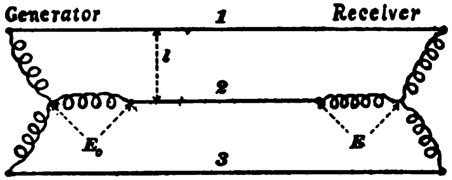


Fig. 204.

Let $\cos\theta$ be the power factor of each receiving circuit, P the total power to be delivered, E the electromotive force between the terminals of each receiving circuit and E_0 the electromotive force of each armature winding on the generator;* all prescribed.

Then $$P = 3EI\cos\theta$$

from which the full load line current I may be calculated.

The difference $E_0 - E$ is due to electromotive force drop in one main. Therefore, looking upon the problem as one in direct currents, we have $E_0 - E = r'I$ where r' is the approximate resistance of one main. From this the approximate size of wire may be found from the table.

Consider one of the mains; say, main No. 2; the other two mains together constitute the return circuit for this main, and the average distance from main 2 to mains 1 and 3 is $1\frac{1}{3}l$ when the mains are crossed as shown in Fig. 203. Find the reactance x of a pair of mains each of the size approximated above and distant $1\frac{1}{3}l$ from each other.

The component of E parallel to I is $E\cos\theta$ and the component of E perpendicular to I is $E\sin\theta$.

The resistance drop in one main is rI and the reactance drop in one main is $\frac{1}{2}xI$, the former being parallel to I and the latter perpendicular to I.

Then the components of E_0 are $E\cos\theta + rI$ and $E\sin\theta + \frac{1}{2}xI$ so that

$$E_0^2 = (E\cos\theta + rI)^2 + (E\sin\theta + \tfrac{1}{2}xI)^2$$

* The electromotive force between mains at receiving station is $\sqrt{3}E$ and the electromotive force between mains at generating station is $\sqrt{3}E_0$.

or

$$r = \frac{\sqrt{E_0^2 - (E \sin \theta + \frac{1}{2}xI)^2} - E \cos \theta}{I} \tag{127}$$

which gives r, the true resistance of one main, from which the correct size of wire is easily found.

The calculation of a transmission line when the electromotive forces between mains is specified instead of the electromotive forces in Y-connected circuits is sufficiently explained in the following example.

Example.—Electromotive force between mains at receiving station to be 20,000 volts. Therefore, electromotive force between terminals of Y-connected receiving circuits would be $20{,}000 \div \sqrt{3}$. Therefore

$$E = 11{,}550 \text{ volts}$$

Electromotive force between mains at generating station to be 23,000 volts. Therefore, $E_0 = 22{,}000 \div \sqrt{3}$, or

$$E_0 = 13{,}280 \text{ volts}$$

Further specifications:

$P = 1{,}000$ kilowatts,
$\cos \theta = .85$,
frequency = 60 cycles per second,
distance = 30 miles,
distance apart of adjacent wires = 15¾ inches (l).

From these data we find

$$I = 34.0 \text{ amperes}$$
$$r' = 50.9 \text{ ohms}$$

Therefore, approximately, a No. 5 wire is needed. The reactance, x, of a 30-mile double line of No. 5 wire at 21 inches ($= 1\frac{1}{3}l$) apart is, from the table,

$$x = 41.2 \text{ ohms}$$

Equation (127) then gives

$$r = 46.5 \text{ ohms}$$

So that a wire between No. 4 and No. 5 would give the prescribed line drop.

ALTERNATING CURRENT MACHINERY.

CHAPTER XVI.

ALTERNATORS.

179. When alternators are to be used solely for the operation of lights, single-phase machines are generally employed. They are simpler than two- or three-phase machines, and perform the work equally well. In fact, in some particulars they are to be preferred, as they do not give rise to the unbalanced voltages often met with in polyphase working. In most modern plants, however, motors as well as lights are operated and if the motor load is at all large, it is best to install polyphase alternators, since the single-phase motor has not as yet proved to be the equal of polyphase motors as regards starting effort and general operation.

180. Construction of alternators.—So far as general construction and appearance are concerned, single-phase and polyphase alternators are practically identical. About the only difference lies in the arrangement and connections of the armature winding and the number of collector rings with which the machines are provided. Three general classes of alternators are in common use, as follows; these have already been briefly described in Chapter II.

(*a*) Machines with a stationary field and revolving armature. In these machines the current is led from the armature by means of collector rings, and the conductors in which the electromotive force is induced are mounted rigidly on the armature core and revolve with it.

(*b*) Machines with a stationary armature and revolving field. In these alternators the armature windings are arranged in slots

around the inner periphery of a stationary armature structure. The revolving field is usually provided with radially projecting poles and revolves within the stationary armature. The exciting current is led into the field windings by means of two collector rings on the shaft and the lines leading from the machine connect directly to the armature winding. This type of machine has of late years come into very extensive use, especially for alternators of large size and high voltage or large current output. The revolving construction allows the armature to be insulated for very high voltages, it being possible to build such machines to generate pressures of twelve thousand volts or higher.

(*c*) Machines in which a revolving mass of iron with polar projections causes the magnetic flux passing through a set of stationary coils to vary, thus setting up an alternating electromotive force in them. In these machines, known as *inductor alternators*, there is no moving wire. The armature coils are stationary, being arranged in very much the same way as those for a revolving field machine. The magnetic flux is set up by a stationary field coil, and while the flux passing through the armature coils varies from zero to a maximum, it does not reverse as in the revolving field or revolving armature machines. This description applies to the Stanley alternator which is the most prominent example of this type as used in America.

181. Revolving armature alternators.—Fig. 205 shows a Westinghouse 300-kilowatt, 133-cycle, belt-driven, single-phase alternator of the revolving armature type. The lower half of the field yoke is in one casting with the bearings. The poles, of which there are thirty-two, project radially inwards, and are laminated, each pole piece being made up of a number of sheet-iron stampings firmly held together between end plates. These pole pieces are cast-welded into the yoke. Fig. 206 (a) shows the armature core and a number of the coils for a single-phase machine of the belt-driven type and Fig. 206 (b) shows a revolving armature of large diameter for a slow-speed direct-connected

machine. It should be noted that the winding, Fig. 206 (a), is of the partially distributed type, there being three coils per pole arranged one inside of the other as indicated by the group of coils

Fig. 205.

at the left. The winding on the armature, Fig. 206 (b), is also partially distributed, there being ten slots per pole. On the end of the shaft, Fig. 205, outside of the right-hand bearing is shown

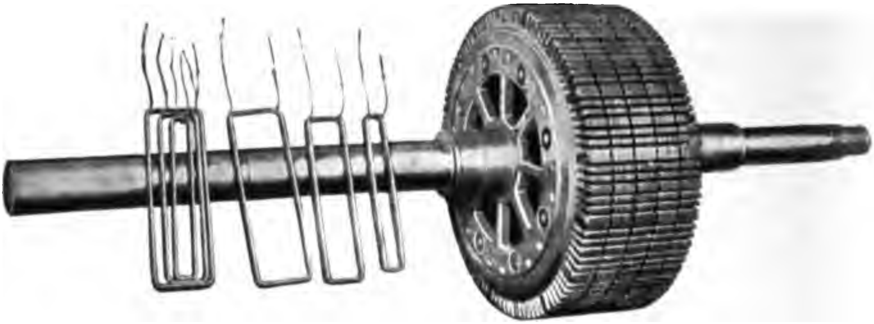


Fig. 206 (a)

Fig. 206 (b).

the rectifier for supplying current to the series field coils. The rectifier connects to the secondary of a small series transformer carried on the armature spider, the primary of the transformer being in series with the armature winding. (See Article 194.) Alternators with revolving armatures are made in a large range of sizes, and may be either of the belt-driven or direct-connected type. However, for large direct-connected machines the tendency seems towards the use of a revolving field and stationary armature.

182. Revolving field alternators.—Fig. 207 (a) shows one arrangement of the field and armature of machines of this class.

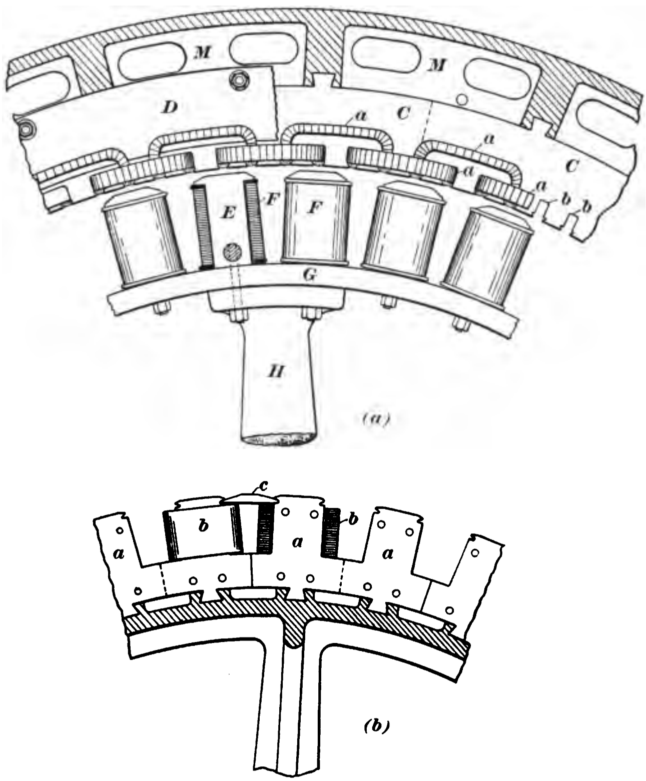


Fig. 207.

The armature coils *a* are held in the slots *b* arranged around the inner periphery of the core *C*, which is built up of stampings clamped between plates *D* and supported by the frame *M*. The

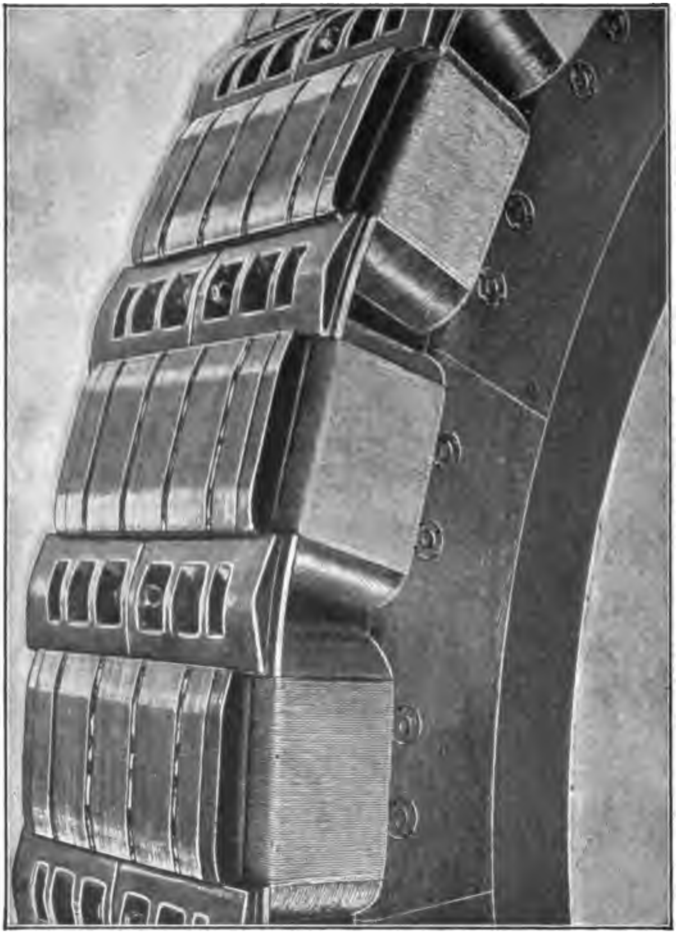


Fig. 208.

revolving field consists of radial laminated pole pieces *E*, carrying exciting coils *F*. These pole pieces are bolted to the steel yoke or rim *G*, which is connected to the shaft by means of the field

spider arms *H*. In the larger machines, the exciting coils *F* are wound with a copper strip, which is coiled on edge as shown. This makes a very solid and substantial coil and one from which the heat is readily radiated. Fig. 207 (a) shows the general construction of the revolving field alternators built by the General Electric Co. The Westinghouse revolving field is constructed

Fig. 209.

as shown in (b) and Fig. 208 shows a portion of a field of this kind. The rim and pole pieces are here built up of stampings *a* which are staggered or overlapped so as to form a continuous structure. The exciting coils *b* are held from flying out by pieces *c* fitted into the notches in the pole pieces. Fig. 209 shows a General Electric alternator of the revolving field type and having

the general construction indicated in Fig. 207 (a). The two collector rings mounted on the shaft are for supplying the exciting current. In this type of machine the stationary armature structure is usually arranged so that it can be slid to one side on the bed plate, thus allowing access to both armature and field coils.

Since the armature windings are stationary, on revolving field machines, there are no collector rings to insulate for high pressures. Also, there is plenty of available room for thorough insulation, so that the armature can readily be insulated for comparatively high pressures. Machines of this type, of 3,500 k. w. capacity, three-phase, are used for operating the surface trolley system in New York City. These alternators are direct-connected to vertical engines running at 75 revolutions per minute. The alternators generate 6,600 volts, and supply current to a number of substations, where the pressure is stepped down by means of transformers. It is then supplied to rotary converters for conversion into direct current at 500 volts for operating the cars.

183. Inductor alternators.—The Stanley alternator is one of the most prominent examples of the inductor type. The general arrangement of this machine is shown in Fig. 210. The machine is double, and has practically two independent armatures, A, A'. The two armature cores are connected by the bars b, which carry the magnetic flux, and the armature coils cc are arranged around the inner periphery of the laminated core structure in practically the same way as for a revolving field machine. The revolving inductor dd carries a crown of projecting laminated pole pieces pp at each end, but, unlike the revolving field machine, these polar projections are not alternately of opposite polarity. All those at one end of the inductor, such as NNN, are of one polarity, and all those at the other end are of opposite polarity. The magnetic flux takes the path shown by the curved dotted line $hfgh$, and, as the inductor revolves, the flux threading

the armature coils alternately increases to a maximum and decreases to zero, but does not reverse. The flux is set up by a large stationary coil *C*, which completely encircles the inductor. The armature coils, as indicated in Fig. 210, are arranged to gen-

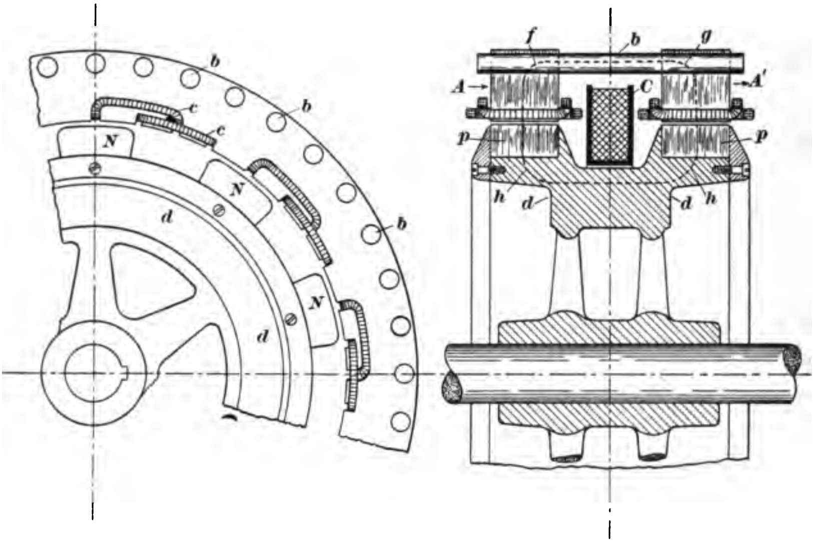


Fig. 210.

erate two electromotive forces differing in phase by 90°, one set of coils being displaced, so that the electromotive force in them is zero at the instant the electromotive force in the other coils is a maximum. Inductor alternators are also made by the Warren and Westinghouse companies. Their machines differ from the Stanley alternator in having only one stationary armature instead of being double. Otherwise the principle of operation is the same.

184. Connections for parallel running of alternators.—When alternators are arranged for parallel running, some means must be provided for telling when the machines are in synchronism. If one alternator is running and it is desired to throw another in parallel with it, the second machine must first be brought up to

synchronism and thrown in with the other at the instant when the electromotive forces of the two machines are in phase. A number of different instruments have been brought out for indicating the condition of synchronism, but in most cases incandescent lamps (see Art. 126), or a synchronizing voltmeter are used. It is essential that the two alternators be running at the same frequency when they are thrown in parallel. A slight difference in phase does not make so much difference, as the machines will pull each other into step, the only disadvantage being a considerable exchange of current between the machines.

Fig. 211 shows the essential connections required for the operation of two compound wound alternators in parallel; *aa* represent the collector rings and rectifier; *bb* are the series field coils. The separately excited field coils and exciter connections are omitted in order to avoid confusing the diagram. When compound wound alternators are run in parallel, it is necessary to connect the series coils in parallel, so that the current can equalize between them. This is accomplished by the equalizing wires *dd* running between the machines. Adjustable compounding resistances *rr'* are connected in parallel with the series coils so that the effect of the coils, and, hence, the degree of compounding may be varied by shunting a portion of the current that would otherwise flow through them. The collector rings are connected, through the main switch, to the bus-bars. In Fig. 211 three-phase alternators are shown and an ammeter is indicated in but one of the line wires. If the system is well balanced only one ammeter is necessary, but if unbalancing is probable an ammeter is placed in each of the three wires. *TT'* are small potential transformers to step down the pressure for synchronizing lamps *ll'*, and the voltmeters *VV'*. These transformers are connected back of the main switch, *i. e.*, between the main switch and the alternator, because it is necessary to know the voltage and phase relation of any of the machines before the main switch is thrown in to connect the machines to the bus-bars. When the synchronizing plugs are inserted at *PP'*, the secondaries of *TT'* are con-

nected in series with the lamps ll'. If the connections are such that the electromotive forces of T and T' oppose each other when the machines are in step, lamps ll' will be dark when synchronism is attained, but if the electromotive forces in the synchronizing circuit are directed as indicated by the arrowheads in Fig. 211 the lamps will burn at full candle power at synchronism because the electromotive forces then aid each other. In some cases the connections are made so that dark lamps indicate synchronism; in other cases light lamps are used. The latter is probably pref-

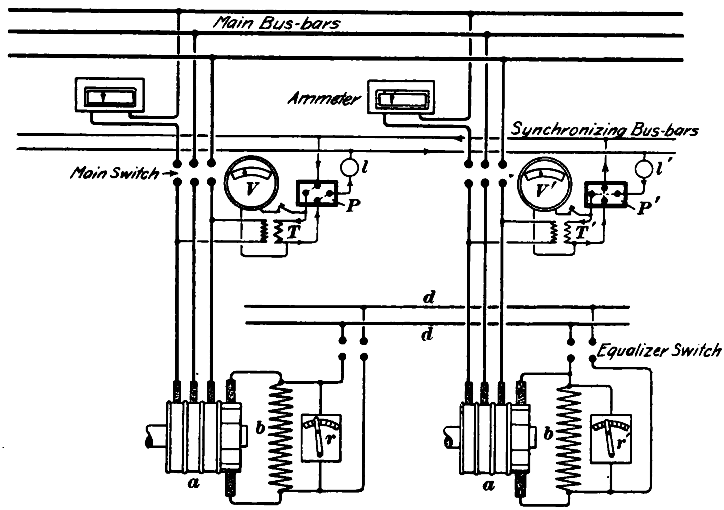


Fig. 211.

erable, because it is easier to tell the point of maximum brightness than maximum darkness. When the pulsations of the lamps become very slow, say one beat in two or three seconds, it indicates that the machines are running at very nearly the same frequency, and the main switch should be closed in the middle of one of the beats when the lamps are light or dark, as the case may be.

Fig. 212 shows one scheme of connections used by the Westinghouse Company for running two, three-phase machines in

parallel. The exciter and field connections are here omitted for the sake of simplicity. The machines are separately excited, no series winding being used on the field. Plain separate excitation is used on most of the larger alternators now installed. By careful designing, the voltage regulation can be made sufficiently close so that whatever regulation is necessary can be obtained by varying the separate field excitation. In Fig. 212, therefore, no

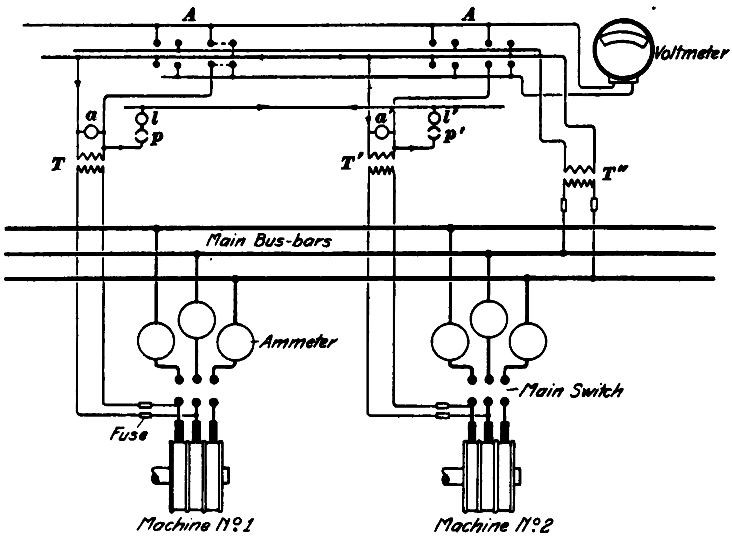


Fig. 212.

equalizing connections are necessary. The lines from the machines lead first to the main switch and then through the ammeters and connect to the main bus-bars. One ammeter is provided in each line although this is not absolutely necessary under ordinary circumstances. One voltmeter is made to serve for any number of machines by providing a set of plug receptacles A, A, for each machine. T and T' are potential transformers, one for each alternator, and connected back of the main switch. T'' is a potential transformer connected to the bus-bars. By inserting a plug in the right-hand receptacle, as indicated by the dotted

lines at A, the voltage of machine No. 1 is indicated. By inserting the plug in the left-hand receptacles, the voltmeter indicates the bus-bar voltage. The attendant can, therefore, readily compare the voltage of the machine which is to be thrown in parallel with the voltage on the bus-bars to which it is to be connected. Pilot lamps are shown at aa', and ll' are the synchronizing lamps. When the synchronizing plugs are inserted at pp' current tends to flow around the circuit indicated by the arrowheads. If the machines were in phase, the electromotive forces of T and T' would oppose each other with the connections shown, and the lamps ll' would be dark at synchronism. By reversing the connections of one of the transformers, the transformer electromotive forces would be added together at synchronism and the two lamps would burn up to full brightness.

CHAPTER XVII.

TRANSFORMERS.

185. Construction.—The construction of a transformer depends to a certain extent upon the use to which it is to be put. Transformers for outdoor use must be protected by water-proof cases,

Fig. 213.

and little provision can be made for ventilation. On the other hand, where transformers are used indoors, as, for example, in substations, the question of ventilation and cooling becomes im-

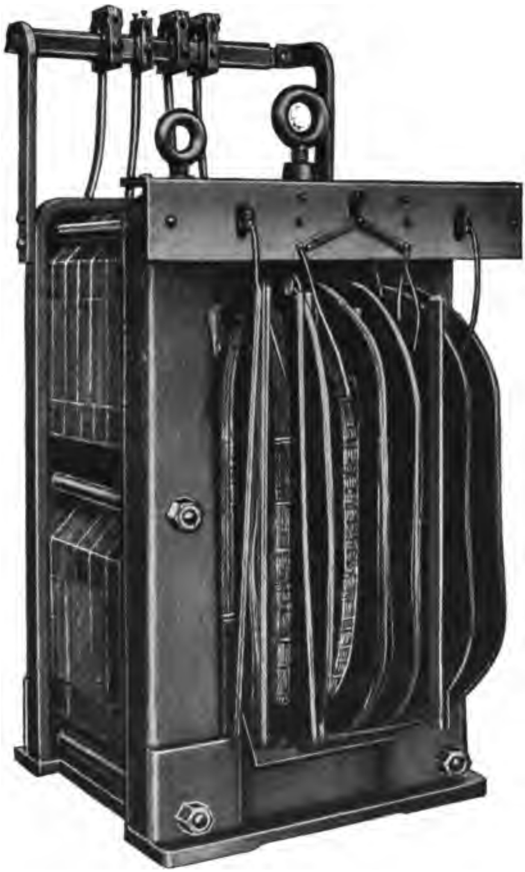


Fig. 214.

portant and the construction of the transformer has to be modified accordingly.

Transformers, used for outdoor work, are of comparatively small size; they are placed in a water-proof iron case and this case is usually designed so that it may be filled with oil, though

in some cases, especially with small transformers, oil is not used. The oil serves two purposes: it improves the insulation and conducts the heat from the coils and core to the outer case, where it is radiated to the surrounding air. Fig. 213 shows the coils and core of a small Westinghouse transformer. It will be noted that this transformer is of the "shell" type; the iron core surrounds the coils which project at each end beyond the laminations. The coils are pancake shaped, the primary and secondary being wound in sections that are sandwiched together. This is done to reduce magnetic leakage to a minimum. Both primary

Fig. 215.

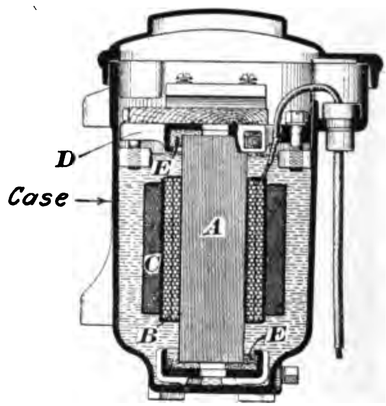


Fig. 216.

and secondary are wound in sections so that the transformer may be adapted to 1,050 or 2,100 volts primary and 52½, 105 or 210 volts secondary by connecting the sections in series for the high voltages and in parallel for the low. Fig. 214 shows the core and coils of a large Westinghouse transformer of a type used for power transmission work. It is of 375 kilowatts capacity and its primary is wound for 15,000 volts. The flat primary and secondary coils are clearly shown; they are flared out at the end so as to give free ventilation. The arrangement of the core and coils is practically the same as that of the small transformer in Fig. 213, the coils being surrounded by the iron.

Fig. 215 shows the arrangement of coils and core for a General Electric Type H transformer of small size, and Fig. 216 shows a section through the transformer in its case. *A* is the core; *B*, one of the secondary coils; *C*, one of the primaries; *D*, the clamp for holding the transformer in the case, and *EE*, insulating blocks to insulate the core from the case. As seen in Fig. 215, the transformer is of the "core" type. The coils surround the iron core which is built up of iron strips.

186. Cooling of transformers.—Transformers of moderate size are capable of radiating the heat generated in them without any special means being provided for radiating the heat. The superficial area of these smaller transformers is sufficient to radiate the heat arising from the various losses. However, as the output of transformers is increased, the radiating area increases in a much lower proportion than the output; hence, large transformers are not capable of getting rid of the heat generated in them without attaining a dangerously high temperature, and special means have to be provided for ventilating them. The fact that special cooling facilities have to be provided for large substation transformers does not, in any sense, mean that these transformers are not efficient. As a matter of fact, large transformers are one of the most efficient, if not the most efficient, pieces of apparatus used for transforming energy. A large substation transformer may have an efficiency of over 98 per cent at full load. The heating does not, therefore, represent an excessive waste of energy; it simply means that the transformer has such small linear dimensions compared with its output that not enough radiating surface is available to get rid of the heat without some special provision for the purpose. It is much more economical to provide special cooling facilities than to make the transformer large enough to radiate the heat itself. Self-cooling transformers of large output are therefore higher in cost per kilowatt than those designed for artificial cooling.

Some of the methods used for cooling are as follows: Use of

oil to conduct the heat to the outer case, oil circulation, water circulation and air blast.

Fig. 217 shows a 250-kilowatt oil-cooled transformer. The case is of cast-iron, and provided with deep corrugations, which present a large surface to the surrounding air. The case is filled

Fig. 217.

with oil, which conducts the heat to the case. This particular transformer is provided with a sectional wound primary and a dial switch, so that the secondary voltage can be changed by changing the ratio of transformation.

Some transformers are cooled by providing an oil-circulating pump which keeps up a circulation of oil around the coils and

core. More often, however, the cooling is accomplished by filling the case with oil and cooling the oil by circulating water through a coiled pipe placed near the top of the case.

In the air-blast type of transformer, openings are provided between the coils and core. The transformer is set over a chamber, into which air is forced by a fan driven by an electric motor;

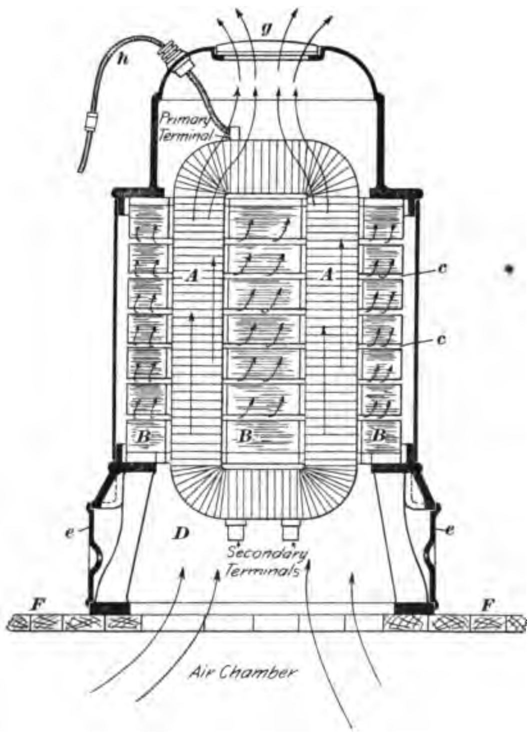


Fig. 218.

the air enters through an opening in the bottom of the transformer casing, passes up through the openings, through the core and between the coils and passes out at the top and sides. The sectional view, Fig. 218, illustrates an air-blast transformer. This method of ventilation is clean and effective, and the amount of power required to run is but a very small fraction (about $\frac{1}{10}$ of 1 per cent.) of the output of the transformers to be cooled. In

Fig. 218, *B* represents the core and *A* one of the coils. The primary and secondary are subdivided into several flat coils that are thoroughly insulated from each other by separating diaphragms. The core is built up in such a way as to leave air-ducts *cc* at regular intervals. The air enters at the bottom, passes up between the coils and through the core and out at the top and back side. The flow of the air between the coils is controlled by the damper *g* at the top, and another damper, not shown, regulates the flow through the core. The lower casing *D* is provided with doors *ee* to allow access to the secondary terminals in case they cannot be reached from the air-chamber below; one of the primary terminals is shown at *h*.

187. Transformer connections on two-phase circuits. — In America it is customary to use separate transformers for each phase of a polyphase system; whereas, in Europe, polyphase transformers are frequently used. For example, in the three-phase

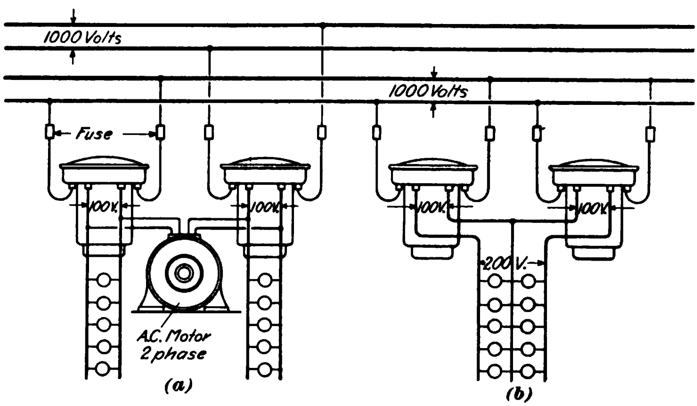


Fig. 219.

transformer, a core with three magnetic circuits is used, each circuit being provided with the coils for one phase. In Figs. 219–222, separate transformers are shown for each phase. Figs. 219 and 220 show connections commonly used for two-phase systems, and Figs. 221 and 222 those for three-phase systems. In all the figures

the primary voltage for the sake of illustration is taken as 1,000 volts, and all the transformers are supposed to have a ratio of 10 to 1, *i. e.*, the primary coils have ten times as many turns as the secondaries. Fig. 219 (a) (b) shows transformers connected on a two-phase four-wire system. In (a) a transformer is connected to each phase and the secondaries supply separate lighting circuits, or in case a motor is operated both phases are run to the motor. Both phases are independent throughout, and this constitutes the commonest scheme of connection used on the two-phase system. In case a single transformer alone is needed for

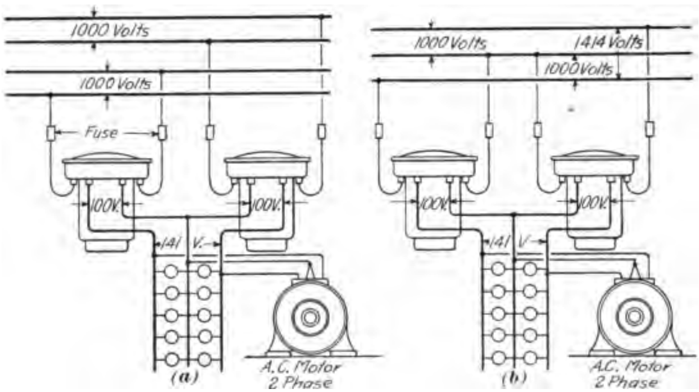
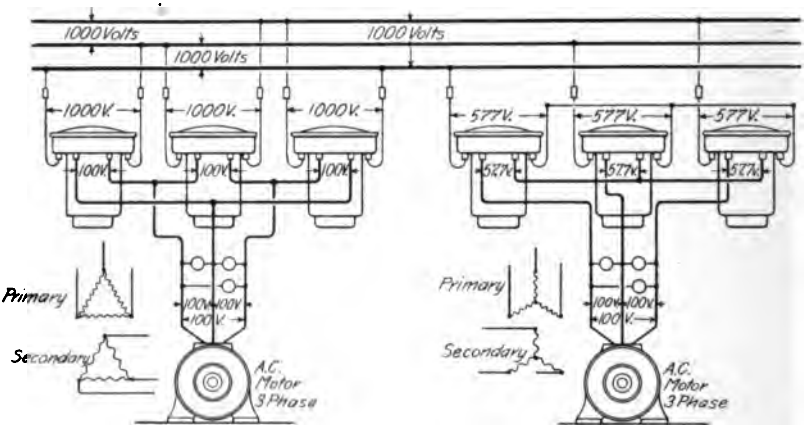


Fig. 220.

lighting or other purposes, it is connected across one of the phases and the balance of the system is maintained by connecting a transformer required at some other point on the other phase. By exercising care in the connecting of scattered transformers the load on the two phases can be kept nearly balanced. Fig. 219 (b) shows two transformers with their primaries connected across the same phase, and their secondaries in series for operating lamps on the three-wire system. This is in reality a single-phase arrangement as only one phase of the system is used. The voltage across the two outside wires is 200 volts or twice that between the middle or neutral wire and either of the outside

wires. This arrangement is sometimes used where a considerable number of lights are to be supplied, or where three-wire secondary mains are used. It would not be used where motors were operated. Fig. 220 (a) shows an arrangement similar to 219 (b) except that the primaries are connected to the two phases. In this case the voltage across the outside secondary wires is only



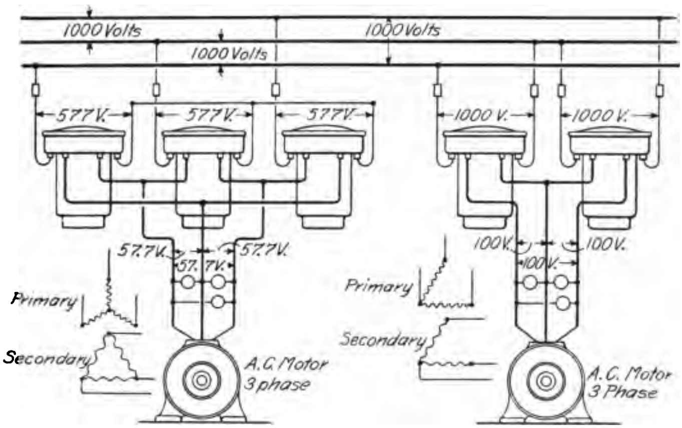
(a) (b)

Fig. 221.

141 volts ($100 \times \sqrt{2}$), instead of 200, because the two pressures of 100 volts on each side are in quadrature with each other. Fig. 220 (b) shows the same arrangement as (a) except that instead of four separate wires three wires are used for the primary, as explained in Article 69. This scheme of connection is not to be recommended where lights are operated, because the voltages on the two sides are liable to become unbalanced to a sufficient extent to seriously affect the lamps. If a three-wire secondary lighting system is to be fed, the connections shown in 219 (b) are much to be preferred.

188. Transformer connections on three-phase circuits.—Owing to the fact that either the primaries or secondaries of transformers operated on a three-phase system may be connected Y or Δ (star or mesh), there are quite a number of different methods of

connection available. Figs. 221 and 222 show four of the methods most commonly used. The connections shown in Fig. 221 (a) are probably used more than any of the others. Here, both primaries and secondaries are delta (Δ) connected. The pressure applied to the primary of each transformer is the same as the line voltage, and the secondary pressure is, in this case, one-tenth of the primary, since we have assumed that the primary coils have ten times as many turns as the secondary. The chief advantages of this scheme of connection are that it admits the



(a) (b)

Fig. 222.

use of transformers having the ordinary ratios of transformation without giving rise to odd secondary voltages; also in case one transformer burns out or in case its primary fuse blows, the service will not be interrupted. By cutting down the load somewhat, it can be carried by the two remaining transformers.

In Fig. 221 (b) the transformers are shown with both their primaries and secondaries Y-connected. The voltage across the primary of each transformer will be equal to the line voltage divided by $\sqrt{3}$, which in this case gives about 577 volts. The pressure across the secondary mains will be equal to the pressure of the secondary of one transformer multiplied by $\sqrt{3}$. Thus,

the secondary voltage of each transformer in (b) will be 57.7, because the winding has a ratio of 10:1 and the voltage between the secondary lines will be $57.7 \times \sqrt{3} = 100$. This scheme of connection requires the primaries and secondaries to be wound for odd voltages, and if one transformer breaks down, the service is crippled. It has the advantage, however, that the voltage across the primary coils is much less than the voltage of the mains, and, consequently, where very high pressure mains are used, the insulation of the primary coils is rendered less difficult. This is also true of the arrangement shown in Fig. 222 (a), where the primaries are Y-connected and the secondaries Δ-connected. The connections shown in Fig. 222 (a) give an odd secondary voltage unless transformers with a special ratio of transformation are used.

Fig. 222 (b) shows the connections for two transformers on a three-phase system, an arrangement sometimes used for the operation of small motors; it is not recommended for large motors. It is equivalent to the delta connection with one side omitted, and if one transformer gives out the service is interrupted. Methods of connection other than those shown are possible; for example, the primaries might be connected delta and the secondaries Y. Also, in some cases where transformers are used for lighting, a common return wire is run from the common connection of the secondaries, so that a return path will be provided in case the system becomes unbalanced. The connections shown are, however, the ones most commonly used.

189. Constant current transformer.—When series arc lamps are to be run on constant potential alternating current systems, it is necessary to use some device whereby the current in the lamp circuit may be kept constant irrespective of the number of lamps in operation. One way in which this may be accomplished is by the use of a constant-current transformer, *i. e.*, a transformer which, when its primary is supplied with current at constant potential, will deliver a constant secondary current. The

General Electric constant-current transformer is one that has been largely used for arc lighting. The principle of operation of this transformer was explained in Article 118, and Fig. 223 shows the working parts of one of the larger sizes. The core is of the same shape as that shown in Fig. 143, but in this case

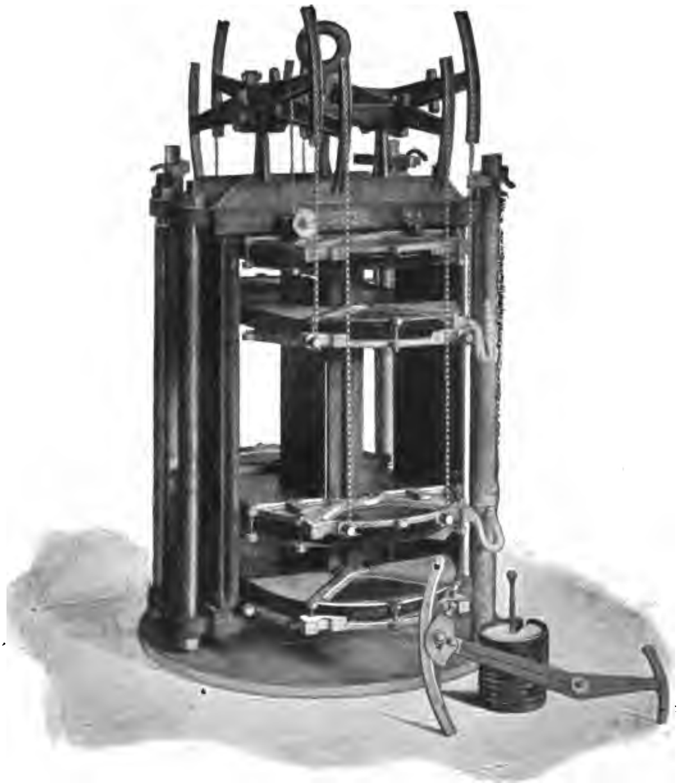


Fig. 223.

there are two movable coils and two fixed coils. The fixed coils are placed at the top and bottom as shown. The movable coils are counterbalanced against each other by means of the levers and move up and down between the fixed coils. When the transformer is fully loaded, the flat movable coils rest on the fixed coils, and the transformer generates its maximum electro-

motive force. If the load is diminished by cutting out lamps, the current in the movable coils tends to increase, thus causing a repulsion between the coils. When the coils become separated, the magnetic leakage between them results in a lowering of the electromotive force, as explained in Article 118. The repulsion between the coils is balanced by a small auxiliary lever shown at the base of the transformer, and by regulating the weight on this lever, the current may be adjusted. The whole mechanism is placed in a corrugated iron case filled with oil, which helps to conduct the heat off, and also steadies the movements of the coils.

CHAPTER XVIII.

INDUCTION MOTORS.

190. Construction.—In most of the induction motors now built, the part into which the currents are led from the line is the stationary member or stator. That in which the induced currents are set up is the rotating member or rotor. The latter is usually

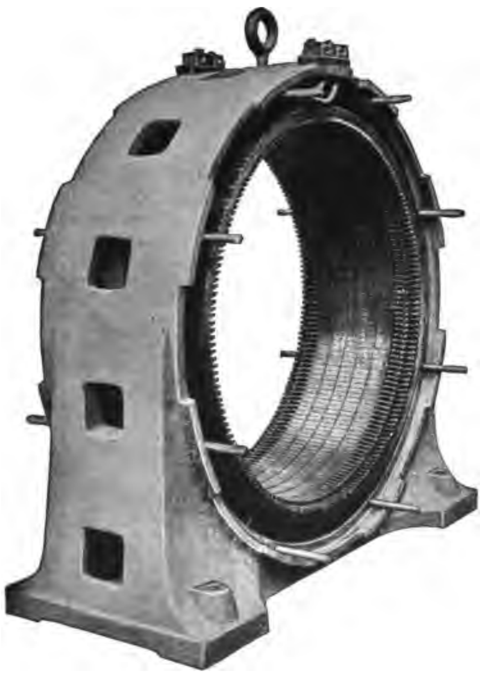


Fig. 224.

called the armature, or secondary, and the former the field, or primary. The construction of an induction motor is, on the whole,

comparatively simple, especially if the squirrel-cage type of armature is used. Fig. 224 shows the field, or primary, of a Westinghouse 150-kilowatt motor. The winding, which is of copper bars in this case, is uniformly distributed in slots, and is connected up in the same way as the windings described for polyphase alternator armatures. Fig. 225 shows the armature or

Fig. 225.

secondary. Each slot is provided with a single rectangular ba and the ends of the bars are all connected together by means of short-circuiting rings at each end of the armature. Fig. 226 shows a General Electric motor in which the squirrel-cage type of armature is used. The general arrangement of the parts of an induction motor with squirrel-cage armature will be understood by referring to Fig. 227. *A* is the cast-iron casing that supports the field stampings *B*. The field coils *C* are generally arranged in two layers and are held in place by strips of wood *d* which engage with notches in the teeth. The armature stampings *e* are supported by the arms *f* of the armature spider. The armature slots are closed with the exception of a small slit *g*, and the bars *h* are pushed in from the end. In many European

motors, slots which are entirely closed are used, but this makes the field coils more difficult to place in position, although it may,

Fig. 226.

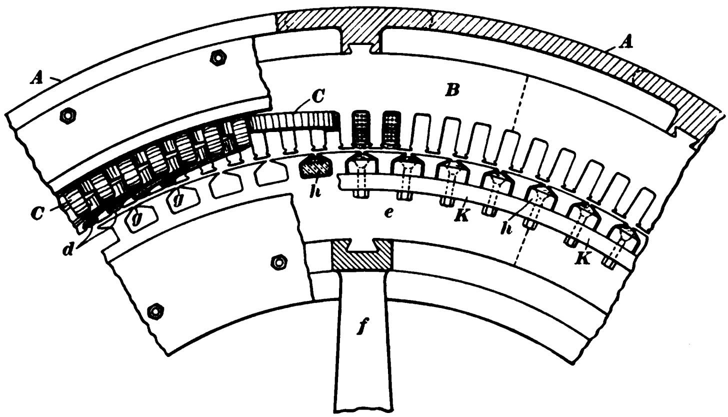


Fig. 227.

in some respects, improve the performance of the motor. *K* is the copper end ring connecting the ends of all the bars, thus forming the squirrel-cage winding.

A motor with a squirrel-cage armature, while it is very simple as regards its construction, is not as desirable for some kinds of work as one with a resistance in the armature. For a large torque at low speed or at starting the low resistance squirrel-cage motor takes more current from the line than one having a starting resistance in the armature circuit. An induction motor with high armature resistance will give a good torque at starting but the high resistance will result in reduced efficiency and poor speed regulation if it is left in permanently while the motor is running. On the other hand a motor with a low resistance squirrel-cage armature will give good speed regulation and a good torque when running, but will not give as good a torque at starting. For cases where a large starting effort is required

Fig. 228.

without taking an excessive current, the motor with a resistance inserted in the armature at starting is to be preferred. Of course, squirrel-cage armatures can be made of fairly high resistance, but this resistance is in circuit permanently and reduces the efficiency somewhat, besides making the speed regulation poor. Large starting currents are especially objectionable on circuits where motors as well as lights are operated.

Fig. 228 shows an armature of a General Electric motor which carries a resistance within the armature spider. The armature is provided with a regular Y-connected bar winding in place of the simple squirrel-cage. A resistance is mounted within the

armature and connected to a sliding switch so that, by pushing in the knob (at the right-hand end of the shaft in Fig. 228), the resistance may be cut out. The resistance is divided into three parts—one section in series with each phase of the armature winding. When the switch is pushed in the resistance is cut out, and the three phases of the winding are short-circuited. By using an armature of this type, a large starting torque may be obtained without an excessive rush of current. No collector rings are necessary in connection with the armature, but the construction is considerably more complicated and expensive than that of a squirrel-cage armature, and the squirrel-cage type is very largely used on account of its simplicity and small amount of attention required.

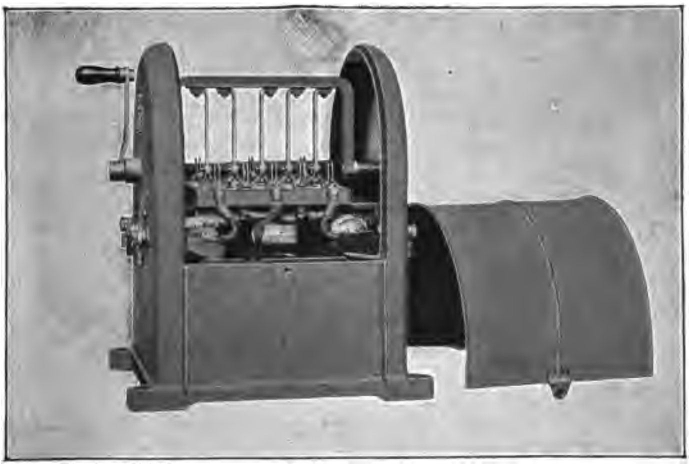


Fig. 229.

191. Starting compensator.—The rush of current with a squirrel-cage motor can be avoided by inserting a starting resistance in series with the field windings or by using other means for cutting down the applied electromotive force. The torque of an induction motor decreases as the square of the applied voltage, so that this method of starting results in a greatly reduced start-

ing effort. However, in a great many cases motors are not started up under full load so that this is not a serious objection; whereas, a large rush of current would be objectionable because of the disturbance of other parts of the system.

While a resistance could be used as described above, it is more economical to use an autotransformer (*i. e.*, a transformer having but one coil, which serves both as primary and secondary) or compensator as it is usually called in this connection. Fig. 229 shows a General Electric starting compensator. The Westinghouse Company also use an arrangement involving the same general features. Fig. 230 shows the compensator connections for a three-phase motor. The compensator consists of coils *a*, *b*, *c*, wound on a laminated iron core, each coil being provided with a number of taps 1, 2, 3, 4. The pressure applied to the motor at starting will depend upon the amount of the coil included in the circuit. For example, in Fig. 230 the use of tap 4

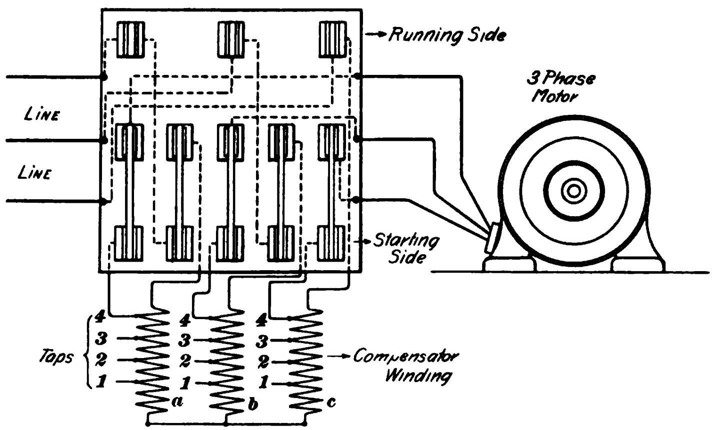


Fig. 230.

would give a higher applied electromotive force than tap 3, so that the starting current may be adapted to the kind of work which the motor is called on to perform.

For light work a small voltage may be applied, in which case the current taken from the line will be small, while for heavy

work, where a stronger starting effort is necessary, a higher electromotive force, with corresponding larger torque, may be used. When the five-blade double-throw switch is in the lower position shown in the figure, a part of the compensator coil is in series with each line leading to the motor, and the voltage applied to the motor is cut down. When the switch is thrown to the upper or running position after the motor has come up to speed, the motor terminals are connected directly to the line and the compensator windings are cut out, so that the motor runs under full voltage. The compensator, therefore, prevents a rush of current and gives the motor a smooth start, although it cuts down the starting torque from what would be obtained if the full voltage were applied. In some cases starting compensators are provided with a switching arrangement by which the coils may be cut out gradually, but for most cases a simple double-throw switch which cuts all the coils out at once is sufficient to give a smooth start. Starting compensators of almost exactly the same construction as the above may also be used for starting synchronous motors and rotary converters when these machines are started by means of current supplied from the alternating current mains. Small induction motors are usually started by simply closing the switch that connects them to the line. One advantage in using the compensator method of starting is that the motor may be controlled from a distant point.

192. Speed regulation of induction motors.—For some classes of work it is desirable to have induction motors arranged so that their speed may be controlled. There are a number of methods by which this may be accomplished, but the two most commonly used are: (*a*) By the insertion of an adjustable resistance in the armature; (*b*) by cutting down the voltage applied to the field either by using a resistance or by means of a compensator. These methods may be used when the range of control is not very wide. Generally speaking the induction motor does not admit of the same range of control as a direct current

motor. Another method that has been used to a limited extent is to have the winding on the field arranged so that by means of a suitable controlling switch, the number of poles on the motor can be changed. Evidently, the fewer the number of poles the faster a motor must run when supplied with current of a given frequency. This is an economical method of control, and can be used when a wide change in speed is desired, but introduces considerable complication. Another method that has been used for electric traction work is applicable where two motors are employed. This consists in connecting the motors as follows, when a low speed is desired. The field of the first motor is connected to line, and the field of the second motor is supplied with current from the armature of the first motor. The armature of the second motor is short-circuited. When the motors are thus connected, they will run at half speed. When a high speed is desired, the motors are connected in parallel directly across the line. Intermediate speeds may be obtained by combining the use of resistances with the above method; on the whole, the method is somewhat analogous to the series-parallel control used with direct-current street-car motors.

Of the first two methods of speed regulation, the one most often used is that by which an adjustable resistance is inserted in the armature. It requires the use of collector rings on the motor, since a resistance designed for continuous use for speed-regulating purposes is too bulky to be placed within the armature and would moreover lead to too much heating in the machine. Fig. 231 (a) shows a motor with a rheostat arranged for speed control. The armature of the motor is provided with a regular three-phase Y winding, and the terminals are brought to the collector rings *a*. The rheostat *R* contains three resistances divided up into sections that can be gradually cut out by moving the three-legged arm *b*. Fig. 231 (b) shows the connections diagrammatically; 1, 2, 3 are the three phases of the winding on the armature, and r_1, r_2, r_3 the three sections of the resistance. When the motor is running at its slowest speed, the windings are

connected together with all the resistance in series by means of the arm *b* which is represented in (b) by means of the circles. As the arm *b* is moved over the resistance contacts, r_1, r_2, and r_3 are cut out and when the motor is running at full speed arm *b* short-circuits the windings as indicated by the full line circle in (b). The wires from the motor field may lead directly to the line

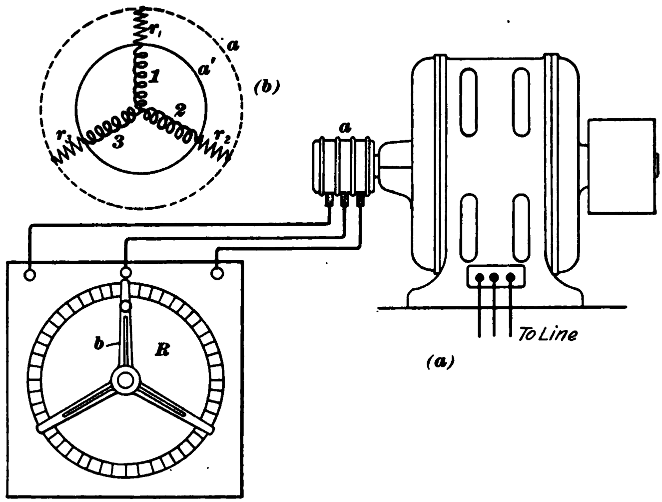


Fig. 231.

through the main switch or, as is frequently done, they may be brought through the rheostat and the arm *b* made to answer both as a starting and regulating switch by letting current into the field when it is placed on the first position.

When fairly large motors are used, as on hoists or pumps, it is customary to have the regulating resistance entirely separate from the controlling switch. In this case, a controller somewhat similar to that used on street cars is employed, and is connected to the resistance and to the motor by means of cables, thus allowing the controller and resistance to be placed in whatever location may be most convenient.

193. Voltage.—Induction motors are usually wound for 110,

220, 440 or 550 volts, though in some cases they are wound for high pressures so as to take current directly from the high-pressure mains without the intervention of transformers. Since the field is stationary, it is capable of being insulated for high pressures. The majority of the motors in common use are, however, operated at the above voltages and are supplied from transformers. It is important that the voltage supplied to an induction motor be kept up to its normal amount. The output of the motor decreases as the square of the voltage, and if the pressure becomes low, the motor will have very little margin for sudden overloads.

194. Current.—The current taken by an induction motor at full load will depend upon the power factor and efficiency of the motor, as well as the output and the voltage of supply. For a three-phase motor the current in each line will be

$$C = \frac{W}{E \times f \times e \times 1.732} \tag{128}$$

where $W =$ the watts output;
$f =$ power factor;
$e =$ commercial efficiency.

The power factor of induction motors of ordinary size and frequency varies from .75 to .90 at full load. Some of the larger motors may run a little higher than this; .85 is a fair average for motors of medium size. The efficiency varies through about the same range. For a two-phase, four-wire motor, the current in each line will be

$$C = \frac{W}{E \times f \times e \times 2} \tag{129}$$

where the quantities W, E, f are the same as before. The following table given by the General Electric Company shows the current taken in each line by three-phase 110-volt motors. For higher voltages the current will be smaller in proportion.

Horse Power of Motor.	Full Load Current.	Horse Power of Motor.	Full Load Current.
1	6.3	20	112
2	12	30	168
3	18	50	268
5	28	75	390
10	54	100	550
15	81	150	780

195. Frequencies.—The common frequencies for which induction motors are built in America are 25, 40, 60, 125 and 133 cycles per second. The high-frequency motors (125 and 133) are not generally built in very large sizes. The high frequency tends to aggravate the effects of lagging currents. These effects may be overcome to a considerable extent by using condensers at the motor to improve its power factor, the wattless current being supplied by the condenser; the Stanley Company have used condensers considerably for this purpose.

196. Transformers for induction motors.—The size of transformers to be used for supplying a given induction motor will, of course, depend upon the power factor and efficiency of the motor as well as on its output. A rule that is commonly followed, and which is a safe one at least for motors of the larger sizes, is to install 1 kilowatt of transformer capacity for every horse power output of the motor. For example, a 75-horse-power three-phase motor would require 75 kilowats in transformers, or three transformers of 25 kilowatts each. The General Electric Company recommends the use of transformers as indicated in the following table:

Size of Motor. Horse Power.	Kilowatts of each Transformer.	
	2 Transformers.	3 Transformers.
1	.6	.6
2	1.5	1
3	2	1.5
5	3	2
7½	4	3
10	6	4
15	7.5	5
20	10	7.5
30	15	10
50	25	15
75	40	25

197. Direction of rotation of induction motors.—The direction of rotation of an induction motor is determined by the direction in which the magnetic field rotates, and this in turn is determined by the relation of the currents in the windings to each other. The direction of rotation of a three-phase motor may be changed by interchanging any two of the wires leading to the field winding. To reverse a two-phase motor, the two wires of one of the phases should be interchanged.

198. Use of alternating current for short-distance transmission.—The simplicity of the induction motor has been largely accountable for the extensive use of alternating current in connection with short-distance transmission plants. While alternating current is preëminently adapted for the transmission of power over long distances because of the ease with which it admits the use of high pressures, yet its use at comparatively low pressure for short-distance transmission, without the intervention of transformers, is by no means limited. Some very large plants have been installed in connection with various manufacturing concerns where polyphase alternating-current transmission has superseded the use of long lines of shafting, thus resulting in a large economy of power and allowing extensions to be readily made. When the machinery is started and stopped frequently, and when a large range of torque and speed is desired, the direct-current motor probably gives the best results, but for other classes of work the induction motor has many advantages and has been installed in preference to the direct-current motor for many kinds of work connected with factories. The principal advantage is the absence of the commutator and sliding contacts, thus allowing the motor to be operated in places where a direct-current motor would give trouble. For example, induction motors have found extensive use in cotton mills and in many places where the sparking of a direct-current machine would be a source of danger.

CHAPTER XIX.

SYNCHRONOUS MOTORS.

199. Synchronous motors are used for work where power is required in comparatively large amounts, and where the motor does not have to be started or stopped frequently. A synchronous motor always runs at the same frequency as the alternator that supplies it with current, and its speed cannot change unless the speed of the generator changes. The speed of the motor may or may not be the same as that of the generator. It will be the same only when the motor has the same number of poles as the generator. The synchronous motor cannot be used where a variable speed is required, and it is not suitable for work where the motor has to start up under a load.

200. Construction of synchronous motors.—Synchronous motors are the same, as regards their construction, as alternators. Fig. 232 shows a two-phase synchronous motor of the revolving field type. The small motor is used for starting purposes as will be explained later. The field of the motor is excited by a small direct-current exciter either direct-driven or belted to the motor. Synchronous motors operate best on fairly low frequencies, most of them being designed for frequencies ranging from 25 to 60 cycles per second. Single-phase synchronous motors are now seldom installed, though they were used to some extent before polyphase motors were introduced.

201. Starting synchronous motors.—If a polyphase synchronous motor is connected to the line, it will run up to synchronism, but in so doing it will take a very large current. When current

flows through the armature, it reacts on the residual magnetism set up in the pole pieces by the current in the preceding phase and produces sufficient torque to start the motor. Of course,

Fig. 232.

this imperfect motor action does not produce sufficient torque to enable the motor to start under load, and the method is objectionable on account of the large lagging current which the motor

takes from the line, and which is likely to create disturbances on other parts of the system. This method of starting is not, therefore, admissible unless the motor is a small one. If the synchronous motor has solid pole pieces, the eddy currents induced in them give rise to considerable starting torque owing to the machine acting as an induction motor—with laminated poles, the torque from this source would be small. A single-phase motor will not start up of its own accord at all, because the torque its exerted rapidly, first in one direction and then in the other, with the result that the armature remains at a standstill. Single-phase machines must be brought up to speed from some outside source.

On account of the large rush of current and large voltage drop caused by connecting the motor directly to the line, polyphase synchronous motors are sometimes started by using a compensator similar to that shown in Figs. 229 and 230. This cuts down the voltage applied to the motor at starting, and is cut out after speed is attained. Where large motors are employed, the method of starting that causes least disturbance is to use a small auxiliary induction-motor, either geared to or mounted on an extension of the main motor-shaft. The motor shown in Fig. 232 is started in this way. After the large motor has been brought up to speed, the small motor is disconnected by means of a clutch. A synchronous motor will run in either direction, depending upon the direction in which it is driven up to synchronism. If, however, it is started by means of alternating current in the armature, its direction of rotation is determined by means of the rotating field set up by the armature windings.

Synchronous motors have the disadvantage of being incapable of starting under load, and of requiring considerable auxiliary apparatus in the shape of an exciter and starting motor or starting compensator. On the other hand, they have some advantages. They are a good motor for work where a constant speed is desirable, because so long as the speed of the generator does not change, the speed of the motor will remain constant. The drop in the line, therefore, does not cause the speed to fall off.

Another valuable feature of the synchronous motor is that its power factor can be controlled by varying the field excitation so that, with care in adjusting the field strength, the power factor may be kept at or near unity. Moreover, if the field is over-excited, the motor acts like a condenser of large capacity and can be used to neutralize the lagging currents that may be set up by induction motors or other pieces of apparatus on the same system. The efficiencies of synchronous motors are the same as those alternators of corresponding size.

CHAPTER XX.

ROTARY CONVERTERS AND MOTOR GENERATORS.

202. General construction of rotary converters.—Two methods are in general use for changing alternating current to direct current, or *vice versa*. These are by means of rotary converters and motor-generators. The principle of the rotary converter has been given in Chapter XIII. In general appearance and construction rotary converters resemble direct-current dynamos very closely. Fig. 233 shows the general arrangement of the various parts of a rotary converter. *AA* is the armature spider carrying the laminated core *B* which carries the conductors *c* in slots on its periphery. These conductors are usually in the form of copper bars and project out at the end so that the end connections between the various bars and to the commutator may be readily made. The projecting bars are supported by means of the end flanges *fg*. *H* is the commutator from which or to which the direct current is supplied; *k* is one of the commutator bars to which the armature winding is attached. *L* shows the collector rings by means of which the alternating current is supplied to the machine or led from it in case the rotary is used to change direct current to alternating. These rings are connected to equidistant points of the winding as described in Chapter XIII., the number of tapping-in points depending upon the number of poles and the number of phases for which the rotary is intended. *P* is one of the pole pieces provided with a laminated pole-shoe *s* and magnetizing coil *M*. No pulley is necessary as these machines simply change the current and are not used as a source of power or driven mechanically.

Rotary converters are usually either shunt- or compound-wound, the latter being used mostly for street railway work. The shunt-wound rotary is, however, more stable in its action than the compound-wound machine and in many cases is to be preferred, especially where the load does not vary suddenly. Rotary converters may also be separately excited, but this is not customary because the machine can supply its own field with current

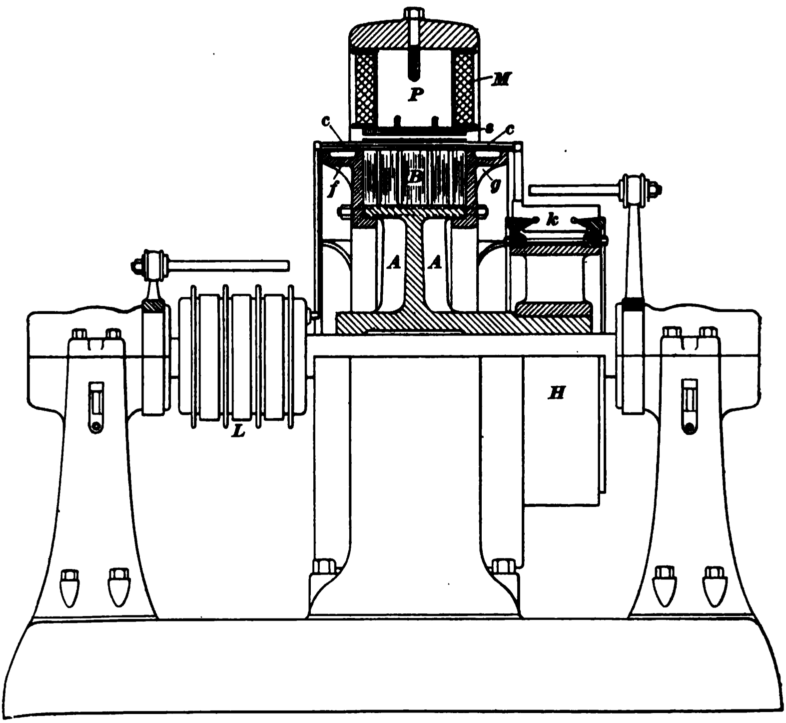


Fig. 233.

from its commutator end. In the great majority of cases rotary converters are used to change alternating current to direct current, though occasionally they are used to change direct current to alternating. When so used they are sometimes called *inverted rotaries*; a somewhat unnecessary term, because there is no essential difference in the machine, the only difference being in the manner in which it is used.

203. Starting of rotary converters.—The rotary converter, when supplied with current at the alternating-current end operates essentially as a synchronous motor; hence, what was said about the starting of synchronous motors applies in large measure to the starting of rotary converters. Since, however, a rotary converter combines the features of a direct-current machine it may also be started by supplying current to the direct-current side. A number of different methods are in use for starting rotary converters, of which the following are some of the more common ones.

(*a*) By connecting the alternating-current side directly to the source of current. The objections to this method are the same as those given in connection with the starting of synchronous motors. There is a rush of current which, on account of its large volume, gives rise to a large drop in the line and is objectionable in its effects on other parts of the system. This method of starting would only be permissible with small machines.

(*b*) By supplying current to the alternating-current side through the medium of a starting compensator. This is practically the same method as described in connection with induction and synchronous motors. The autotransformer supplies a large current at low potential to the machine and takes a comparatively small current at high potential from the line, thus cutting down the line current and causing comparatively little disturbance.

(*c*) By means of a small auxiliary induction motor mounted on or geared to the shaft of the rotary. This arrangement is largely used by the Westinghouse Company; it is a simple method of starting, and one that takes but a small current from the line. Fig. 234 shows a Westinghouse rotary converter with an induction motor for starting, the armature of the starting motor being mounted on an extension of the shaft.

(*d*) By supplying direct current to the commutator end of the machine and starting the rotary as a direct-current motor. This

method is the one generally used where direct current is available for starting purposes. In some cases the direct current is obtained from another rotary already in operation or from a storage battery which may be at hand. Sometimes a small motor-generator set, consisting of an induction motor coupled to a direct-current machine, is installed to supply current for starting purposes. Starting from the direct-current side is preferable

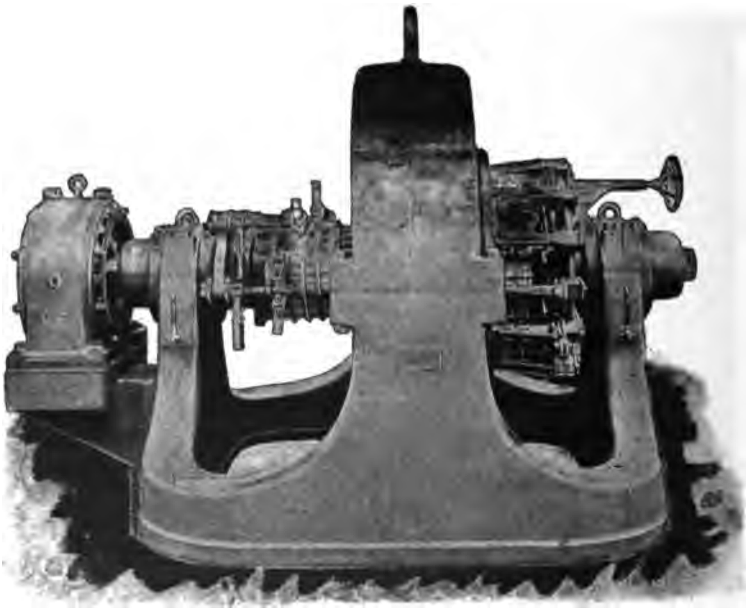


Fig. 234.

on account of the fact that when a rotary is started from the alternating-current side, by allowing alternating current to flow through the armature, and has to excite its own fields, the polarity of either of the direct-current terminals may be positive or negative; that is, one direct-current terminal may be positive when the machine is started at one time, and the next time it is started the same terminal may be negative. This does not occur if the

rotary is started from the direct current side or if its fields are separately excited. Also, when either a rotary converter or synchronous motor is started by allowing alternating currents to flow in the armature coils, the alternating magnetism set up through the field coils induces very high electromotive forces in them so that it has been found necessary to use special switches to disconnect the field coils from each other at starting. This prevents the induced electromotive forces from being added together and causing a breakdown of the field-spool insulation. After the machine has attained synchronism, this switch is closed and the coils supplied with exciting current from the direct-current side. When running a rotary converter as a direct-current motor at starting, care must be taken to see that the field circuit is intact, otherwise the armature may attain a dangerously high speed, because under this condition the machine no longer runs as a synchronous motor but as a direct-current shunt motor.

(*e*) By starting the generator and motor together. This method cannot often be applied in practice, but where it is possible, it makes a good method of starting, especially where large units are concerned. The machines are in this case always in phase, and the starting current required is comparatively small.

204. Transformation from direct current to alternating current.—The behavior of a rotary converter when used to change from direct current to alternating is considerably different than when it is used in the ordinary way. When supplied with direct current, it runs as a direct-current motor; hence, its speed will vary with changes in the applied voltage, and also with changes in the field strength. If the field becomes weakened, the rotary will speed up, and a break in the field-exciting circuit may result in damage, due to racing. Changing the field excitation will not change the voltage of the alternating current, because the ratio of transformation is fixed and changing the field strength merely makes the speed of the rotary change. The voltage of the alternating current may be changed by changing the voltage of the applied

direct current, or it may be increased or decreased by using alternating-current potential regulators, as described later.

Under certain conditions, a rotary fed from the direct-current side may be subject to wide variations in speed, unless special means are taken to prevent it. For example, if the alternating-current side delivers a large lagging current, the armature exerts a powerful demagnetizing action on the field, which may result in the speed running dangerously high. One method used by the Westinghouse Company to prevent this is as follows: The rotary is separately excited by a small direct-current machine directly coupled to the rotary in the same way as in the small induction motor in Fig. 234. This small shunt-wound machine is operated with an under-saturated magnetic circuit, so that any increase of speed will cause a rapid building up of the field and a rapid increase in voltage. If the speed of the rotary tends to increase, the voltage of the exciter at once increases and strengthens the field-magnet, thus checking the tendency to race.

205. Transformer connections for rotary converters.—Rotary converters are usually required to deliver current at comparatively low voltage, generally 600 volts or lower. The alternating current is, as a rule, transmitted at high voltage and moreover, as shown in Article 144, the ratio of transformation between the alternating current and direct current sides is fixed. For these reasons it is nearly always necessary to use transformers to step-down the line pressure. Fig. 235 shows two three-phase rotaries connected to their transformers. It will be noted that the transformers are Δ-connected. This method of connection is generally preferred to the Y scheme, because in case one of the transformers should burn out or its fuse blow, the service would not be entirely crippled. In this figure T is a small potential transformer used to step-down the pressure for the voltmeter V, and also to supply current for the synchronizing lamps l, l. By inserting a plug at p or p', the lamps will indicate when the electromotive force of either rotary is in synchronism with the

line electromotive force. V' and V'' are voltmeters to indicate when the rotary is up to voltage. In this figure, the transformer switches are connected in the high-tension side, though in some

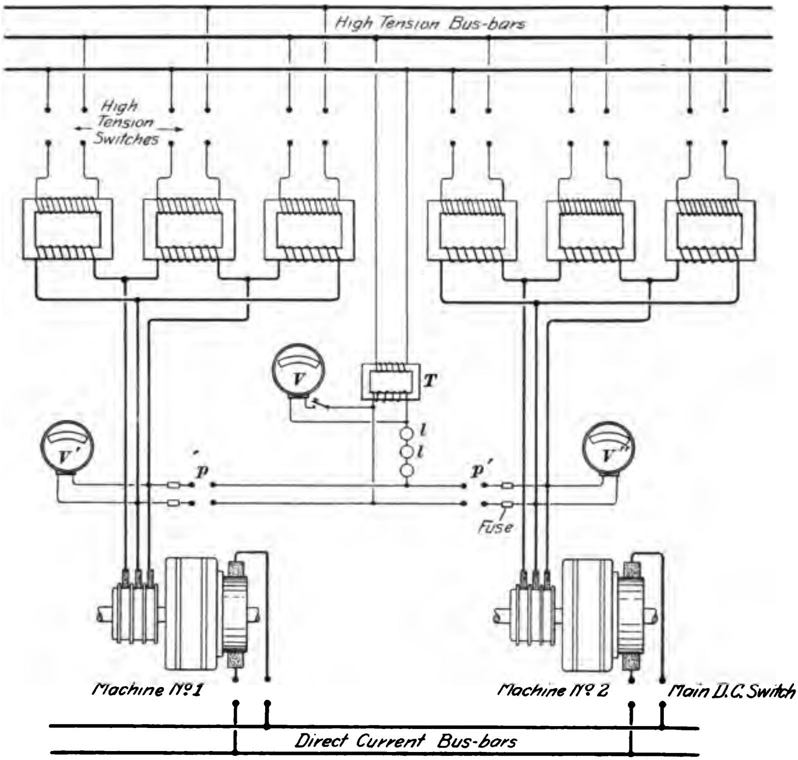


Fig. 235.

cases the switching is done on the low-tension side between the transformer secondaries and the rotary. Switching on the high-tension side avoids the use of switches carrying large currents.

206. Connections for six-phase converter.—It was shown in Article 143 that the output of a rotary is greater if supplied with six phases than if supplied with three only, because the armature heating is reduced. The use of six phases necessitates six collector rings on the rotary and makes the connections somewhat

more complicated. At the same time the gain in output is large enough (40 to 50 per cent. over that of a three-phase converter) to warrant the use of six-phase converters, especially where large units are concerned. Six-phase converters are extensively used in connection with the Metropolitan Street Railway System, New York.

Fig. 236 (a) shows transformer connections for supplying a six-phase rotary. The current is supplied from a three-phase high-tension line and transformed into six phases by using a double Δ system of connections on the secondaries of the transformers.

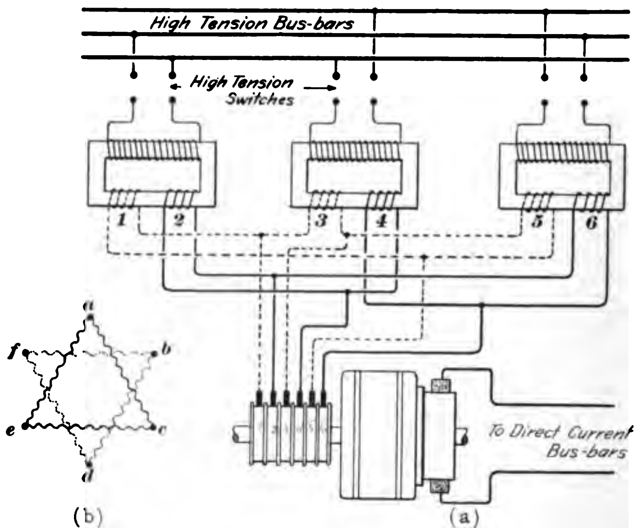


Fig. 236.

Each of the transformers is provided with two secondary coils. Coils 1, 3 and 5 are connected in the Δ or mesh arrangement, and coils 2, 4 and 6 are also connected in the same way, except that they are reversed with regard to the first set. This gives the arrangement indicated diagrammatically in Fig. 236 (b). The points *a*, *b*, *c*, *d*, *e*, *f* are connected to the collector rings of the rotary and from there connect to equidistant points of the armature winding.

207. Frequency of rotary converters.—Rotary converters have been most successful on frequencies ranging from 25 to 40 cycles. They have been built in comparatively large sizes for frequencies as high as 60 cycles, but some of these large 60-cycle machines have given trouble and have been eventually displaced by motor-generator sets. The fact that a rotary combines both the features of a direct current generator and an alternating current synchronous motor makes it difficult to construct large machines to operate on high frequencies. With a high frequency, the distance from center to center of the poles becomes so small that it is difficult to get enough commutator segments of sufficient thickness between brushes of opposite polarity without making the diameter of the commutator and armature very large, and thus running the peripheral speed dangerously high. For these reasons in transmission plants where the bulk of the power is supplied to rotary converters, it is customary to install low-frequency alternators. A frequency of 25 cycles is commonly used in America for this kind of work. Frequencies from 25 to 40 usually give satisfactory results.

208. Voltage regulation of rotary converters.—As shown in Article 144, the ratio of transformation of a rotary converter is practically a fixed quantity. Assuming that the machine is fed from the alternating-current side, changes in field strength will produce changes in the direct-current voltage within certain limits, if there is considerable reactance present on the alternating-current side of the rotary. Strengthening the field of the rotary makes the current lead the electromotive force, and it has already been shown that when a leading current is delivered over an inductive line, the voltage at the distant end of the line is increased, and the increased voltage thus supplied to the rotary causes an increase in the pressure of the direct-current side. Also, if the field of the rotary be weakened, the current will lag, and, consequently, the alternating-current voltage will fall off, thus decreasing the direct-current voltage. Changing the field-

strength, however, affects the power-factor of the rotary, and renders this method somewhat objectionable unless the required adjustment of voltage is not very large.

Where a comparatively wide range of voltage adjustment is needed, a number of methods are available. One that has been used considerably is to have the primary or secondary of the step-down transformer divided into a number of sections which can be cut in or out by means of a dial-switch, as shown on the transformer, Fig. 217. This allows the voltage applied to the alternating-current side of the rotary to be varied through a wide range, and causes a corresponding variation in the direct-current voltage.

Another method of securing a considerable range of regulation is to connect *potential regulators* in each line running between the rotary converters and the transformer secondaries. These potential regulators are made in a variety of forms, but they are usually provided with two windings, the primary of which is connected across the secondary of the static transformer, and the secondary of which is in series between the transformer and the rotary. Whatever voltage is generated in this series coil is, therefore, added to or subtracted from that of the main generator, depending upon the relation of the electromotive force generated in the series coil to that generated in the secondary of the main transformer. Fig. 237 shows a General Electric three-phase regulator. This regulator is designed for use with rotary converters, and is fitted for hand control. Some of the larger regulators are moved by means of a small auxiliary motor. The construction of this regulator is similar to that of an induction motor, the secondary of which is limited to small range of movement. The primary winding is placed in slots on the periphery of a stationary core in exactly the same way as the field or stator winding of an induction motor. The series winding is placed on a toothed core which can rotate through a limited range within the stationary core, and the current is led through the series winding by means of flexible leads. The phase relation of the secondary to that of

the primary (or the electromotive force of the main transformer) can be varied by moving the core, and, hence, the pressure added to or subtracted from that of the main transformer can easily be

Fig. 237.

regulated by turning the hand wheel shown in the figure. A number of other devices have been brought out for regulating the voltage of rotary converters, but the above will give an idea of some of the more common methods.

209. Motor generators.—For some kinds of work it has been found more advantageous to use motor-generator sets than rotary transformers for changing alternating current to direct. A motor generator set consists of an alternating-current motor coupled to one or more direct-current generators. One of these sets is shown in Fig. 238. The motor may be either of the synchronous or induction type. The latter is the less troublesome to start, but the former has the advantage that it maintains a constant speed irrespective of line drop, and, hence, has found favor

in some lighting stations. Motor-generator sets are generally more reliable on circuits operating at 60 cycles or higher than rotary transformers. They also admit of ready regulation of the direct-current voltage, because the direct-current machine is

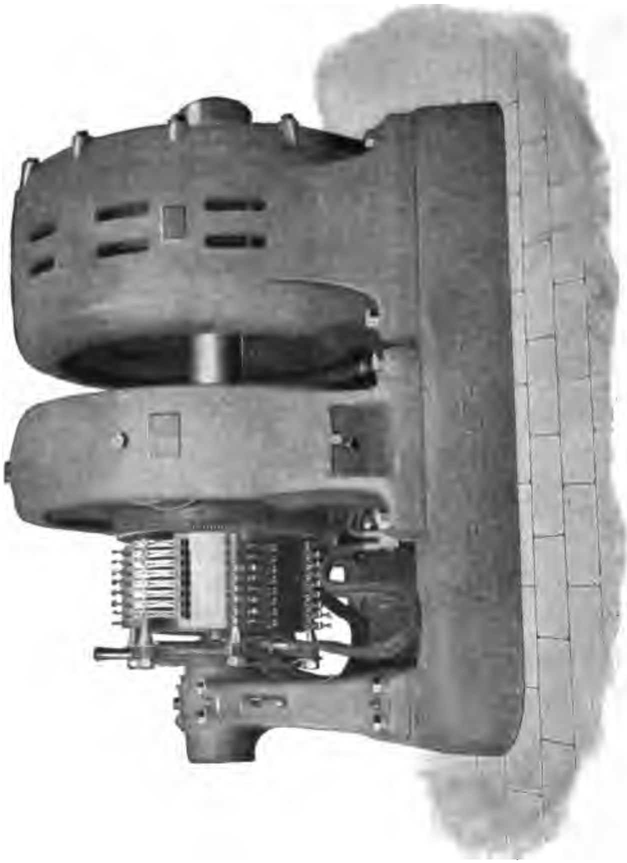


Fig. 238.

entirely separate from the alternating current side of the circuit and its voltage may be easily regulated by changing the shunt field excitation. Motor-generator sets are used quite extensively in connection with arc lighting where alternating-current motors

are used to drive arc-light dynamos. They are also used in some places for supplying three-wire direct-current systems from alternating current. They are somewhat more expensive than rotary converters, and are not as efficient. At the same time it must be remembered that, when they are used, the alternating-current motor can often be wound so as to take the high-pressure current direct from the line, thus doing away with the static transformers, and the elimination of the static transformer helps somewhat to even up the difference in cost and efficiency. Moreover, the motor-generator is not so liable to the peculiar effects met with in connection with rotary converters such as hunting and sparking.

THE END.

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